The Konishi operator from string sigma model

Romuald A. Janik

Jagellonian University
Krakow

Recent Developments in String/M Theory
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Outline

1. **Motivation**
2. **Integrability in AdS/CFT**
3. **The Konishi operator**
4. **Spin chains vs. worldsheet string sigma models**
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   - Multiparticle Lüscher corrections
5. **The Konishi computation**
   - Bound states
   - The S_{Q-1} S-matrix
   - The final computation
6. **Conclusions**

Motivation

$\boxed{\mathcal{N} = 4 \text{ Super Yang-Mills theory}}$	$\equiv$	$\boxed{\text{Superstrings on } AdS_5 \times S^5}$
strong coupling nonperturbative physics very difficult		(semi-)classical strings or supergravity 'easy'
weak coupling 'easy'		highly quantum regime very difficult

Goal: Interpolate from strong to weak coupling to reach perturbative results staying on the string theory side of the correspondence

Anomalous dimensions

- $\mathcal{N} = 4$ SYM is an exact CFT
- One can define dimensions Δ of operators through two-point functions

$$\langle O(x)O(y)\rangle = \frac{const}{|x-y|^{2\Delta}}$$

- Main example: the Konishi operator $O_{Konishi} = \operatorname{tr} \Phi_i^2$

$$\Delta = 2 + \underbrace{12g^2 - 48g^4 + 336g^6 + \ldots}_{\text{anomalous dimension}} \qquad g^2 \equiv \frac{\lambda}{16\pi^2}$$

- A lot is known about long operators

$$\operatorname{tr} \underbrace{\Phi_1\Phi_2\Psi DFD\Phi_2\Phi_3\Phi_3\Phi_1\Psi \ldots \Phi_5}_{\text{length} = \text{no. of operators}}$$

- For short operators, like the Konishi, very little is known $\rightarrow$ this talk.

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String theory side

- $\Delta \equiv$ energy of a string state in $AdS_5 \times S^5$
- Goal:

Find anomalous dimensions of all operators for all values of the coupling	$\equiv$	Find energy levels of quantized superstring in $AdS_5 \times S^5$

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- Fourier transform in position space $\longrightarrow$ associate a momentum p_i to each excitation.
- String side: excitations of string worldsheet with definite worldsheet momentum ($\equiv$ magnons)
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- Scattering encoded in $S(p_1, p_2)$

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Asymptotic Bethe Ansatz

- Solve Bethe equations

$$e^{ip_i L} = \prod_{k \neq i} S(p_i, p_k)$$

- Find the momenta $\{p_k\}_{k=1}^{M}$
- Obtain the dimension (energy) from

$$\Delta - J = E = \sum_{k=1}^{M} \sqrt{1 + 16 g^2 \sin^2 \frac{p_k}{2}}$$

Caution: The answer is not exact!
Asymptotic Bethe Ansatz incorporates all graphs of the type

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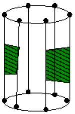

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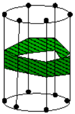

The Konishi operator

- It is convenient to consider $\operatorname{tr} Z^2 X^2 + \ldots$ in the $\mathrm{su}(2)$ sector. Has equal anomalous dimension to the original Konishi operator
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations
- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16 g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3} g^2 + \frac{9\sqrt{3}}{2} g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3} g^6 + \ldots$$

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$$E_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The result can be trusted only up to g^{2L} where L is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$)
- Results up to 3-loop order have been verified perturbatively
- The formidable perturbative 4-loop computation has been undertaken by several groups
- Results can be parametrized as

$$E = E_{Bethe} + \Delta_{wrapping} E$$

- Final result of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon] is

$$\Delta_{wrapping} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

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- Goal: How to compute the effects of wrapping interactions?

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Spin chains vs. worldsheet string sigma models

- Bethe ansatz quantization appears in both formulations:
  - For conventional spin chains like Heisenberg $XXX_{\frac{1}{2}}$ Bethe ansatz is exact!
  - For integrable 2D QFT it appears for large values of the charges ($\sim$ size of the worldsheet cylinder)
- The construction of the S-matrix based on $su(2|2) \times su(2|2)$ symmetry is completely paralel in both cases
  - determines the S-matrix up to a scalar factor
  - fixes the form of Bethe equations
- In order to fix the scalar factor ($\equiv$ dressing factor) one has to use a field-theoretic ingredient – crossing symmetry
- On the gauge theory (spin-chain) side it is extremely difficult to guess the way that deviations from Bethe ansatz ($\equiv$ wrapping interactions) may be computed
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Deviations from Bethe ansatz

- From the worldsheet QFT perspective, deviations from Bethe ansatz arise due to
  virtual particles circulating around the circumference of the worldsheet cylinder

- For large sizes of the cylinder these effects are supressed and Bethe ansatz works
- One can estimate the magnitude of these corrections from the Thermodynamic Bethe Ansatz perspective Ambjorn,RJ,Kristjansen

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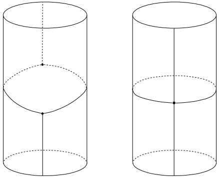

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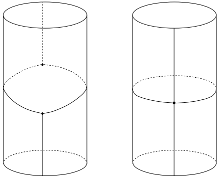

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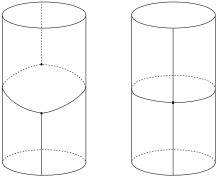

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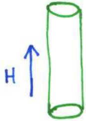

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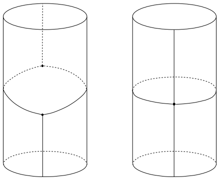

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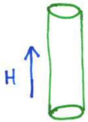

- The typical magnitude of these corrections is

$$magnitude \sim e^{-E_{TBA}(q)\cdot L}$$

where $E_{TBA} = ip$, $q = iE$ are the energy momenta in the space-time interchanged 'mirror' theory

- Start from the original dispersion relation

$$E = \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- Perform space-time interchange...

$$iq = \sqrt{1 - 16g^2 \sinh^2 \frac{E_{TBA}}{2}}$$

$$16g^2 \sinh^2 \frac{E_{TBA}}{2} = 1 + q^2$$

$$E_{TBA} = 2 \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}$$

- Finally we get for the magnitude of corrections

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$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow \frac{4^L g^{2L}}{(1+q^2)^L}$$

- At low orders in g Bethe ansatz results are uncorrected!
- For the Konishi operator one may expect a contribution at four loops ($L = 4$)
- We may expect to get exact answer for 4-loop Konishi from the leading correction
- Caveats:
  - Prefactor ...
  - What is L? In the light-cone quantized string sigma model it is more natural to expect $L_{string} = J$. This gives $L_{string} = J = 2$
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$$\Delta E = m \cosh \hat{\theta}_n - \underbrace{m \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \frac{\cosh(\theta - \hat{\theta}_n)}{\cosh \hat{\theta}_n} \left(S(\frac{i\pi}{2} + \theta - \hat{\theta}_n) - 1 \right) e^{-mL \cosh \theta}}_{F-term}$$

$$+ \underbrace{residues}_{\mu-term}$$

- This can be generalized to the AdS case (F-term given here) [RJ,Lukowski]

$$\Delta E = - \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left(1 - \frac{\varepsilon'(p)}{\varepsilon'(q_*)} \right) \cdot e^{-iq_* L} \cdot \sum_b (-1)^{F_b} \left(S_{ba}^{ba}(q_*, p) - 1 \right)$$

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- In order to find corrections to the Konishi operator we have to generalize these formulas to multiparticle states
- Start from sinh-Gordon (or SLYM) theory where the exact finite size spectrum is encoded in TBA integral equations [Dorey,Tateo;Teschner]

$$\epsilon(\theta) = mL\cosh\theta + \sum_{j=1}^{N} \log S(\theta - \theta_j - \frac{i\pi}{2}) - \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \phi(\theta - \theta') \log(1 + e^{-\epsilon(\theta')})$$

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- If L is large we may self-consistently neglect the integral and set

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- Consider the consistency condition

$$1 + e^{-\epsilon\left(\theta_j + \frac{i\pi}{2}\right)} = 0$$

- Substituting the leading piece of $\epsilon(\theta)$ gives

$$e^{im\sinh\theta_j L} \prod_{k\neq j} S(\theta_j - \theta_k) = 1$$

- The energy at this order follows just from the first term

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- Two distinct effects arise:
  1. The Bethe ansatz quantization gets modified

$$\hat{\theta}_k \longrightarrow \hat{\theta}_k + \delta\hat{\theta}_k$$

  2. There is an additional (direct) contribution to the energy

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Finite size corrections to AdS multimagnon states

- Consider a multimagnon state in a closed sector (e.g. `su(2)` or `sl(2)`) made up of magnons with label a
- Then the energy of an N-magnon state including the leading finite size correction is

$$E^j(L) = \sum_k \epsilon(p_k) - \sum_k \frac{d\epsilon(p_k)}{dp_k}\delta p_k + $$
$$- \int_{-\infty}^{\infty} \frac{d\tilde{p}}{2\pi} \sum_{a_1,\ldots,a_N} (-1)^F \left[S_{a_1 a}^{a_2 a}(\tilde{p}, p_1) S_{a_2 a}^{a_3 a}(\tilde{p}, p_2) \ldots S_{a_N a}^{a_1 a}(p, p_N) \right] e^{-\tilde{\epsilon}_{a_1}(\tilde{p})L}$$

where $\tilde{\epsilon}_{a_1}(\tilde{p})$ is the energy in the mirror model of the virtual particle

Apply this formula to the Konishi operator!

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where $\tilde{\epsilon}_{a_1}(\tilde{p})$ is the energy in the mirror model of the virtual particle

Apply this formula to the Konishi operator!

Finite size corrections to AdS multimagnon states

- Consider a multimagnon state in a closed sector (e.g. `su(2)` or `sl(2)`) made up of magnons with label a
- Then the energy of an N-magnon state including the leading finite size correction is

$$E^j(L) = \sum_k \epsilon(p_k) - \sum_k \frac{d\epsilon(p_k)}{dp_k}\delta p_k + \\ - \int_{-\infty}^{\infty} \frac{d\tilde{p}}{2\pi} \sum_{a_1,\ldots,a_N} (-1)^F \left[S_{a_1 a}^{a_2 a}(\tilde{p}, p_1) S_{a_2 a}^{a_3 a}(\tilde{p}, p_2) \ldots S_{a_N a}^{a_1 a}(p, p_N) \right] e^{-\tilde{\epsilon}_{a_1}(\tilde{p})L}$$

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Apply this formula to the Konishi operator!

- Start with the `su(2)` or `sl(2)` representative
- Set the worldsheet length $L_{string} = J \longrightarrow L_{string} = 2$
- Exponential factors appear as

$$\left(\frac{z^-}{z^+}\right)^2 \longrightarrow \frac{16g^4}{(Q^2+q^2)^2}$$

- Additional factor of g^4 have to arise from the prefactor
- At weak coupling we have to include the contributions of bound states ($Q > 1$ above). Two choices:
  1. `su(2)` bound states – symmetric representation – physical in the original theory
  2. `sl(2)` bound states – antisymmetric representation – physical in the mirror theory
- Novel feature w.r.t. relativistic integrable theories...
- Our conclusion is that `sl(2)` bound states should be used

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The Konishi computation

- At weak coupling it turns out that the effect of the modification of the Bethe quantization is subleading
- We have to compute

$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left(\frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

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  - summation over $Q \equiv$ summation over bound states in the antisymmetric representation
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- For su(2) (symmetric reps.) the scalar part is

$$S_{Q-1}^{su(2)}(z^\pm, x^\pm) = \frac{z^+ - x^-}{z^- - x^+} \frac{1 - \frac{1}{z^+ x^-}}{1 - \frac{1}{z^- x^+}} \cdot \frac{z^+ - x^+}{z^- - x^-} \frac{1 - \frac{1}{z^+ x^+}}{1 - \frac{1}{z^- x^-}}$$

  and does not depend on the choice of constituent magnons
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- We adopt the choice such that $z_Q^+ \sim g$, and all others $\sim 1/g$
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$$\left(\frac{z^-}{z^+} \right)^2 = \frac{16 g^4}{(Q^2 + q^2)^2} + \dots$$

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$$S_{Q-1}^{scalar, sl(2)} = \frac{3q^2 - 6iQq + 6iq - 3Q^2 + 6Q - 4}{3q^2 + 6iQq - 6iq - 3Q^2 + 6Q - 4} \cdot \frac{16}{9q^4 + 6(3Q(Q+2)+2)q^2 + (3Q(Q+2)+4)^2}$$

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- 'String frame' phase factors of AFZ promote the sigma model length J to the effective 'spin-chain' length. Total order of the correction is g^8 as expected from gauge theory

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- The result is

$$\sum_{Q=1}^{\infty}\left\{-\frac{num(Q)}{(9Q^4-3Q^2+1)^4(27Q^6-27Q^4+36Q^2+16)}+\frac{864}{Q^3}-\frac{1440}{Q^5}\right\}$$

where

$$num(Q) = 7776Q(19683Q^{18}-78732Q^{16}+150903Q^{14}-134865Q^{12}+ \\ +1458Q^{10}+48357Q^8-13311Q^6-1053Q^4+369Q^2-10)$$

- Two last terms give at once $864\,\zeta(3)-1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping}E=(324+864\zeta(3)-1440\zeta(5))g^8$$

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Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

is far from obvious. A similar computation using su(2) bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and Q bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT

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