Fermi liquid near Pomeranchuk quantum criticality

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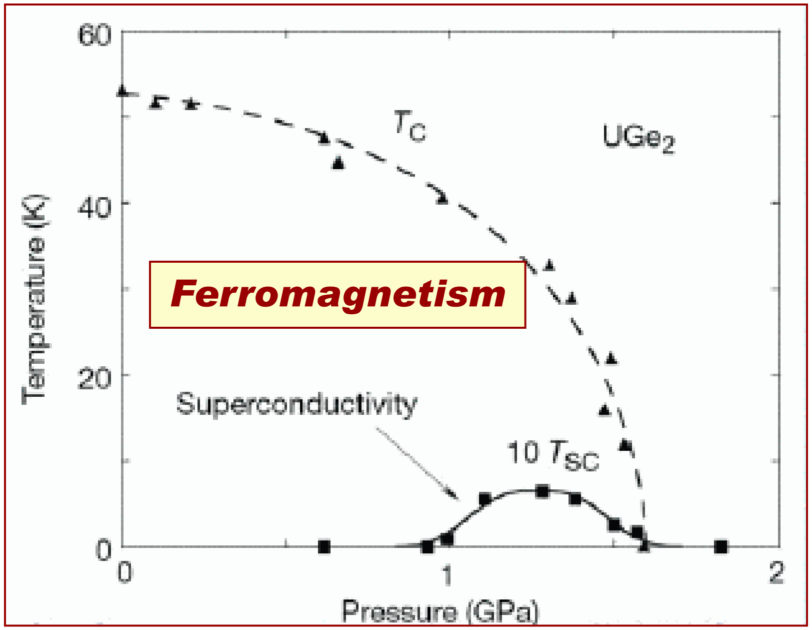
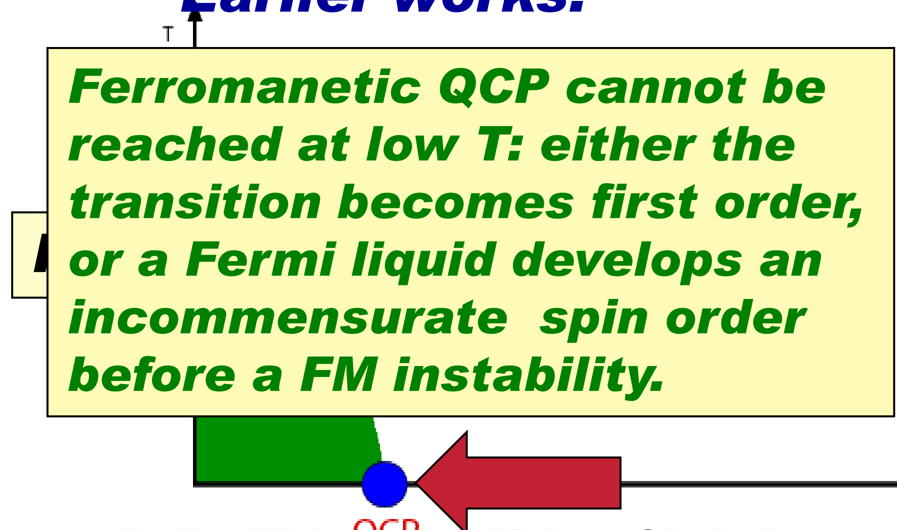
Result:

Consider a 2D system of itinerant fermions at T=0, near a ferromagnetic instability.

ZrZn2, UGe_2



Earlier works:



Ferromanetic QCP cannot be reached at low T: either the transition becomes first order, or a Fermi liquid develops an incommensurate spin order before a FM instability.

Belitz, Kirkpatrick, Vojta, Chubukov, Maslov, Millis, Pepin, Rech, A. Green….

We found another pre-emptive instability of a Fermi liquid: towards a p-wave spin nematic (a magnetic analog of ^{3}He)
In a wide parameter range, this instability comes first.

Issue: what is Femi liquid theory near a Pomeranchuk instability in a charge or a spin channel

Introduction: Fermi liquid theory, Pomeranchuk instabilities

Critical Fermi liquid near a Pomeranchuk QCP:

effective interaction,
Landau function
multi-channel criticality

Specifics of the spin case:

the consequence of SU(2) spin invariance
a new instability of a near-ferromagnetic Fermi liquid

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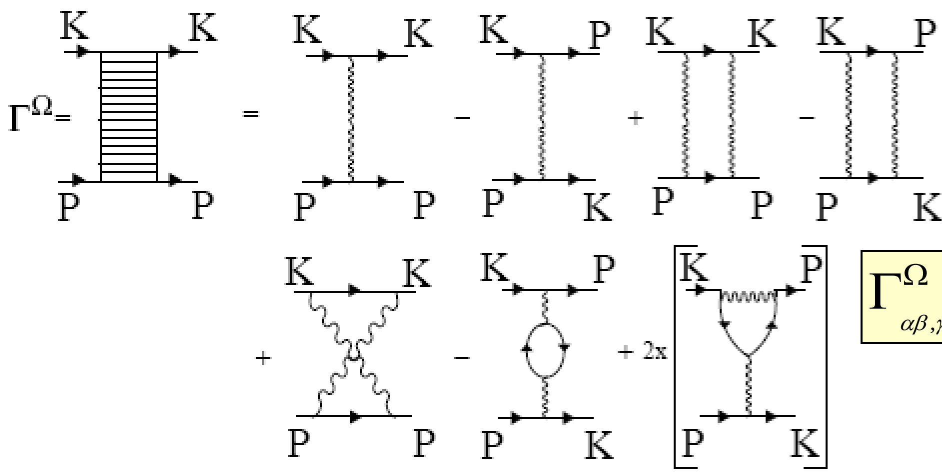
Landau Fermi liquid theory uses conservation laws and Galilean invariance to capture effects of interactions, which come from the immediate vicinity of the Fermi surface

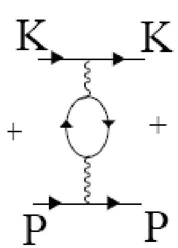
The input for the theory is a vertex

$$\Gamma^{\Omega}_{\alpha\beta,\gamma\delta}(\vec{k}_F - \vec{p}_F) = \Gamma^{\Omega}_{\alpha\beta,\gamma\delta}(\theta)$$



$$\Gamma^{\Omega}_{\alpha\beta,\gamma\delta}(\theta) = \Gamma^{\Omega}_{\alpha\beta,\gamma\delta}(\theta, q=0, \Omega \Rightarrow 0)$$

Includes everything except for soft bubbles with vanishing energy/momentum transfer. These come from the Fermi surface



$$\int G(P)\, G(P+Q)\, \frac{d^3P}{(2\pi)^3} = \frac{m}{2\pi}\left(1 - \frac{\Omega}{\sqrt{\Omega^2 + (v_F q)^2}}\right) = 0 \ \text{at} \ \frac{v_F q}{\Omega} \to 0$$

$$Q = (q, \Omega)$$

All effects due to interactions near the Fermi surface can be captured by applying conservation laws (conservation of the number of particles, SU(2) spin conservation, gauge invariance, and Galilean invariance

No need for further diagrammatics once Γ^{Ω} is known

How it works:

$$\Gamma^{\Omega}{}_{\alpha\beta,\gamma\delta}(\theta) = A\, \mathrm{g}_{\alpha\beta,\gamma\delta}(\theta)$$

Landau function

$$\mathrm{g}_{\alpha\beta,\gamma\delta}(\theta) = \mathrm{g}_{\mathrm{c}}(\theta)\, \delta_{\alpha\gamma}\, \delta_{\beta\delta} + \mathrm{g}_{\mathrm{s}}(\theta)\, \vec{\sigma}_{\alpha\gamma}\, \vec{\sigma}_{\beta\delta}$$

$$\mathrm{g}_{\mathrm{c}}(\theta) = \sum_{\mathrm{n}} g_{\mathrm{c,n}} \cos(\mathrm{n}\ \theta)$$

$$\mathrm{g}_{\mathrm{s}}(\theta) = \sum_{\mathrm{n}} g_{\mathrm{s,n}} \cos(\mathrm{n}\ \theta)$$

in 2D

$$\mathrm{C(T)} = \gamma\, \mathrm{T}, \quad \chi_{\mathrm{s}} = \mathrm{const}; \quad \frac{\gamma}{\gamma_0} = \frac{\mathrm{m}^*}{\mathrm{m}} = 1 + \mathrm{g}_{\mathrm{c},1}, \quad \frac{\chi_{\mathrm{s}}}{\chi_0} = \frac{1 + \mathrm{g}_{\mathrm{c},1}}{1 + \mathrm{g}_{\mathrm{s},0}}$$

It is actually a bit more involved....

Landau function

$$g_{\alpha\beta,\gamma\delta}(\theta) = \frac{m^*}{\pi} Z^2 \Gamma^{\Omega}{}_{\alpha\beta,\gamma\delta}(\theta)$$

$$\frac{m^*}{m} = 1 + g_{c,1}$$

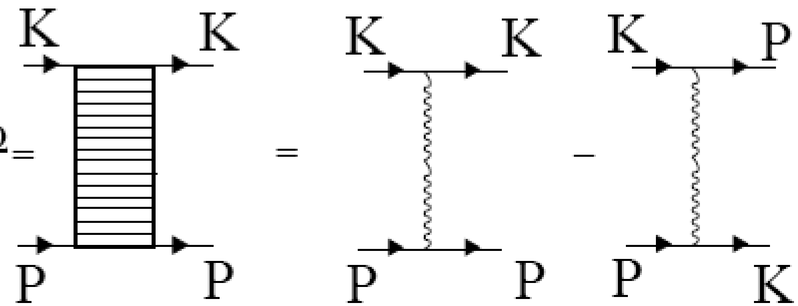
$$Z^{-1} = 1 - i \sum_{\beta} \Gamma^{\Omega}_{\alpha\beta,\alpha\beta}(K_F, P)\, G(P)\, G(P+Q) \frac{d^3P}{(2\pi)^3}, \quad Q = (q, \Omega), \quad P = (p, \omega),$$

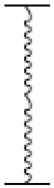
$$G(P) = \frac{Z}{i\,\omega - v_F \dfrac{m}{m^*}(p - p_F)}$$

Pitaevskii & Landau

How it works:

constant interaction, weak coupling

$\Gamma^{\Omega} =$ 

 **=U**

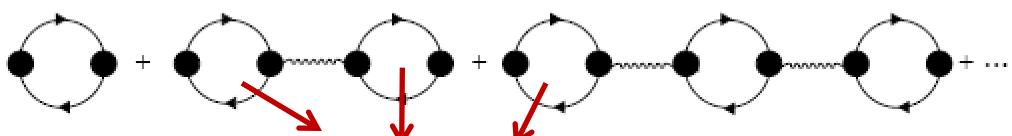
$$\boxed{\Gamma^{\Omega}{}_{\alpha\beta,\gamma\delta}\,(\theta) = \frac{U}{2}\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \frac{U}{2}\,\vec{\sigma}_{\alpha\gamma}\,\vec{\sigma}_{\beta\delta}}$$

Use Fermi liquid theory:

$$\boxed{g_{c,0} = \frac{m\,U}{\pi},\;\; g_{s,0} = -\frac{m\,U}{\pi},\;\; \chi_{s,0} = \frac{m}{\pi}\,\frac{1}{1+g_{so}} = \frac{m}{\pi}\,\frac{1}{1-\dfrac{m\,U}{\pi}}}$$

the same

Or use diagrammatics:

$\boxed{\chi_s}$ $=$ 

$\boxed{\text{soft bubbles,}\quad \Pi(\Omega = 0,\, q \Rightarrow 0)}$

$$\boxed{\chi_{s,0} = \frac{m}{\pi}\,\frac{1}{1-\dfrac{m\,U}{\pi}}}$$

$$\chi_{s.0} = \frac{m}{\pi} \frac{1}{1 - \frac{m\,U}{\pi}}$$

$$1 - \frac{m\,U}{\pi} = 0$$

Stoner criterion of a ferromagnetic instability

This can be done for any spin or charge channel:

$$\chi_{a,n} = \frac{m}{\pi} \frac{1}{1 + g_{a,n}}$$

$$\chi_{a,n} > 0 \Rightarrow 1 + g_{a,n} > 0$$

Pomeranchuk condition

A Fermi liquid instability at $g_{a,n}$ =-1 is called a Pomeranchuk instability

Examples: phase separation ($g_{c,0}$ =-1), a ferromagnetic transition ($g_{s,0}$ =-1), charge quadrupolar transition ($g_{c,2}$ =-1), spin dipolar ($g_{s,1}$ =-1) and quadrupolar ($g_{s,2}$ =-1) transitions

Castellani, Di Castro, Metzner, Dell'Anna, Belitz, Vojta, Kirkpatrick, Rosch, Oganesyan, Kivelson, Fradkin, Garst, Wolfle, Kee, Kim, S-C. Zhang, C.Wu, Hirsch, Pepin, Senthil, P. Lee, S-S Lee, Green, Betouras, Hooley, Maslov, A.C. .....

According to a textbook reasoning, only one, critical channel is relevant near a QCP: one Landau component approaches -1, others are irrelevant.

Indeed, if

$$\Gamma^{\Omega}{}_{\alpha\beta,\gamma\delta}(\theta) = \frac{U}{2}\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \frac{U}{2}\,\vec{\sigma}_{\alpha\gamma}\,\vec{\sigma}_{\beta\delta}$$

$$g_{c,0} = \frac{m\,U}{\pi},\ \ g_{s,0} = -\frac{m\,U}{\pi},\ \ g_{a,n>0} = 0$$

$$\chi_{s,0} = \frac{m}{\pi}\,\frac{1}{1+g_{s,0}} = \frac{m}{\pi}\,\frac{1}{1-\frac{m\,U}{\pi}}$$

$$\chi_{c,0} = \frac{m}{\pi}\,\frac{1}{1+g_{c,0}} = \frac{m}{\pi}\,\frac{1}{1+\frac{m\,U}{\pi}}$$

$$\frac{m^*}{m} = 1+g_{c,1} = 1,\quad \chi_{a,n>0} = \frac{m}{\pi}\,\frac{1+g_{c,1}}{1+g_{a,n}} = \frac{m}{\pi}$$

OK in dimension D >3, but not in D=2

What if we just do diagrammatics (e.g., large N)

Effective mass diverges

$$\boxed{\Sigma(\omega) \sim \frac{\omega}{(1+g_{s,0})^{1/2}}, \quad \frac{m^*}{m} \sim \frac{1}{(1+g_{s,0})^{1/2}} \Rightarrow \infty}$$

$$\boxed{\Sigma(\omega) \sim \omega^{2/3} \text{ at a QCP}}$$

m*/m = 1 + $g_{c,1}$

At the same time,

$$\boxed{\chi = \chi_{s,0} = \frac{m}{\pi}\frac{1}{1+g_{s,0}}}$$

no extra renormalization

So, the question is, what is the Landau function near a 2D nematic QCP

Here the story begins….

Critical Fermi liquid near a Pomeranchuk QCP:

effective nteraction,
Landau function
multi-channel criticality

Specifics of the spin case:

the consequence of SU(2) spin invariance
a new instability of a near-ferromagnetic Fermi liquid

For definiteness, consider a charge quadrupolar (d-wave) instability $g_{c,2}$ -> -1

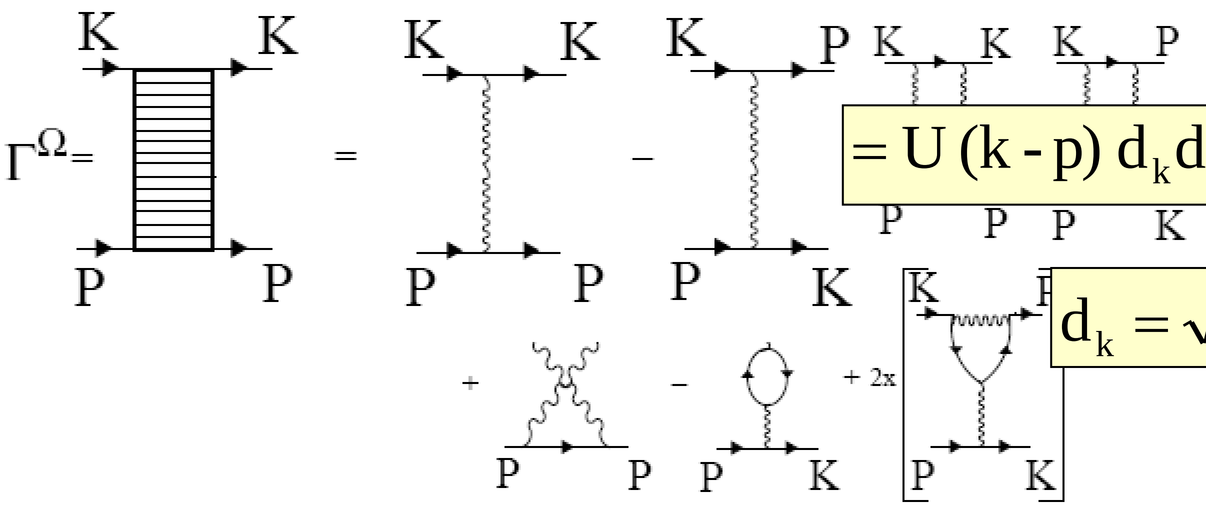
Metzner, Yamada, Kee, Kim, Oganesyan, Kivelson, Fradkin, Garst, Woelfle, Rosch, del'Anna

"Zero" order theory --same as before:

$$g^{(0)}_{\alpha\beta,\gamma\delta}(\theta) = \mathrm{g}_{c,2}\ \cos 2\theta\, \delta_{\alpha\gamma}\, \delta_{\beta\delta} + \mathrm{g}_{s,2}\ \cos 2\theta\, \vec{\sigma}_{\alpha\gamma}\, \vec{\sigma}_{\beta\delta}$$

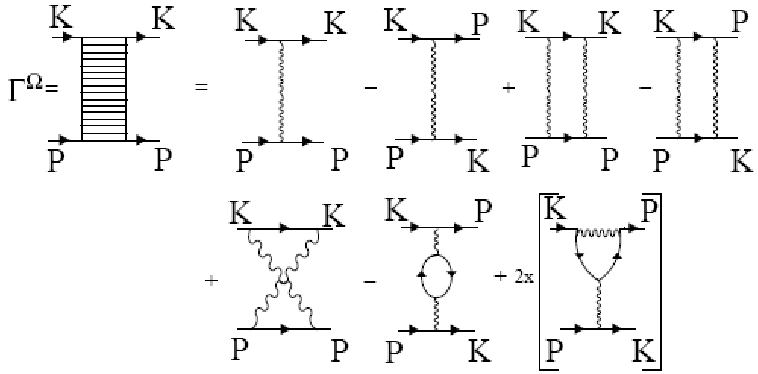
$$\frac{\mathrm{m}^*}{\mathrm{m}} = 1,\ \chi^{(0)}_{c,2} \propto \frac{1}{(1+\mathrm{g}_{c,2})}$$

Next



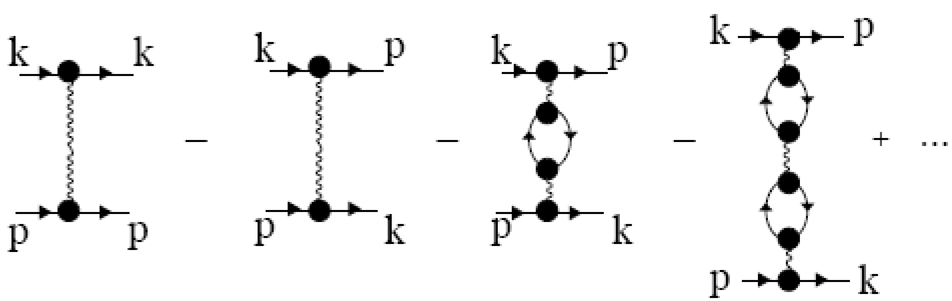
$$= \mathrm{U}(\mathrm{k}-\mathrm{p})\, \mathrm{d}_{\mathrm{k}} \mathrm{d}_{\mathrm{p}}$$

$$\mathrm{d}_{\mathrm{k}} = \sqrt{2}\cos(2\varphi_{\mathrm{k}})$$



$$U(\mathbf{q}) = U_0(1-(qa)^2+\dots) \qquad k_{\mathrm{F}}a \gg 1$$

$$\Gamma^{\Omega,\mathrm{RPA}}_{\alpha\beta;\gamma\delta} =$$



$$\Gamma^{\Omega,\mathrm{RPA}}_{\alpha\beta;\gamma\delta} \qquad \boxed{\Gamma^{\Omega} \propto \frac{1}{1+\mathrm{U}(\mathrm{q})\,\Pi_{\mathrm{d}}(\mathrm{q},\Omega)}} \qquad \boxed{\chi_{c,2} \propto \frac{1}{1+\mathrm{U}(0)\,\Pi_{\mathrm{d}}(0,0)}}$$

$$\nu = m/2\pi$$

Interaction mediated by soft collective charge nematic excitations

Berk & Schrieffer

$$\boxed{\Gamma^{\Omega}_{\alpha\beta,\gamma\delta}(\theta) \Rightarrow \bar{g}_{\alpha\beta,\gamma\delta}(\theta) \Rightarrow \bar{g}_{c}(\theta),\ \bar{g}_{s}(\theta) \Rightarrow \bar{g}_{c,n}, \bar{g}_{s,n}}$$

The form of $\bar{g}_{c,n}$, $\bar{g}_{s,n}$ depends on a single parameter

$$\boxed{\lambda = \frac{1}{2\,(a\,k_F)\sqrt{1+g_{c,2}}}}$$

$$\boxed{\lambda << 1 \ \text{weak coupling,} \qquad \bar{g}_{c,n},\ \bar{g}_{c,n} \approx 1}$$

$$\boxed{\lambda << 1 \ \text{strong coupling,} \qquad \bar{g}_{c,n},\ \bar{g}_{c,n} \approx \lambda >> 1}$$

Critical Fermi liquid

$$\lambda = \frac{1}{2ak_{\mathrm{F}}\sqrt{1+g_{c,2}}}.$$

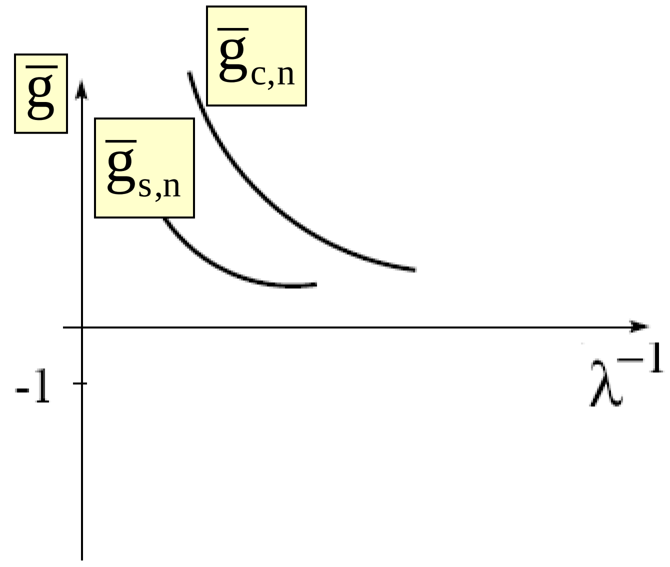
In a critical Fermi liquid (when λ >> 1)

$$\bar{g}_{a,n} = \lambda$$

for a= c,s, and ALL n < λ $(ak_F)^2$

The textbook assumption that only one Landau component is singular near a Pomeranchuk transition is incorrect in 2D (and 3D)

Consequences of $\bar{g}_{a,n} = \lambda,$



"Zero" order theory for a critical Fermi liquid:

$$\boxed{\frac{m^*}{m} = 1,\ \chi_{c,2}^{(0)} \propto \frac{1}{(1+g_{c,2})}}$$

Interaction with soft collective modes, acts on top of zero order theory and gives:

$$\boxed{\frac{m^*}{m} \sim 1 + \bar{g}_{c,1} = 1 + \lambda \propto \frac{1}{(1+g_{c,2})^{1/2}}}$$

effective mass diverges

$$\boxed{\chi_{c,2} = \chi_{c,2}^{(0)} \frac{1+\bar{g}_{c,1}}{1+\bar{g}_{c,2}} \approx \chi_{c,2}^{(0)} \frac{1+\lambda}{1+\lambda} = \chi_{c,2}^{(0)} = \frac{m}{\pi} \frac{1}{1+g_{c,2}}}$$

susceptibility stays intact

This is fully consistent with diagrammatics

We can also introduce the full Landau function, which acts on top of free fermion theory, i.e., include "zero order" renormalization into Fermi liquid

$$\chi^{(0)}_{c,2} = \frac{m}{\pi}, \quad \chi_{c,2} = \frac{m}{\pi}\,\frac{1+g^{*}_{c,1}}{1+g^{*}_{c,2}}$$

$$g^{*}_{a,n} = g_{a,n} + \bar{g}_{a,n}\left(1 + g_{a,n}\right) \qquad g_{a,n} = g_{c,2}\,\delta_{a,c}\,\delta_{n,2}$$

$g^{*}_{a,n} = \bar{g}_{a,n}$ ***for all components except for n=2 charge component***

$$\chi_{c,2} = \frac{m}{\pi}\,\frac{1}{1+g_{c,2}} = \frac{m}{\pi}\,\frac{1+g^{*}_{c,1}}{1+g^{*}_{c,2}} = \frac{m}{\pi}\,\frac{(1+g_{c,2})^{-1/2}}{(1+g_{c,2})^{1/2}}$$

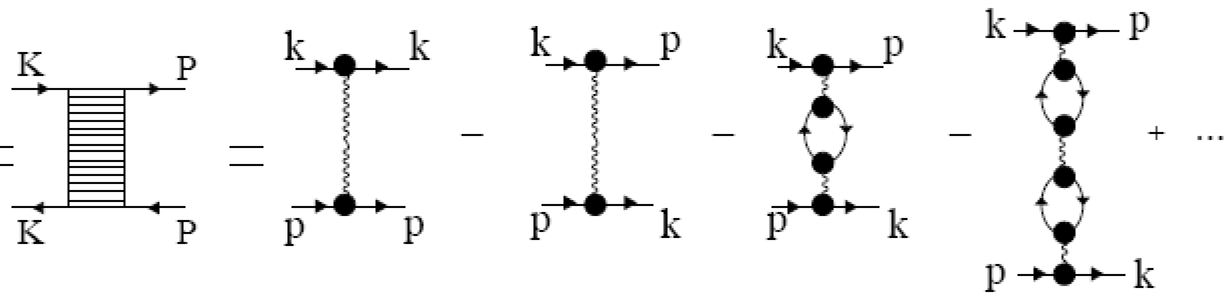
oaches -1,
exponent
than 1

Divergence of χ is split between m*/m and nematic "g-factor"

As usual, full story is a bit more messy:

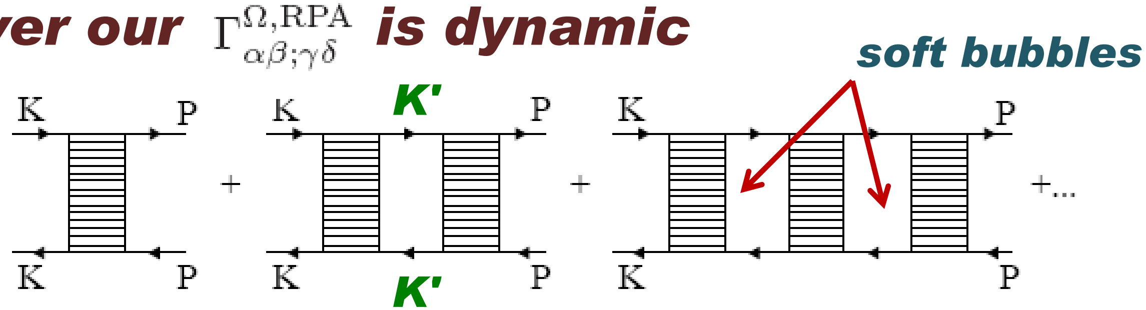
$$\Gamma^{\Omega}_{\alpha\beta;\gamma\delta} = \frac{1}{Z_\Gamma}\Gamma^{\Omega,\mathrm{RPA}}_{\alpha\beta;\gamma\delta}$$

$\Gamma^{\Omega,\mathrm{RPA}}_{\alpha\beta;\gamma\delta} =$



We neglected soft bubbles, as textbooks told us to do.

This is fine as long as effective interaction is static, however our $\Gamma^{\Omega,\mathrm{RPA}}_{\alpha\beta;\gamma\delta}$ is dynamic



This yields, after summation of ladder series

$$\Gamma^{\Omega}_{\alpha\beta;\gamma\delta} = \frac{1}{Z_\Gamma}\Gamma^{\Omega,\mathrm{RPA}}_{\alpha\beta;\gamma\delta}$$

$$\frac{1}{Z_\Gamma} = \frac{2}{\sqrt{1-\frac{Z^2 m*}{m}\lambda}\left(1+\sqrt{1-\frac{Z^2 m*}{m}\lambda}\right)}$$

$$\boxed{\lambda = \frac{1}{2ak_\mathrm{F}\sqrt{1+g_{c,2}}}.}$$

Use

$$\boxed{g_{\alpha\beta,\gamma\delta} = 2\,\nu\,\frac{\mathrm{m}*}{\mathrm{m}}\,\mathrm{Z}^2\,\Gamma^{\Omega}_{\alpha\beta,\gamma\delta}}$$

$$\bar{g}_{\alpha\beta;\gamma\delta}(\theta) = 2\nu Z^2\frac{m^*}{m}\Gamma^{\Omega}_{\alpha\beta;\gamma\delta}(\theta) = \frac{Z^2 m^*}{4Z_\Gamma m}\frac{d_\mathbf{k}d_\mathbf{p}}{1+g_{c,2}+(ak_\mathrm{F}\theta)^2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta}+\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\delta}\right)$$

$$\frac{m^*}{m} = 1+\frac{\lambda}{4}\frac{Z^2}{Z_\Gamma}\frac{m^*}{m}$$

$$\boxed{\mathrm{Z}^{-1} = 1 - \mathrm{i}\sum_\beta \Gamma^{\Omega}_{\alpha\beta,\alpha\beta}\,(\mathrm{K_F},\mathrm{P})\,\mathrm{G(P)}\,\mathrm{G(P+Q)}\,\frac{\mathrm{d}^3\mathrm{P}}{(2\pi)^3}}$$

$$\Rightarrow \quad \frac{1}{Z} = 1+\frac{\lambda}{4}\frac{Z^2}{Z_\Gamma}\frac{m^*}{m}$$

Three coupled equations

$$\frac{1}{Z_\Gamma} = \frac{2}{\sqrt{1-\frac{Z^2 m*}{m}\lambda}\left(1+\sqrt{1-\frac{Z^2 m*}{m}\lambda}\right)} \qquad \frac{1}{Z} = 1 + \frac{\lambda}{4}\frac{Z^2}{Z_\Gamma}\frac{m^*}{m} \qquad \frac{m^*}{m} = 1 + \frac{\lambda}{4}\frac{Z^2}{Z_\Gamma}\frac{m^*}{m}$$

with $\lambda = \frac{1}{2ak_F\sqrt{1+g_{c,2}}}.$ **as a parameter**

$$Z\frac{m^*}{m} = 1.$$

Weak coupling $\lambda \ll 1$

$$\frac{m^*}{m}\Big|_{\lambda\ll 1} = 1+\lambda/4;\; Z\Big|_{\lambda\ll 1} = 1-\lambda/4;\; Z_\Gamma\Big|_{\lambda\ll 1} = 1-3\lambda/4$$

Strong coupling

$$\frac{m^*}{m}\Big|_{\lambda\gg 1} = \lambda + 1/4\lambda;\;\; ;Z\Big|_{\lambda\gg 1} = 1/\lambda - 1/4\lambda^3$$

$$Z_\Gamma\Big|_{\lambda\gg 1} = 1/4\lambda + 1/4\lambda^2$$

$$Z_\Gamma \approx Z/4$$

$$\bar{g}_{\alpha\beta;\gamma\delta}(\theta) \approx \frac{1}{1+g_{c,2}+(ak_F\theta)^2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} + \vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\delta}\right) \Rightarrow \boxed{\bar{g}_{a,n} = \lambda}$$

The story continues....

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Specifics of the spin case:

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Apply the same reasoning to a ferromagnetic transition

$$\Gamma^{\Omega}_{\alpha\beta;\gamma\delta}(\mathbf{k},\omega_k;\mathbf{p},\omega_p) = \frac{1}{2\nu Z}\frac{1}{\delta + (aq)^2 + \frac{|\Omega|}{v_F q}}\vec{\sigma}_{\alpha\delta}\cdot\vec{\sigma}_{\beta\gamma}$$

$$\mathbf{q} = \mathbf{k} - \mathbf{p},\ \ \Omega = \omega_k - \omega_p$$

$$\nu = m/2\pi$$

This is Hertz-Millis theory

$$\delta = 1 + g_{s,0}$$

$$\bar{g}_{\alpha\beta;\gamma\delta}(\mathbf{k},\mathbf{p}) = Z^2(m^*/m)\Gamma^{\Omega}_{\alpha\beta;\gamma\delta}(\mathbf{k},0;\mathbf{p},0)$$

$$\bar{g}_{\alpha\beta;\gamma\delta} = \bar{g}_c\delta_{\alpha\gamma}\delta_{\beta\delta} + \bar{g}_s\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\delta}$$

$$m^*/m \ = \ 1 + \bar{g}_{c,1} \qquad \chi_s \ = \ \chi_s^0(1+\bar{g}_{c,1})/(1+\bar{g}_{s,0})$$

$$\frac{m^*}{m} = \lambda \qquad \lambda \equiv \frac{3}{4ak_F\sqrt{\delta}}$$

$$\chi_s = \chi_s^0\frac{1+\lambda}{1-\frac{\lambda}{3}}$$

Divergence before a QCP?

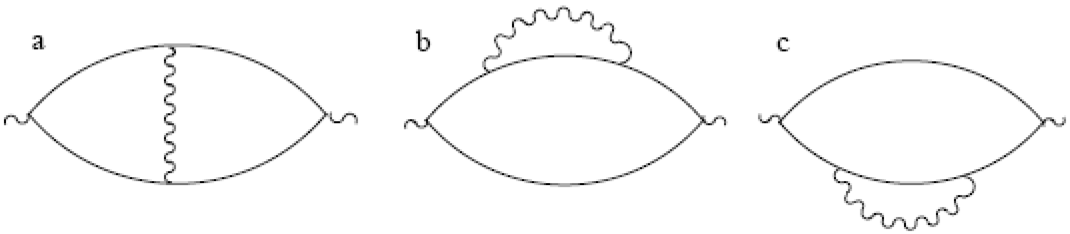
Diagrammatics: $\frac{m^*}{m} \approx \frac{1}{Z} = \lambda \qquad \chi_s = \chi_s^0 \propto \frac{1}{\delta},$

Fermi liquid theory is based on conservation laws. Let's verify if we are using a conserving approach in the spin case

Check Ward identities:

the total number of particles and SU(2) spin are conserved, hence

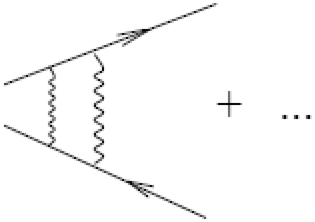
$$\chi_{c,s}(q = 0, \Omega) = 0$$



$$\Lambda_{c,s} = 1$$

for free fermions

$$\Lambda_{c,s} =$$



Maki-Thompson diagrams

Charge susceptibility:

Series for Λ_c are geometric, the n^th term is $(\lambda/(1+\lambda))^n$

$$\Lambda_c = 1 + \frac{\lambda}{1+\lambda} + \left(\frac{\lambda}{1+\lambda}\right)^2 + \left(\frac{\lambda}{1+\lambda}\right)^3 + \ldots = \frac{1}{1-\frac{\lambda}{1+\lambda}} = 1+\lambda$$

$$\chi_{c,s}(q=0,\Omega) = \chi^0_{c,s}\left(1 - \frac{\Lambda_{c,s}}{1+\lambda}\right)$$

$$\chi_c(q=0,\Omega) = 0$$

as it indeed should

Spin susceptibility:

Series for Λ_c ***are geometric, the n^th^ term is*** $(-1/3)^n(\lambda/(1+\lambda))^n$

$$\Lambda_c = 1 - \left(\frac{1}{3}\right)\frac{\lambda}{1+\lambda} + \left(\frac{1}{9}\right)\left(\frac{\lambda}{1+\lambda}\right)^2 - \left(\frac{1}{27}\right)\left(\frac{\lambda}{1+\lambda}\right)^3 + \ldots$$

$$= \frac{1}{1+\left(\frac{1}{3}\right)\frac{\lambda}{1+\lambda}} = \frac{3(1+\lambda)}{3+4\lambda}$$

$$\chi_{c,s}(q=0,\Omega) = \chi^0_{c,s}\left(1 - \frac{\Lambda_{c,s}}{1+\lambda}\right)$$

$$\chi_s(q=0,\omega) \neq 0$$

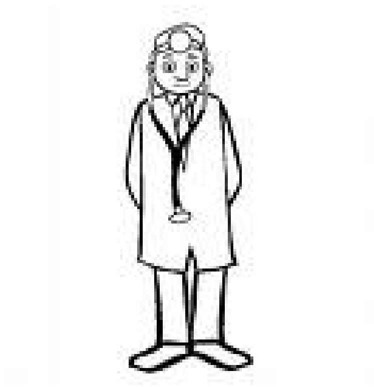
Physics:

We have a model with spin-spin interaction between itinerant fermions, mediated by collective spin excitations

An electron spin is split into a spin of an itinerant fermion $\mathrm{s} = c^\dagger_\alpha \vec{\sigma}_{\alpha\beta} c_\beta$ and a spin of a collective boson S .

For SU(2) invariant case, the interaction is $\mathrm{s} \cdot \mathrm{S}$, which can flip s^z. As a result, fermionic s is not conserved separately from S.

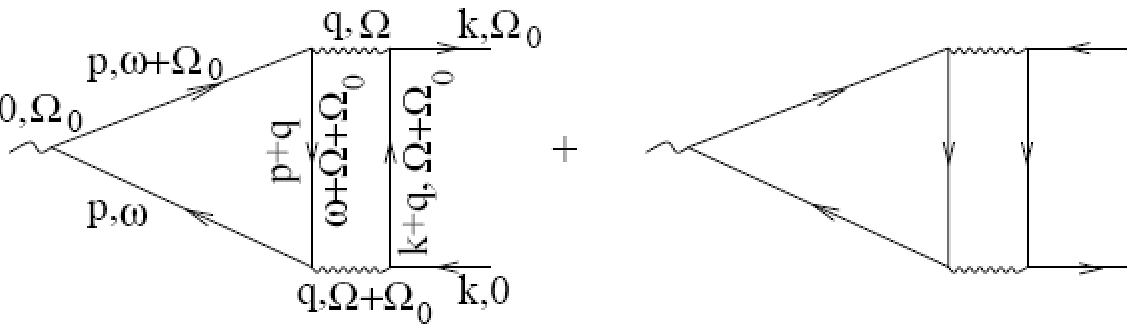
N.B. For Ising spins, Ward identities are satisfied.



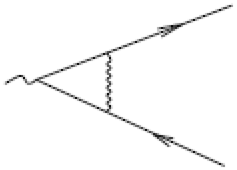
How to cure?

Need backflow terms which flip s^z back

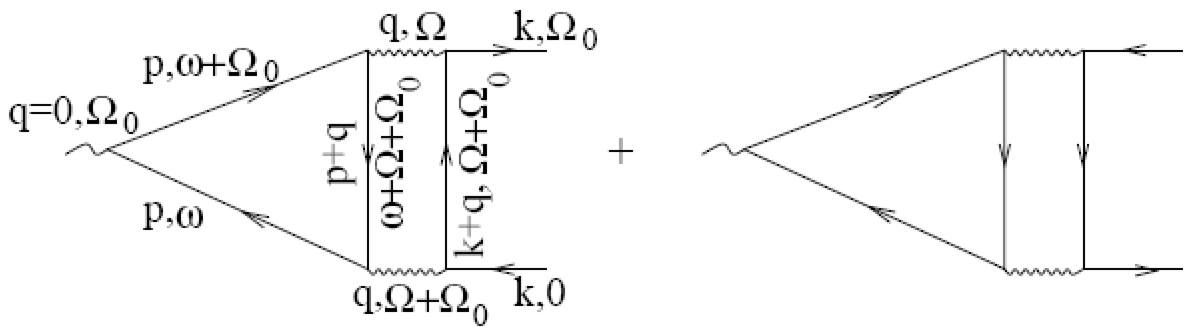
Candidates: Aslamazov-Larkin processes



Charge case: the two diagrams cancel each other.
Spin case: the two add up



$I_{MT} = \ (-1/3)\ (\lambda/(1+\lambda))$



$$I_{\rm AL} \ = \ \frac{1}{2\nu Z^2}\int\frac{dqq}{2\pi}\int\frac{d\Omega}{2\pi}\int\frac{d\theta_{\mathbf{pq}}}{2\pi}\int\frac{\theta_{\mathbf{kq}}}{2\pi}\int\frac{d\omega}{2\pi}\int d\epsilon_p G(\mathbf{p},\omega)G(\mathbf{p},\omega+\Omega_0)G(\mathbf{k}_F+\mathbf{q},\Omega+\Omega_0)$$
$$\times\left[G(\mathbf{p}+\mathbf{q},\omega+\Omega+\Omega_0)-G(\mathbf{p}-\mathbf{q},\omega-\Omega)\right]\bar\Gamma(\mathbf{q},\Omega)\bar\Gamma(\mathbf{q},\Omega+\Omega_0)$$

$$I_{\rm AL} \ = \ (4/3)\lambda/(1+\lambda)$$

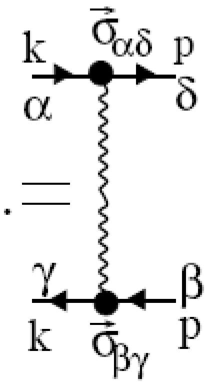
$I_{MT} + I_{AL} = (\lambda/(1+\lambda))$ ***the same as for the charge case***

Series now hold in $(\lambda/(1+\lambda))^n$ ***and*** $\Lambda_s = 1+\lambda$ $\quad \chi_s(q=0,\omega) = 0$

Now back to $\Gamma^{\Omega}_{\alpha\beta;\gamma\delta}(\mathbf{k},0;\mathbf{p},0)$

Before

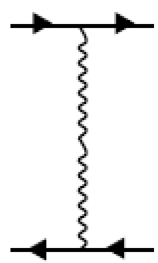
$\Gamma^{\Omega} \doteq$



spin-fluctuation exchange
Hertz-Millis model

Now

$\Gamma^{\Omega,\mathrm{full}} =$



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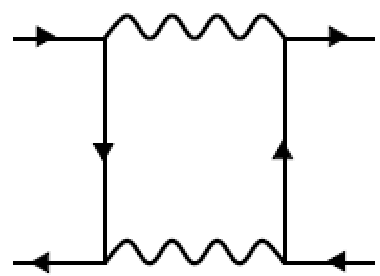

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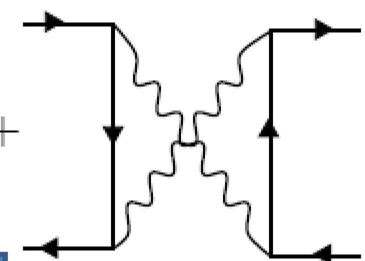


$$\Gamma^{\Omega,\mathrm{full}}_{\alpha\beta;\gamma\delta}(\mathbf{k},0;\mathbf{p},0) = \frac{1}{2\nu Z}\left(\frac{\vec{\sigma}_{\alpha\delta}\cdot\vec{\sigma}_{\beta\gamma}}{\delta + a^2(\mathbf{k}-\mathbf{p})^2} + \frac{3\delta_{\alpha\delta}\delta_{\beta\gamma} - \vec{\sigma}_{\alpha\delta}\cdot\vec{\sigma}_{\beta\gamma}}{\delta + a^2(\mathbf{k}+\mathbf{p})^2}\right)$$

spin only **spin AND charge**

Fermi liquid revisited

$$g^{\text{full}}_{\alpha\beta;\gamma\delta}(\mathbf{k},\mathbf{p}) \quad = $$
$$Z^2(m^*/m)\Gamma^{\omega,\text{full}}_{\alpha\beta;\gamma\delta}(\mathbf{k},0;\mathbf{p},0) = g_c^{\text{full}}\delta_{\alpha\gamma}\delta_{\beta\delta} + g_s^{\text{full}}\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\delta}$$

$$g^{\text{full}}_{\alpha\beta;\gamma\delta}(\theta) = \frac{3}{2}\frac{\delta_{\alpha\gamma}\delta_{\beta\delta}}{\delta + 4(ak_F)^2\sin^2\theta/2} + \frac{1}{2}\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\delta}\times$$
$$\left(\frac{4}{\delta + 4(ak_F)^2\cos^2\theta/2} - \frac{1}{\delta + 4(ak_F)^2\sin^2\theta/2}\right),$$

$$g^{\text{full}}_{c,1} = \bar{g}^{\text{full}}_{s,0} = \lambda \text{ for } \lambda \gg 1$$

$$m^*/m = \lambda; \;\; \chi_s = \chi_s^0\frac{1+\lambda}{1+\lambda} = \chi_s^0$$

All Landau components again diverge upon approaching a ferromagnetic QCP

Charge components:

$$g^{\rm full}_{c,n} = (3/2\pi) \int_0^\pi d\theta \cos n\theta / (\delta + 4a^2 k_F^2 \sin^2 \theta/2) \approx \lambda$$

charge susceptibilities $\chi_{c,n} = \chi^0_{c,n}(m^*/m)/(1+g^{\rm full}_{c,n}) = \chi^0_{c,n}(1+\lambda)/(1+\lambda) = \chi^0_{c,n}$ remain intact

Spin components:

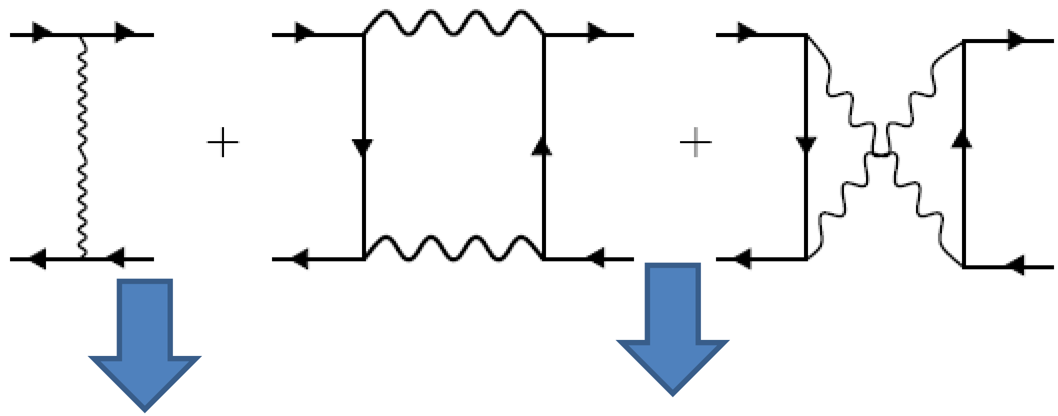
$$g^{\rm full}_{s,n} = \lambda \delta_{n,2m} - \frac{5}{3}\lambda \delta_{n,2m+1}$$

For $n = 2m$ $\quad g^{\rm full}_{s,n} = \lambda$ ***same as charge components***

For $n = 2m+1$, $g^{\rm full}_{s,n} = -5\lambda/3$

$g^{\rm full}_{s,n}$ approaches -1 at $\lambda = 3/5$

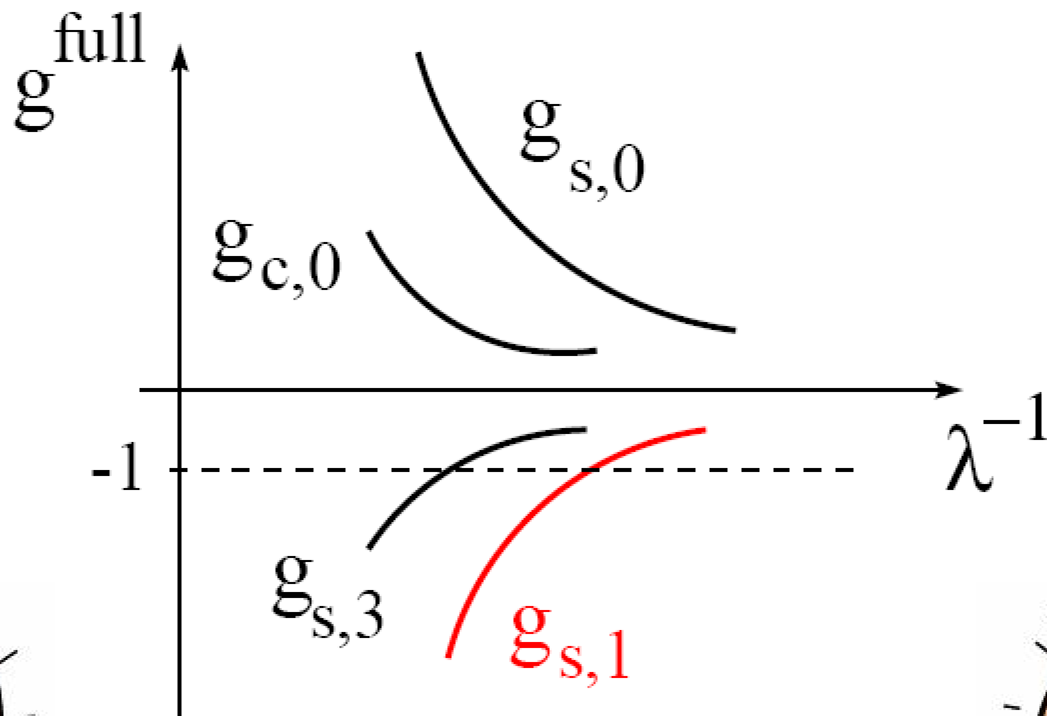
$\Gamma^{\Omega,\text{full}} =$



$$\Gamma^{\Omega,\text{full}}_{\alpha\beta;\gamma\delta}(\mathbf{k},0;\mathbf{p},0) = \frac{1}{2\nu Z}\left(\frac{\vec{\sigma}_{\alpha\delta}\cdot\vec{\sigma}_{\beta\gamma}}{\delta + a^2(\mathbf{k}-\mathbf{p})^2} + \frac{3\delta_{\alpha\delta}\delta_{\beta\gamma} - \vec{\sigma}_{\alpha\delta}\cdot\vec{\sigma}_{\beta\gamma}}{\delta + a^2(\mathbf{k}+\mathbf{p})^2}\right)$$

forward scattering ***back scattering***

Beyond leading approximation: $g_{s,1}^{\mathrm{full}}$ approaches -1 first



A pre-emptive spontaneous instability into a p-wave spin-nematic state!

A p-wave spin nematic.

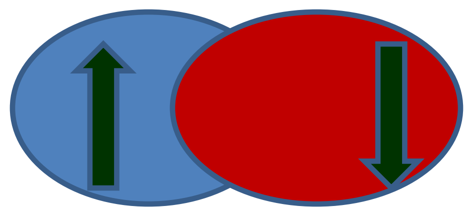
Hirsch, Kee, Kim, S-C Zhang,
C. Wu, Fradkin, Kivelson….

Can appear in two phases, A and B.

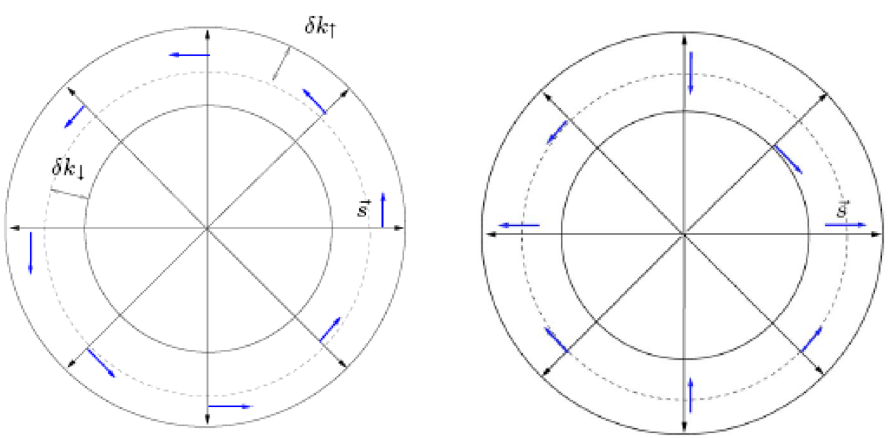
A phase -- stable

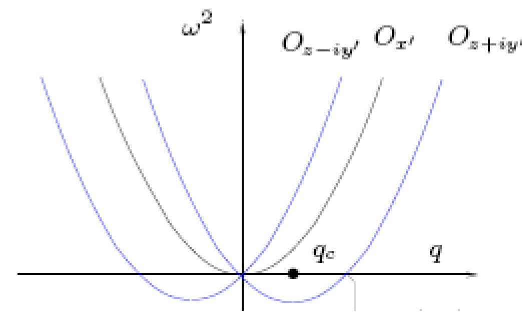
$$\Delta_s = \sum_{\mathbf{k}} f(k)(c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow} - c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}\downarrow}) \cos\theta_{\mathbf{k}}$$

M =0



B phase, a vortex state with a winding number 1 (-1)





Unstable: negative modes

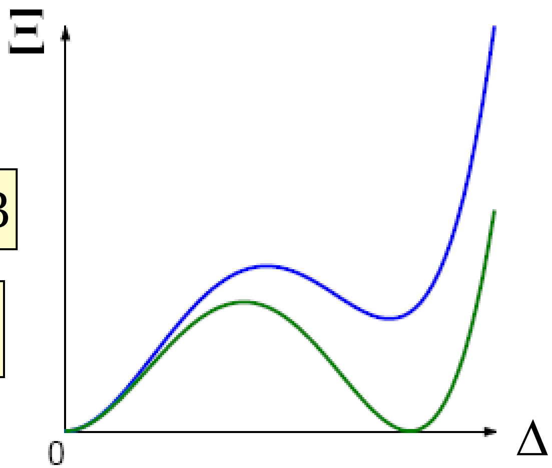
Comparison with other pre-emptive instabilities of a Fermi liquid near a FM QCP

Belitz et al,
Maslov et al
Betouras et al
Green et al…

First-order transition

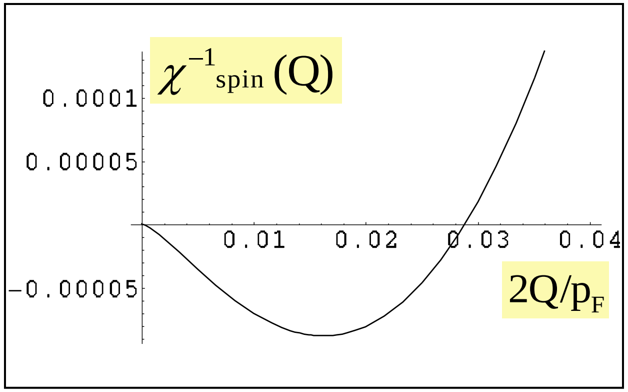
$$\Xi(T=0) = \delta_F\,\Delta^2 + b\,\Delta^4 - a\,\Delta^4\,|\mathrm{Log}\,\Delta|, \quad D=3$$

$$\Xi(T=0) = \delta_F\,\Delta^2 + b\,\Delta^4 - a\,\Delta^3, \quad D=2$$



A spiral instability

$$\chi_{\mathrm{spin}}(Q) = (Q^2 - a\,|Q|\,p_F)^{-1}$$



If $ak_F < 7.4$, nematic instability comes first!

Conclusions:

To construct a Fermi liquid near a nematic transition requires some efforts

All Landau components except for a critical one diverge upon approaching a charge QCP. Effective mass diverges, but susceptibilities in non-critical channels are not affected and remain finite.

Hertz spin-fermion model is NOT the correct theory near a ferromagnetic QCP: in the conserving approximation, the interaction has both spin and charge components

A nearly ferromagnetic Fermi liquid develops a spontaneous instability towards a p-wave spin nematic.

THANK YOU