M-Theory Branes as BPS Configurations in Bagger-Lambert Theory

Nakwoo Kim

Physics Department
Kyung Hee University, Seoul Korea

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Recent Developments in String/M Theory

Based on

- I. Jeon, J. Kim, N. Kim, S. Kim, J.H. Park *Classification of BPS states in Bagger-Lambert theory*, arXiv:0805.3236
- I. Jeon, J. Kim, N. Kim, B.H. Lee, J.H. Park *M-brane bound states and the supersymmetry of the BPS solutions in the Bagger-Lambert theory*, arXiv:0809.0856

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- Conjectured to exist as a certain dual of string theory.
- 11d, Maximal susy, M2-branes, M5-branes etc.
- Unlike string theory, we don't have the action for multiplet M2-branes (nor M5-branes)

BLG theory

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- Based on a new gauge symmetry : 3-algebra.
- Finally, do we have gauge theory dual of $AdS_4 \times S^7$?

Myers effect: D-branes

- Theory of multiple D-branes is Yang-Mills theory. For D0-branes,

$$L \sim \mathrm{Tr} \sum_{I,J}^{9} [X^I, X^J]^2$$

- When transverse NS-NS flux is present, we have another type of coupling, usually dubbed Myers term.

$$L \sim \mathrm{Tr} \sum_{I,J}^{9} [X^I, X^J]^2 + m^2 F_{IJK} X^I [X^J, X^K]$$

- Describes how D-branes arrange themselves under the effect of *transverse* external field.

Myers effect: fuzzy spheres

- As a consequence, the BPS equation becomes

$$\dot{X}^I + mX^I + iF_{IJK}[X^J, X^K] = 0$$

- Solved by representations of $SU(2)$ algebra: Fuzzy sphere, fuzzy funnel
- Now what will happen analogously in M-theory?

M-theory Myers effect

- Through T-duality, one can argue that M2-branes with external magnetic flux will be blown into a fuzzy 3-sphere (M5-brane)
- Basu and Harvey wrote down a generalized Nahm equation.

$$\dot{X}^i + \epsilon_{ijkl}[G_5, X^j, X^k, X^l] = 0$$

realized by 4d gamma matrices and their higher dimensional reps.

- Some kind of 3-product above!

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- Inspired by Basu-Harvey equation. Bagger and Lambert (2007) wrote down a classical gauge theory action for multiple M2-branes, based on 3-algebra structure.
- Classically superconformal. Bosonic symmetry $SO(3,2)$ from 3d conformal theory, $SO(8)$ from 8 scalars and 8 Majorana spinors. Sextic scalar potential etc.

3-algebra

- Vector space for gauge symmetry generators. $T^a, a = 1, \cdots, N$
- 3-algebra : Trilinear antisymmetric producs

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d$$

- Metric defined as $h^{ab} = \mathrm{Tr}(T^a, T^b)$.
- Total antisymmetry for f, and fundamental identity from

$$[T^a, T^b, [T^c, T^d, T^e]] = [[T^a, T^b, T^c], T^d, T^e] +$$

$$[T^c, [T^a, T^b, T^d], T^e] + [T^c, T^d, [T^a, T^b, T^e]]$$

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- Only 1 example for finite dimensional, positive-definite metric space. $SO(4)$
- Otherwise, either one should take the continuum limit, or allow negative-norm gauge generators!

Gauge symmetry

- Scalar fields in fundamental rep : X_d

$$\delta X_a = \Lambda_{ab} f^{abc}{}_d X_c$$

- Covariant derivative :

$$D_\mu X_d = \partial_\mu X_d - \tilde{A}_\mu{}^c{}_d X_c, \quad \delta \tilde{A}_\mu{}^c{}_d = D_\mu \tilde{\Lambda}^c{}_d$$

Action

BL theory lagrangian :

$$\begin{aligned} L &= -\frac{1}{2}D^\mu X^{aI} D_\mu X^I_a + \frac{i}{2}\bar{\Psi}_a \Gamma_\mu D_\mu \Psi_a \\ &+ \frac{i}{4}\bar{\Psi}_b \Gamma_{IJ} X^I_c X^J_d \Psi_a f^{abcd} - V(X) \\ &+ \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd} A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef}\right) \end{aligned}$$

Scalar potential :

$$V = \frac{1}{12}\mathrm{Tr}([X^I, X^J, X^K], [X^I, X^J, X^K])$$

Properties of BL theory

- 3d gauge theory with CS term for gauge field.
- Gauge group $SU(2) \times SU(2)$, matter in bifundamental rep.
- Gauge coupling is quantized due to the usual CS level quantization.
- No continuously tunable coupling constant.

Infinite dimensional 3-algebra

- Can be realized as Nambu-Poisson bracket of metric 3-space Σ. For $f, g, h : \Sigma \to \mathbb{R}$,

$$\{f, g, h\} = \frac{\epsilon_{ijk}}{\sqrt{g}} \partial_i f \partial_j g \partial_k h$$

- Effectively $X(x^\mu) \to X(x^\mu; \sigma^i)$. M5 instead of M2?

BPS Solitons of BLG theory

- If BLG theory gives a faithful description of M2-branes, various M-brane configurations (intersecting with M2 in general) should appear as BPS solitons.
- Also appear as central extension of the supersymmetry algebra.

$$\begin{aligned}\{Q^\alpha, Q^\beta\} &= -2P_\mu(\Gamma^\mu\Gamma^0)^{\alpha\beta} + Z_{IJ}(\Gamma^{IJ}\Gamma^0)^{\alpha\beta} \\ &\quad + Z_{iIJKL}(\Gamma^{IJKL}\Gamma^i\Gamma^0)^{\alpha\beta} + Z_{IJKL}(\Gamma^{IJKL})^{\alpha\beta}\end{aligned}$$

Supercharges

- vortex, domain wall, spacetime-filling :

$$
\begin{aligned}
Z_{IJ} &= -\int d^2x \mathsf{Tr}(D_i X^I D_j X^J \varepsilon^{ij} - D_0 X^K F^{KIJ}) \\
Z_{iIJKL} &= \frac{1}{3}\int d^2x \mathsf{Tr}(D_j X^{[I} F^{JKL]} \varepsilon^{ij}) \\
Z_{IJKL} &= \frac{1}{4}\int d^2x \mathsf{Tr}(F^{M[IJ} F_M{}^{KL]})
\end{aligned}
$$

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- BPS solutions are supersymmetric intersections (or composites) of such M-branes.

1/2-BPS equations : vortex type

- Type I : with gamma matrix projector : $P = 1 \pm \Gamma^{xy}\Gamma^{12}$
- BPS equations

$$D_x X^1 \pm D_y X^2 = 0, \quad D_y X^2 \pm D_x X^1 = 0, \quad D_t X^I - F^{I12} = 0.$$

- Simple solution is given as holomorphic curves :

$$W = X^1 + iX^2, z = x + iy, \quad W = W(z)$$

- Vortex solution from intersecting M2's. But in general when $D_t X^I, F^{I12}$ are nonzero, we also have MW and M5.

MW-M2-M2-M5 system

Suppose we have $D_i X^1, D_i X^2, D_t X^3, [X^1, X^2, X^3]$ non-vanishing. Then the interpretation is:

M2:	t	x	y	-	-	-	-	-	-	-	-
M2:	t	-	-	1	2	-	-	-	-	-	-
MW:	t	-	-	-	-	3	-	-	-	-	-
M5:	t	x	y	1	2	3	-	-	-	-	-

1/2-BPS equations : fuzzy funnel type

- Type II : $P = 1 \pm \Gamma^y \Gamma^{1234}$
- BPS eq : $D_y X^1 \pm [X^2, X^3, X^4] = 0$ plus cyclic.
- Solution given as planar M5, intersecting with M2 over a string. (Or fuzzy funnel) : $X \sim 1/\sqrt{c-y}$.

More general BPS equation of domain wall type:

$$(D_t - D_x)X^I = 0, \quad D_\mu X^I - \frac{1}{6}\mathcal{C}^{IJKL}F_{JKL} = 0$$

$$\mathcal{C}_{1234} = \mathcal{C}_{5678} = 1$$

MW:	t	x	-	-	-	-	-	-	-	-	-
M2:	t	x	y	-	-	-	-	-	-	-	-
M5:	t	x	-	1	2	3	4	-	-	-	-
M5:	t	x	-	-	-	-	-	5	6	7	8

1/2-BPS equations: Planar M5

- Type III : $P = 1 \pm \Gamma^{1234}$
- Realized with infinite dimensional 3-algebra only.
- BPS eq :

$$
\begin{aligned}
[X^1, X^2, X^5] \pm [X^3, X^4, X^5] &= 0 \\
[X^1, X^3, X^5] \pm [X^4, X^2, X^5] &= 0 \\
[X^1, X^4, X^5] \pm [X^2, X^3, X^5] &= 0
\end{aligned}
$$

- Interpretation : M5 instanton, containing M2 and intersecting with another M5. Eventually brings about KKM.

Composite: M2-M5-M5-KKM

Nonvanishing 3-products are related to M5. Now sharing M2's, the intersecting M5's give rise to KKM.

M2:	t	x	y	-	-	-	-	-	-	-	-
M5:	t	x	y	1	2	-	-	5	-	-	-
M5:	t	x	y	-	-	3	4	5	-	-	-
KK:	t	x	y	1	2	3	4	-	-	-	-

Generic BPS equations

- Less supersymmetric solutions can be interpreted as more complicated web of intersecting branes.
- Least susy is 1/16.
- Can work out nontrivial BPS conditions for 1-12 times 1/16.
- BPS equations governed by the relevant symmetry-invariant tensors.
- For instance, one 1/8 class : $\Psi_{IJKL}[X^J, X^K, X^L] = 0$, where Ψ is the Cayley tensor of $SO(7)$ structure in 8d.

Liouville and Sinh-Gordon from Vortex Ansatz

- Vortex solution with electromagnetic charge : For BL theory with SO(4) gauge group, we set

$$X^1 - iX^2 = (f_1, f_2, 0, 0), \quad X^3 = (0, 0, v, 0)$$

$$\tilde{A}_{i12} \neq 0, \quad \tilde{A}_{t34} \neq 0$$

- SUSY condition plus Gauss law guarantee EOM is satisfied.

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- SUSY condition plus Gauss law guarantee EOM is satisfied.
- Turns out v is constant, and $f_1^2 + f_2^2 = \alpha(z,t)$: for simplicity consider constant α.

Liouville eq

- When $\alpha = 0$, the Gauss law leads to $(Y = |f_1|/v)$

$$\partial_z \partial_{\bar{z}} Y - v^4 e^{2Y} = 0$$

- One can express the most general solution in terms of holomorphic function :

$$e^Y = \frac{1}{v^2} \left| \frac{\partial_z h}{1 - h\bar{h}} \right|$$

- Singular where $h\bar{h} = 1$.

Sinh-Gordon eq

- In general if $\alpha \neq 0$ we have ($\Phi = 2\mathrm{Im}\Upsilon, f_2/f_1 = \tan\Upsilon$)

$$2\partial_z\partial_{\bar{z}}\Phi - \alpha v^2 \sinh(2\Phi) = 0$$

- This is integrable, and one can use Bäcklund transform to construct nontrivial solutions from a known solution.

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$$\cosh\Psi = \coth(\beta z + \frac{\alpha v^2}{\beta}\bar{z} + z_0)$$