Fermi liquid near Pomeranchuk quantum criticality

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Result:

Consider a 2D system of itinerant fermions at $T=0$, near a ferromagnetic instability.

We found another pre-emptive instability of a Fermi liquid: towards a $p$-wave spin nematic (a magnetic analog of $^3$He) In a wide parameter range, this instability comes first.
Issue: what is Femi liquid theory near a Pomeranchuk instability in a charge or a spin channel

Introduction: Fermi liquid theory, Pomeranchuk instabilities

Critical Fermi liquid near a Pomeranchuk QCP:

- effective interaction,
- Landau function
- multi-channel criticality

Specifics of the spin case:

- the consequence of SU(2) spin invariance
- a new instability of a near-ferromagnetic Fermi liquid
**Introduction:** Fermi liquid theory, Pomeranchuk instabilities

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Landau Fermi liquid theory uses conservation laws and Galilean invariance to capture effects of interactions, which come from the immediate vicinity of the Fermi surface.

**The input for the theory is a vertex**

\[
\Gamma^\Omega_{\alpha\beta,\delta} (\vec{k}_F - \vec{p}_F) = \Gamma^\Omega_{\alpha\beta,\delta} (\theta)
\]

Includes everything except for soft bubbles with vanishing energy/momentum transfer. These come from the Fermi surface.

\[
\int G(P) G(P + Q) \frac{d^3P}{(2\pi)^3} = \frac{m}{2\pi} \left( 1 - \frac{\Omega}{\sqrt{\Omega^2 + (v_F q)^2}} \right) = 0 \text{ at } \frac{v_F q}{\Omega} \to 0
\]

\[Q = (q, \Omega)\]
All effects due to interactions near the Fermi surface can be captured by applying conservation laws (conservation of the number of particles, SU(2) spin conservation, gauge invariance, and Galilean invariance).

No need for further diagrammatics once $\Gamma^\Omega$ is known.

How it works:

$$\Gamma^\Omega_{\alpha\beta,\gamma\delta}(\theta) = A \, g_{\alpha\beta,\gamma\delta}(\theta)$$

Landau function

$$g_{\alpha\beta,\gamma\delta}(\theta) = g_{c}(\theta) \, \delta_{\alpha\gamma} \, \delta_{\beta\delta} + g_{s}(\theta) \, \tilde{\sigma}_{\alpha\gamma} \, \tilde{\sigma}_{\beta\delta}$$

in 2D

$$g_{c}(\theta) = \sum_n g_{c,n} \cos(n \theta)$$

$$g_{s}(\theta) = \sum_n g_{s,n} \cos(n \theta)$$

C(T) = $\gamma$ T, $\chi_s = \text{const}; \quad \frac{\gamma}{\gamma_0} = \frac{m^*}{m} = 1 + g_{c,1}, \quad \frac{\chi_s}{\chi_0} = \frac{1 + g_{c,1}}{1 + g_{s,0}}$
It is actually a bit more involved....

**Landau function**

\[ g_{\alpha\beta,\gamma\delta}(\theta) = \frac{m^*}{\pi} Z^2 \Gamma^\Omega_{\alpha\beta,\gamma\delta}(\theta) \]

\[ \frac{m^*}{m} = 1 + g_{c,1} \]

\[ Z^{-1} = 1 - i \sum_\beta \Gamma^\Omega_{\alpha\beta,\alpha\beta}(K_F, P) G(P) G(P + Q) \frac{d^3P}{(2\pi)^3}, \quad Q = (q, \Omega), \quad P = (p, \omega), \]

\[ G(P) = \frac{Z}{i \omega - v_F} \frac{m}{m^*} \frac{m}{(p - p_F)} \]

*Pitaevskii & Landau*
How it works:

constant interaction, weak coupling

\[ \Gamma^\Omega_{\alpha\beta,\gamma\delta} (\theta) = \frac{U}{2} \delta_{\alpha\gamma} \delta_{\beta\delta} - \frac{U}{2} \tilde{\sigma}_{\alpha\gamma} \tilde{\sigma}_{\beta\delta} \]

Use Fermi liquid theory:

\[ g_{c,0} = \frac{mU}{\pi}, \quad g_{s,0} = -\frac{mU}{\pi}, \quad \chi_{s,0} = \frac{m}{\pi} \frac{1}{1 + g_{s,0}} = \frac{m}{\pi} \frac{1}{1 - \frac{mU}{\pi}} \]

Or use diagrammatics:

\[ \chi_s = \sum \text{bubbles, } \Pi(\Omega = 0, q \to 0) \]

the same
This can be done for any spin or charge channel:

\[ \chi_{s,0} = \frac{m}{\pi} \frac{1}{1 - \frac{m U}{\pi}} \]

\[ 1 - \frac{m U}{\pi} = 0 \]

**Stoner criterion of a ferromagnetic instability**

\[ \chi_{a,n} = \frac{m}{\pi} \frac{1}{1 + g_{a,n}} \]

\[ \chi_{a,n} > 0 \Rightarrow 1 + g_{a,n} > 0 \]

**Pomeranchuk condition**

*A Fermi liquid instability at* \( g_{a,n} = -1 \) *is called a Pomeranchuk instability*

**Examples:** phase separation \( (g_{c,0} = -1) \), a ferromagnetic transition \( (g_{s,0} = -1) \), charge quadrupolar transition \( (g_{c,2} = -1) \), spin dipolar \( (g_{s,1} = -1) \) and quadrupolar \( (g_{s,2} = -1) \) transitions

According to a textbook reasoning, only one, critical channel is relevant near a QCP: one Landau component approaches -1, others are irrelevant.

Indeed, if

$$
\Gamma^\Omega_{\alpha\beta,\gamma\delta}(\theta) = \frac{U}{2} \delta_{\alpha\gamma} \delta_{\beta\delta} - \frac{U}{2} \vec{\sigma}_{\alpha\gamma} \vec{\sigma}_{\beta\delta}
$$

we have

$$
g_{c,0} = \frac{m U}{\pi}, \quad g_{s,0} = -\frac{m U}{\pi}, \quad g_{a,n>0} = 0
$$

and

$$
\chi_{s,0} = \frac{m}{\pi} \frac{1}{1 + g_{s,0}} = \frac{m}{\pi} \frac{1}{1 - \frac{m U}{\pi}}
$$

$$
\chi_{c,0} = \frac{m}{\pi} \frac{1}{1 + g_{c,0}} = \frac{m}{\pi} \frac{1}{1 + \frac{m U}{\pi}}
$$

$$
\frac{m^*}{m} = 1 + g_{c,1} = 1, \quad \chi_{a,n>0} = \frac{m}{\pi} \frac{1 + g_{c,1}}{1 + g_{a,n}} = \frac{m}{\pi}
$$
Effective mass diverges

\[ \Sigma(\omega) \sim \frac{\omega}{(1 + g_{s,0})^{1/2}}, \quad \frac{m^*}{m} \sim \frac{1}{(1 + g_{s,0})^{1/2}} \Rightarrow \infty \]

\[ \Sigma(\omega) \sim \omega^{2/3} \text{ at a QCP} \]

\[ m^*/m = 1 + g_{c,1} \]

At the same time,

\[ \chi = \chi_{s,0} = \frac{m}{\pi} \frac{1}{1 + g_{s,0}} \]

no extra renormalization

So, the question is, what is the Landau function near a 2D nematic QCP

OK in dimension D >3, but not in D=2

What if we just do diagrammatics (e.g., large N)
Critical Fermi liquid near a Pomeranchuk QCP:

effective interaction,
Landau function
multi-channel criticality

Specifics of the spin case:

the consequence of SU(2) spin invariance
a new instability of a near-ferromagnetic Fermi liquid

Here the story begins....
For definiteness, consider a charge quadrupolar (d-wave) instability $g_{c,2} \rightarrow -1$

**“Zero” order theory --same as before:**

$$g^{(0)}_{\alpha \beta, \gamma \delta}(\theta) = g_{c,2} \cos 2 \theta \delta_{\alpha \gamma} \delta_{\beta \delta} + g_{s,2} \cos 2 \theta \vec{\sigma}_{\alpha \gamma} \vec{\sigma}_{\beta \delta}$$

Next

$$\Gamma^\Omega = \frac{m^*}{m} = 1, \chi_{c,2}^{(0)} \propto \frac{1}{(1 + g_{c,2})}$$

$$= U (k - p) d_k d_p$$

$$d_k = \sqrt{2} \cos (2 \varphi_k)$$

Metzner, Yamada, Kee, Kim, Oganesyan, Kivelson, Fradkin, Garst, Woelfle, Rosch, del’Anna
Interaction mediated by soft collective charge nematic excitations

Berk & Schrieffer
The form of $\tilde{g}_{c,n}$, $\tilde{g}_{s,n}$ depends on a single parameter

$$\lambda = \frac{1}{2 (a k_F) \sqrt{1 + g_{c,2}}}$$

$\lambda << 1$ weak coupling, $\tilde{g}_{c,n}$, $\tilde{g}_{c,n} \approx 1$

$\lambda << 1$ strong coupling, $\tilde{g}_{c,n}$, $\tilde{g}_{c,n} \approx \lambda >> 1$

**Critical Fermi liquid**
In a critical Fermi liquid (when $\lambda \gg 1$)

\[ \lambda = \frac{1}{2ak_F \sqrt{1 + g_{c,2}}} \]

The textbook assumption that only one Landau component is singular near a Pomeranchuk transition is incorrect in 2D (and 3D) for $a = c, s$, and ALL $n < \lambda (ak_F)^2$
Consequences of \( \bar{g}_{a,n} = \lambda \),

“Zero” order theory for a critical Fermi liquid:

\[
\frac{m^*}{m} = 1, \quad \chi_{c,2}^{(0)} \propto \frac{1}{(1 + g_{c,2})}
\]

Interaction with soft collective modes, acts on top of zero order theory and gives:

\[
\frac{m^*}{m} \sim 1 + \bar{g}_{c,1} = 1 + \lambda \propto \frac{1}{(1 + g_{c,2})^{1/2}}
\]

\[
\chi_{c,2} = \chi_{c,2}^{(0)} \frac{1 + \bar{g}_{c,1}}{1 + \bar{g}_{c,2}} \approx \chi_{c,2}^{(0)} \frac{1 + \lambda}{1 + \lambda} = \chi_{c,2}^{(0)} = \frac{m}{\pi} \frac{1}{1 + g_{c,2}}
\]

This is fully consistent with diagrammatics
We can also introduce the full Landau function, which acts on top of free fermion theory, i.e., include “zero order” renormalization into Fermi liquid

\[ \chi^{(0)}_{c,2} = \frac{m}{\pi}, \quad \chi_{c,2} = \frac{m}{\pi} \frac{1 + g^*_{c,1}}{1 + g^*_{c,2}} \]

\[ g_{a,n}^* = g_{a,n} + \bar{g}_{a,n} (1 + g_{a,n}) \quad g_{a,n} = g_{c,2} \delta_{a,c} \delta_{n,2} \]

\[ g_{a,n}^* = \bar{g}_{a,n} \quad \text{for all components except for } n=2 \text{ charge component} \]

\[ \chi_{c,2} = \frac{m}{\pi} \frac{1}{1 + g_{c,2}} = \frac{m}{\pi} \frac{1 + g^*_{c,1}}{1 + g^*_{c,2}} = \frac{m}{\pi} \frac{(1 + g_{c,2})^{-1/2}}{(1 + g_{c,2})^{1/2}} \]

Divergence of \( \chi \) is split between \( m^*/m \) and nematic “g-factor”
As usual, full story is a bit more messy:

\[ \Gamma^{\Omega}_{\alpha\beta;\gamma\delta} = \frac{1}{Z_{\Gamma}} \Gamma^{\Omega,\text{RPA}}_{\alpha\beta;\gamma\delta} \]

We neglected soft bubbles, as textbooks told us to do.

This is fine as long as effective interaction is static, however our \( \Gamma^{\Omega,\text{RPA}}_{\alpha\beta;\gamma\delta} \) is dynamic
This yields, after summation of ladder series

$$\Gamma_{\alpha \beta; \gamma \delta} = \frac{1}{Z_{\Gamma}} \Gamma_{\alpha \beta; \gamma \delta}^{\text{RPA}}$$

$$\frac{1}{Z_{\Gamma}} = \frac{2 \alpha \beta \gamma \delta \Gamma_{\alpha \beta; \gamma \delta}^{\text{RPA}}}{\sqrt{1 - \frac{Z^2 m^*}{m} \lambda} \left(1 + \sqrt{1 - \frac{Z^2 m^*}{m} \lambda}\right)}$$

$$\lambda = \frac{1}{2 a k_F \sqrt{1 + g_{c,2}}}$$

Use

$$g_{\alpha \beta, \gamma \delta} = 2 \nu \frac{m^*}{m} Z^2 \Gamma_{\alpha \beta, \gamma \delta}^{\Omega}$$

$$\bar{g}_{\alpha \beta; \gamma \delta}(\theta) = 2 \nu Z^2 \frac{m^*}{m} \Gamma_{\alpha \beta; \gamma \delta}^{\Omega}$$

$$\frac{m^*}{m} = 1 + \frac{\lambda}{4 Z_{\Gamma}} Z^2 \frac{m^*}{m}$$

$$Z^{-1} = 1 - i \sum_\beta \Gamma_{\alpha \beta, \alpha \beta}^{\Omega} (K_F, P) G(P) G(P + Q) \frac{d^3 P}{(2\pi)^3}$$

$$\frac{1}{Z} = 1 + \frac{\lambda Z^2 m^*}{4 Z_{\Gamma} m}$$
Three coupled equations

\[
\frac{1}{Z_\Gamma} = \frac{2}{\sqrt{1 - \frac{Z^2 m^*}{m}} \lambda \left(1 + \sqrt{1 - \frac{Z^2 m^*}{m}} \lambda\right)}
\]

\[
\frac{1}{Z} = 1 + \frac{\lambda}{4} \frac{Z^2}{Z_\Gamma} \frac{m^*}{m}
\]

\[
\frac{m^*}{m} = 1 + \frac{\lambda}{4} \frac{Z^2}{Z_\Gamma} m
\]

with \( \lambda = \frac{1}{2ak_F \sqrt{1 + g_{c,2}}} \) as a parameter

Weak coupling \( \lambda \ll 1 \)

\[
\left. \frac{m^*}{m} \right|_{\lambda \ll 1} = 1 + \lambda/4; \quad \left. Z \right|_{\lambda \ll 1} = 1 - \lambda/4; \quad \left. Z_\Gamma \right|_{\lambda \ll 1} = 1 - 3\lambda/4
\]

Strong coupling

\[
\left. \frac{m^*}{m} \right|_{\lambda \gg 1} = \lambda + 1/4\lambda; \quad ; \quad \left. Z \right|_{\lambda \gg 1} = 1/\lambda - 1/4\lambda^3
\]

\[
\left. Z_\Gamma \right|_{\lambda \gg 1} = 1/4\lambda + 1/4\lambda^2
\]

\[
\bar{g}_{\alpha\beta;\gamma\delta}(\theta) \approx \frac{1}{1 + g_{c,2} + (ak_F \theta)^2} (\delta_{\alpha\gamma} \delta_{\beta\delta} + \tilde{\sigma}_{\alpha\gamma} \cdot \tilde{\sigma}_{\beta\delta})
\]

\[
\bar{g}_{\alpha,n} = \lambda
\]
Critical Fermi liquid near a Pomeranchuk QCP:

effective interaction,
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multi-channel criticality

Specifics of the spin case:

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Apply the same reasoning to a ferromagnetic transition

\[
\Gamma_{\alpha\beta;\gamma\delta}(k, \omega_k; p, \omega_p) = \frac{1}{2\nu Z} \frac{1}{\delta + (aq)^2 + \frac{|\Omega|}{v_F q}} \vec{\sigma}_{\alpha\delta} \cdot \vec{\sigma}_{\beta\gamma}
\]

\[\delta = 1 + g_{s,0}\]

\[q = k - p, \quad \Omega = \omega_k - \omega_p\]

\[\nu = m/2\pi\]

This is Hertz-Millis theory

\[
\bar{g}_{\alpha\beta;\gamma\delta}(k, p) = Z^2 (m^*/m) \Gamma_{\alpha\beta;\gamma\delta}(k, 0; p, 0)
\]

\[
\bar{g}_{\alpha\beta;\gamma\delta} = \bar{g}_c \delta_{\alpha\gamma} \delta_{\beta\delta} + \bar{g}_s \vec{\sigma}_{\alpha\gamma} \cdot \vec{\sigma}_{\beta\delta}
\]

\[
m^*/m = 1 + \bar{g}_{c,1}\]

\[
\chi_s = \chi_s^0 / (1 + \bar{g}_{c,1})
\]

\[
\frac{m^*}{m} = \lambda
\]

\[
\lambda \equiv \frac{3}{4ak_F\sqrt{\delta}}
\]

\[
\chi_s = \chi_s^0 \frac{1 + \lambda}{1 - \frac{\lambda}{3}}
\]

Divergence before a QCP?

Diagrammatics:

\[
\frac{m^*}{m} \approx \frac{1}{Z} = \lambda\]

\[
\chi_s = \chi_s^0 \propto \frac{1}{\delta}
\]
Fermi liquid theory is based on conservation laws. Let’s verify if we are using a conserving approach in the spin case.

Check Ward identities:

the total number of particles and SU(2) spin are conserved, hence

\[ \chi_{c,s}(q = 0, \Omega) = 0 \]

\[ \Lambda_{c,s} = 1 \]

for free fermions

\[ \Lambda_{c,s} = \]

Maki-Thompson diagrams
Charge susceptibility:

Series for $\Lambda_c$, are geometric, the $n^{\text{th}}$ term is

$$\Lambda_c = 1 + \frac{\lambda}{1+\lambda} + \left( \frac{\lambda}{1+\lambda} \right)^2 + \left( \frac{\lambda}{1+\lambda} \right)^3 + \ldots = \frac{1}{1 - \frac{\lambda}{1+\lambda}} = 1 + \lambda$$

$$\chi_{c,s}(q = 0, \Omega) = \chi^0_{c,s} \left( 1 - \frac{\Lambda_c}{1+\lambda} \right)$$

$$\chi_{c}(q = 0, \Omega) = 0 \quad \text{as it indeed should}$$
Spin susceptibility:

Series for $\Lambda_c$ are geometric, the $n^{th}$ term is $(-1/3)^n(\lambda/(1+\lambda))^n$

\[
\Lambda_c = 1 - \left(\frac{1}{3}\right) \frac{\lambda}{1+\lambda} + \left(\frac{1}{9}\right) \left(\frac{\lambda}{1+\lambda}\right)^2 - \left(\frac{1}{27}\right) \left(\frac{\lambda}{1+\lambda}\right)^3 + \ldots
\]

\[
= \frac{1}{1 + \left(\frac{1}{3}\right) \frac{\lambda}{1+\lambda}} = \frac{3(1+\lambda)}{3+4\lambda}
\]

\[
\chi_{c,s}(q = 0, \Omega) = \chi_{c,s}^0 \left(1 - \frac{\Lambda_{c,s}}{1+\lambda}\right)
\]

\[
\chi_s(q = 0, \omega) \neq 0
\]
Physics:

We have a model with spin-spin interaction between itinerant fermions, mediated by collective spin excitations.

An electron spin is split into a spin of an itinerant fermion \( s = c_\alpha \sigma_\alpha c_\beta \) and a spin of a collective boson \( S \).

For SU(2) invariant case, the interaction is \( s \cdot S \), which can flip \( s^\parallel \). As a result, fermionic \( s \) is not conserved separately from \( S \).

N.B. For Ising spins, Ward identities are satisfied.
How to cure?

Need backflow terms which flip $s^z$ back

Candidates: Aslamazov-Larkin processes

Charge case: the two diagrams cancel each other.  
Spin case: the two add up
\[ I_{\text{MT}} = \frac{-1}{3} \left( \frac{\lambda}{1 + \lambda} \right) \]

\[ I_{\text{AL}} = \frac{1}{2 \nu Z^2} \int \frac{dq}{2\pi} \int \frac{d\Omega}{2\pi} \int \frac{d\theta_{pq}}{2\pi} \int \frac{d\theta_{kq}}{2\pi} \int \frac{d\omega}{2\pi} \int d\epsilon_p G(p, \omega) G(p, \omega + \Omega_0) G(k_F + q, \Omega + \Omega_0) \]

\[ \times \left[ G(p + q, \omega + \Omega + \Omega_0) - G(p - q, \omega - \Omega) \right] \tilde{\Gamma}(q, \Omega) \tilde{\Gamma}(q, \Omega + \Omega_0) \]

\[ I_{\text{AL}} = \frac{4}{3} \lambda / (1 + \lambda) \]

\[ I_{\text{MT}} + I_{\text{AL}} = \left( \frac{\lambda}{1 + \lambda} \right) \text{ the same as for the charge case} \]

Series now hold in \((\frac{\lambda}{1 + \lambda})^n\) and \(\Lambda_s = 1 + \lambda \quad \chi_s(q = 0, \omega) = 0\)
Now back to $\Gamma_{\alpha\beta;\gamma\delta}^{\Omega}(k, 0; p, 0)$

Before

spin-fluctuation exchange
Hertz-Millis model

Now

$\Gamma_{\alpha\beta;\gamma\delta}^{\Omega, \text{full}}(k, 0; p, 0) = \frac{1}{2\nu Z} \left( \frac{\vec{\sigma}_{\alpha\delta} \cdot \vec{\sigma}_{\beta\gamma}}{\delta + a^2 (k - p)^2} + \frac{3\delta_{\alpha\delta} \delta_{\beta\gamma} - \vec{\sigma}_{\alpha\delta} \cdot \vec{\sigma}_{\beta\gamma}}{\delta + a^2 (k + p)^2} \right)$

spin only

spin AND charge
Fermi liquid revisited

\[ g_{\alpha\beta;\gamma\delta}^{\text{full}}(k, p) = Z^2(m^*/m) \Gamma^{\omega,\text{full}}_{\alpha\beta;\gamma\delta}(k, 0; p, 0) = g_c^{\text{full}} \delta_{\alpha\gamma} \delta_{\beta\delta} + g_s^{\text{full}} \bar{\sigma}_{\alpha\gamma} \cdot \bar{\sigma}_{\beta\delta} \]

\[
g_{\alpha\beta;\gamma\delta}^{\text{full}}(\theta) = \frac{3}{2} \delta_{\alpha\gamma} \delta_{\beta\delta} + \frac{1}{2} \bar{\sigma}_{\alpha\gamma} \cdot \bar{\sigma}_{\beta\delta} \times \left( \frac{4}{\delta + 4 (ak_F)^2 \cos^2 \theta/2} - \frac{1}{\delta + 4 (ak_F)^2 \sin^2 \theta/2} \right), \]

\[ g_{c,1}^{\text{full}} = g_{s,0}^{\text{full}} = \lambda \text{ for } \lambda \gg 1 \]

\[ m^*/m = \lambda; \quad \chi_s = \chi_s^0 \frac{1 + \lambda}{1 + \lambda} = \chi_s^0 \]
All Landau components again diverge upon approaching a ferromagnetic QCP

**Charge components:**

\[ g_{c,n}^{\text{full}} = \left( \frac{3}{2\pi} \right) \int_0^\pi d\theta \cos n\theta / \left( \delta + 4a^2k_F^2\sin^2\theta / 2 \right) \approx \lambda \]

charge susceptibilities \[ \chi_{c,n} = \chi_{c,n}^0 \left( m^*/m \right) / \left( 1 + g_{c,n}^{\text{full}} \right) = \chi_{c,n}^0 (1 + \lambda) / (1 + \lambda) = \chi_{c,n}^0 \] remain intact

**Spin components:**

\[ g_{s,n}^{\text{full}} = \lambda \delta_{n,2m} - \frac{5}{3} \lambda \delta_{n,2m+1} \]

For \( n = 2m \), \( g_{s,n}^{\text{full}} = \lambda \) \hspace{1cm} \text{same as charge components}

For \( n = 2m + 1, \) \( g_{s,n}^{\text{full}} = -\frac{5\lambda}{3} \)

\( g_{s,n}^{\text{full}} \) approaches \(-1\) at \( \lambda = 3/5 \)
\[ \Gamma_{\Omega,\text{full}}^\alpha; \gamma; \delta (k, 0; p, 0) = \frac{1}{2\nu Z} \left( \frac{\vec{\sigma}_\alpha \cdot \vec{\sigma}_\beta \gamma}{\delta + a^2 (k - p)^2} + \frac{3\delta_{\alpha\delta} \delta_{\beta\gamma} - \vec{\sigma}_\alpha \cdot \vec{\sigma}_\beta \gamma}{\delta + a^2 (k + p)^2} \right) \]
Beyond leading approximation: $g_{s,1}^{\text{full}}$ approaches -1 first

A pre-emptive spontaneous instability into a p-wave spin-nematic state!
A p-wave spin nematic.

Hirsch, Kee, Kim, S-C Zhang, C. Wu, Fradkin, Kivelson....

Can appear in two phases, A and B.

A phase -- stable

\[ \Delta_s = \sum_k f(k) (c_{k\uparrow}^\dagger c_{k\uparrow} - c_{k\downarrow}^\dagger c_{k\downarrow}) \cos \theta_k \]

\[ M = 0 \]

B phase, a vortex state with a winding number 1 (-1)

Unstable: negative modes
Comparison with other pre-emptive instabilities of a Fermi liquid near a FM QCP

First-order transition

\[ \Xi(T = 0) = \delta_F \Delta^2 + b \Delta^4 - a \Delta^4 |\log \Delta|, \quad D = 3 \]

\[ \Xi(T = 0) = \delta_F \Delta^2 + b \Delta^4 - a \Delta^3, \quad D = 2 \]

A spiral instability

\[ \chi_{\text{spin}}(Q) = (Q^2 - a |Q| p_F)^{-1} \]

If \( ak_F < 7.4 \), nematic instability comes first!
Conclusions:

To construct a Fermi liquid near a nematic transition requires some efforts

All Landau components except for a critical one diverge upon approaching a charge QCP. Effective mass diverges, but susceptibilities in non-critical channels are not affected and remain finite.

Hertz spin-fermion model is NOT the correct theory near a ferromagnetic QCP: in the conserving approximation, the interaction has both spin and charge components

A nearly ferromagnetic Fermi liquid develops a spontaneous instability towards a p-wave spin nematic.
THANK YOU