Interacting anyons in topological quantum liquids

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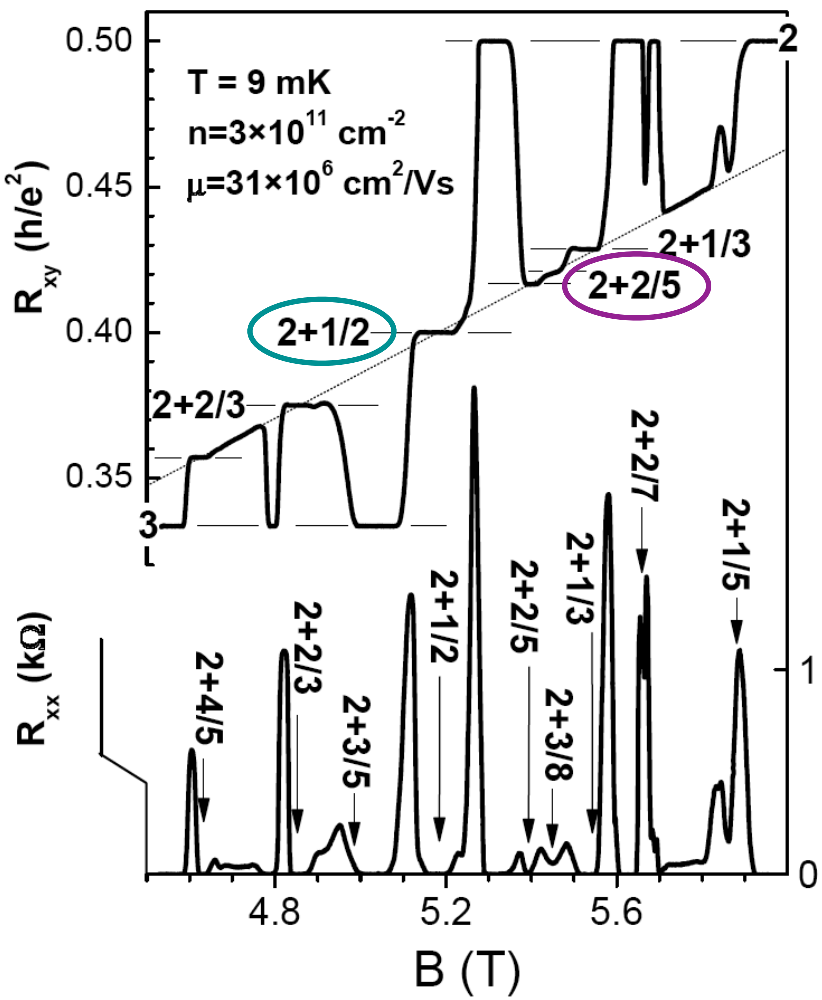
Zhenghan Wang, Station Q

Fractional quantum Hall liquids



J.S. Xia *et al.*, PRL (2004)

“Pfaffian” state

Moore & Read (1994)

Charge e/4 quasiparticles
Ising anyons

Nayak & Wilczek (1996)

$SU(2)_2$

“Parafermion” state

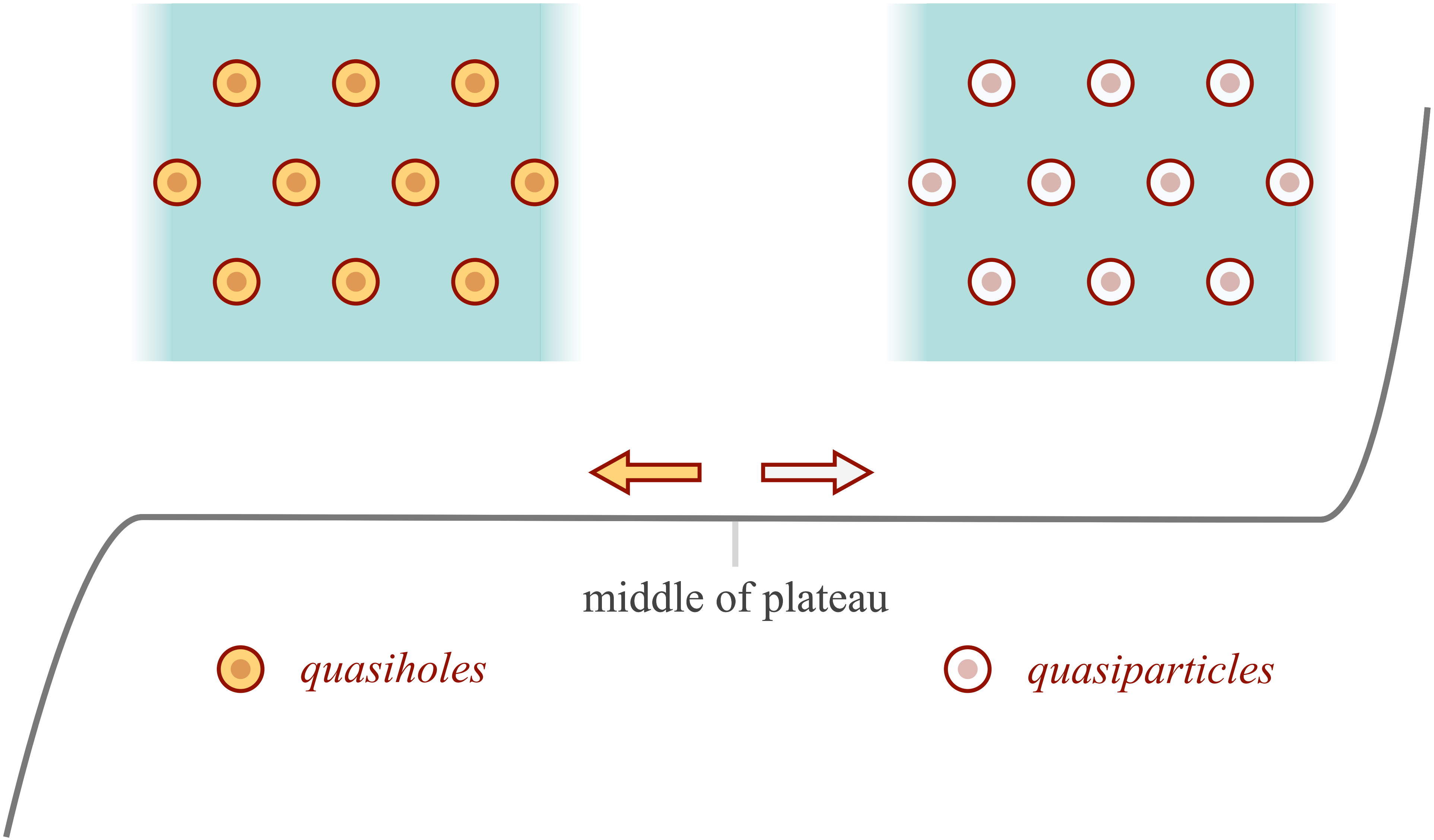
Read & Rezayi (1999)

Charge e/5 quasiparticles
Fibonacci anyons

Slingerland & Bais (2001)

$SU(2)_3$

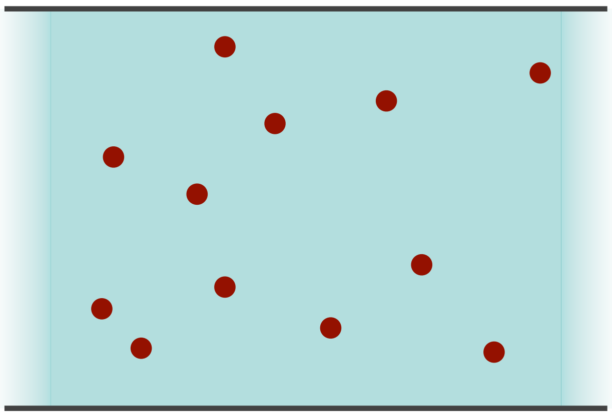
Quantum Hall plateaus



Abelian vs. non-Abelian anyons

Consider a set of ‘**pinned**’ anyons at fixed positions.

Abelian

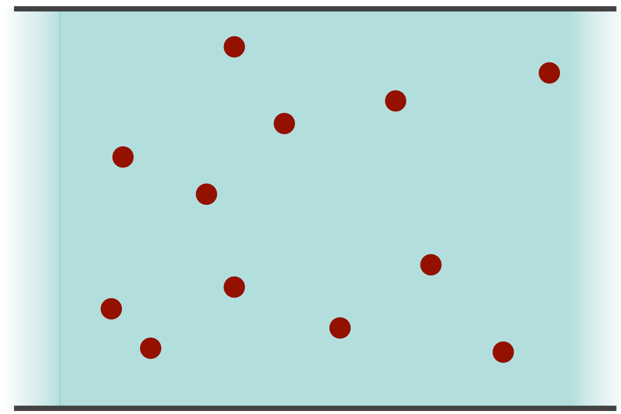


single state

$$\psi(x_2, x_1) = e^{i\pi\theta} \cdot \psi(x_1, x_2)$$

fractional phase

non-Abelian



(degenerate) **manifold** of states

matrix

$$\psi(x_1 \leftrightarrow x_3) = M \cdot \psi(x_1, \ldots, x_n)$$

$$\psi(x_2 \leftrightarrow x_3) = N \cdot \psi(x_1, \ldots, x_n)$$

In general *M* and *N* do not commute!

Non-Abelian anyons

Ising anyons = Majorana fermions

Moore-Read quantum Hall state
topological insulators
A_1 phase of ^{3}He films
p-wave superconductors
Kitaev's honeycomb model

$SU(2)_2$

Fibonacci anyons

Read-Rezayi quantum Hall state
Levin-Wen model

$SU(2)_3$

$SU(2)_k$

ordinary spins

quantum magnets

$SU(2)_\infty$

$SU(2)_k$ = 'deformations' of SU(2)

Quantum numbers in $SU(2)_k$

$$0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ 2,\ \ldots,\ \frac{k}{2}$$

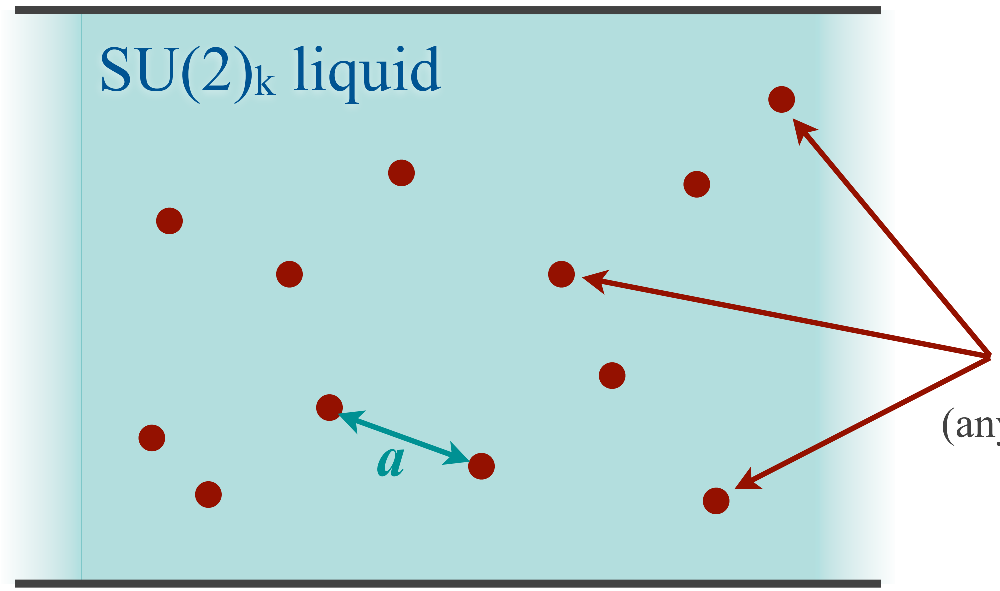
cutoff level *k*
"quantization"

Fusion rules

$$j_1 \times j_2 \quad = \quad |j_1 - j_2| + (|j_1 - j_2| + 1) + \ldots + \min(j_1 + j_2, k - j_1 - j_2)$$

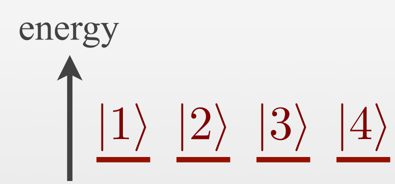
for all k ≥ 2	for all k ≥ 4
$\frac{1}{2} \times \frac{1}{2} = 0 + 1$	$1 \times 1 = 0 + 1 + 2$

A soup of non-Abelian anyons

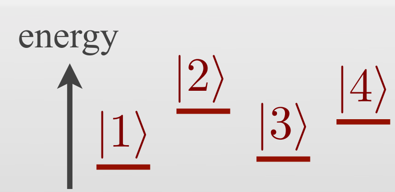


finite density of anyons
(anyons are at fixed positions or '**pinned**')

$a \gg \xi_m$ The ground state has a **macroscopic degeneracy**.

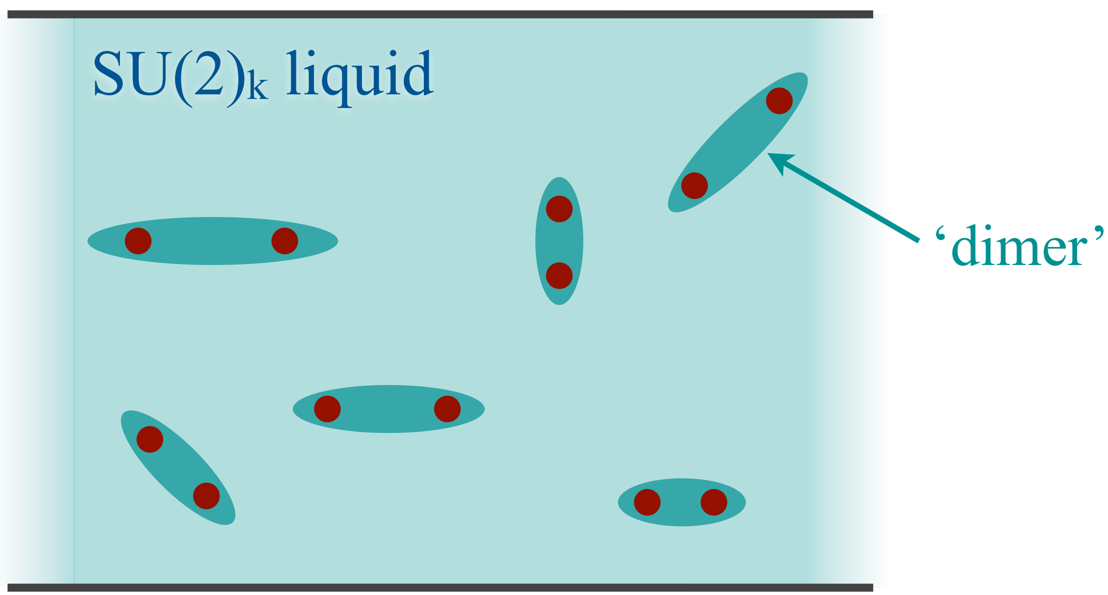


$a \ll \xi_m$ Anyons approach each other and interact. The interactions will **lift the degeneracy**.



What is the **collective state** of a set of interacting anyons?

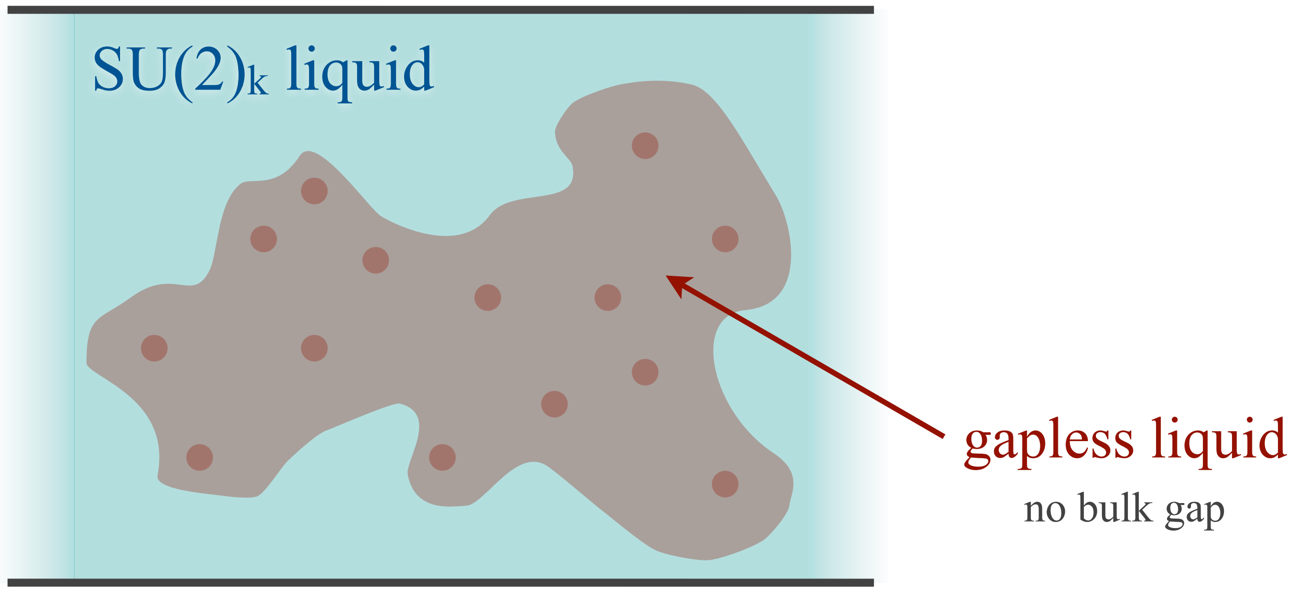
Collective states: possible scenarios



The collective state of anyons is **gapped**.

The parent liquid remains **unchanged.**

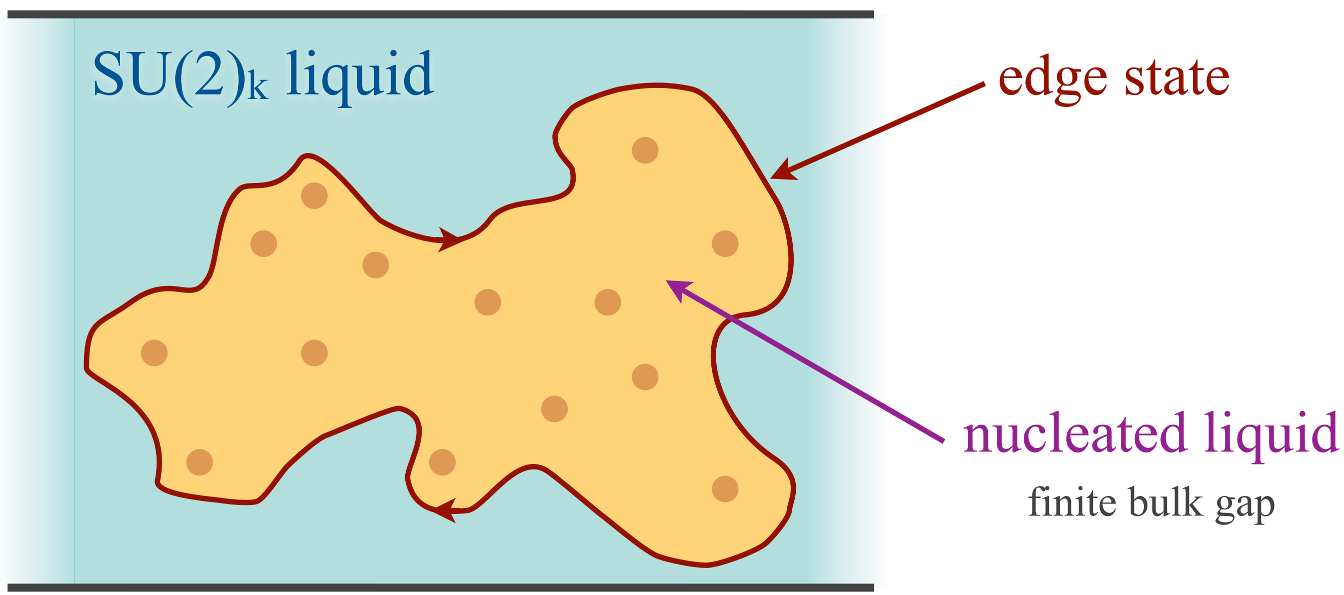
Collective states: possible scenarios



The collective state of anyons is a **gapless quantum liquid**.

A **gapless phase nucleates** within the parent liquid.

Collective states: possible scenarios

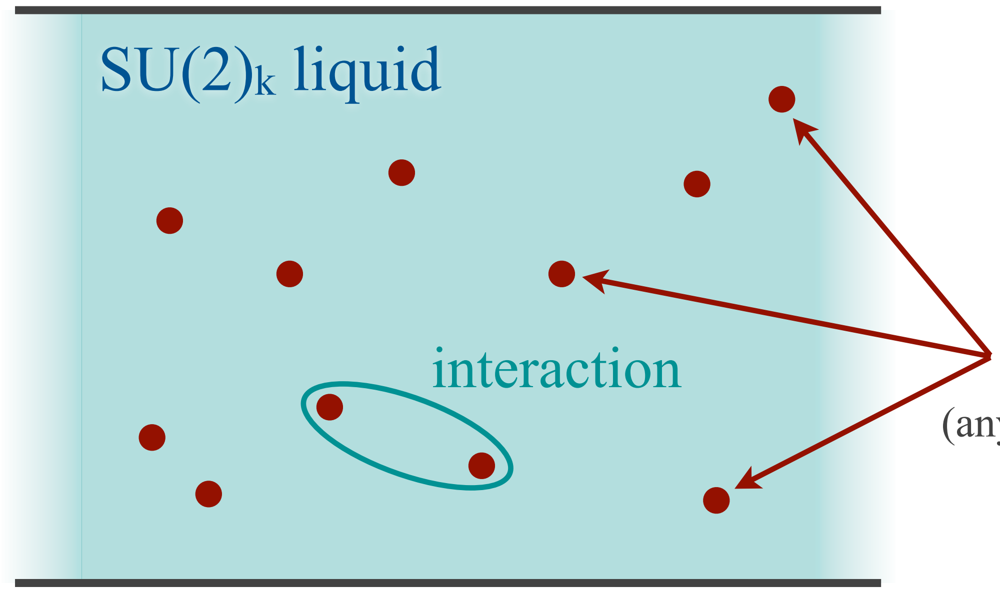


The collective state of anyons is a **gapped quantum liquid**.

A **novel liquid is nucleated** within the parent liquid.

A soup of non-Abelian anyons

Phys. Rev. Lett. **98**, 160409 (2007).



finite density of anyons
(anyons are at fixed positions or '**pinned**')

$SU(2)_k$ fusion rules

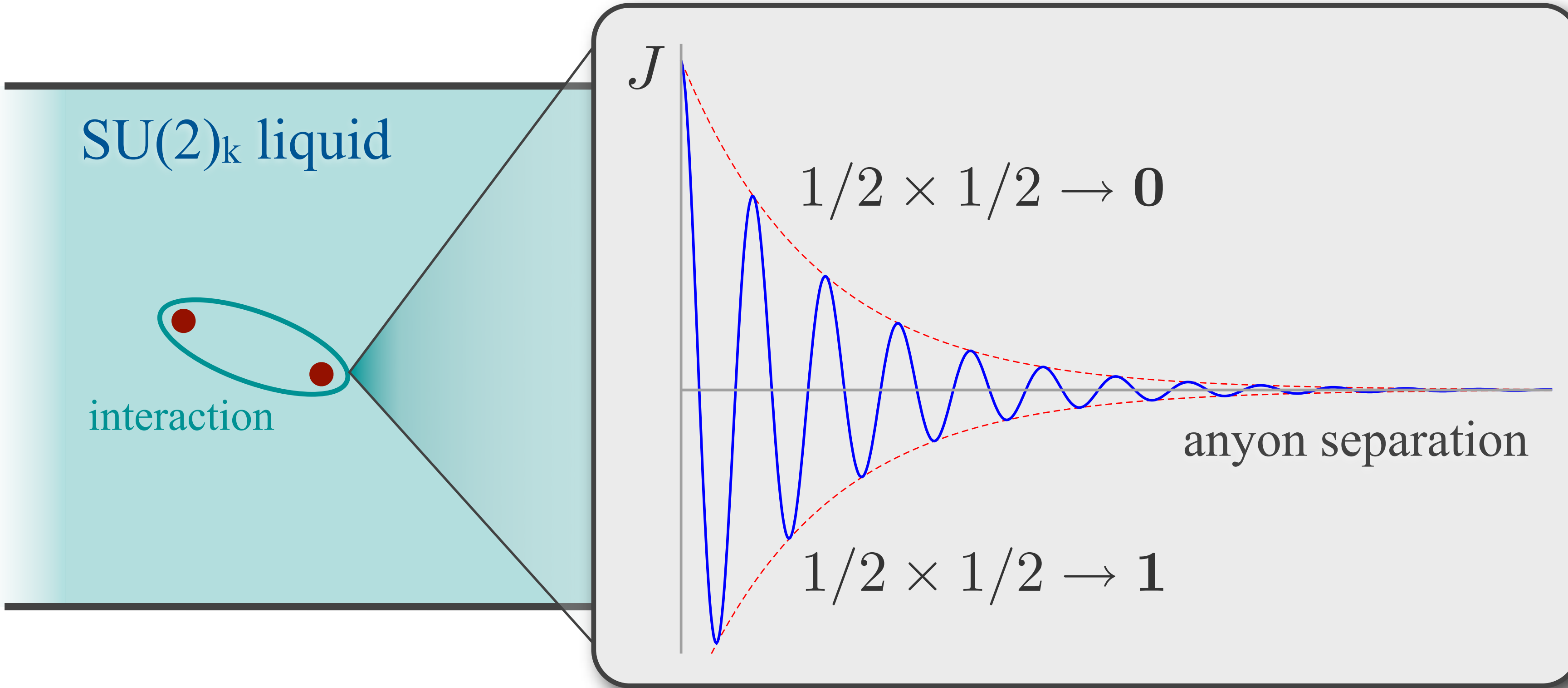
$$\frac{1}{2} \times \frac{1}{2} = 0 + 1$$

$\longrightarrow$ energetically split multiple fusion outcomes

"Heisenberg" Hamiltonian

$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

Microscopic splitting



$SU(2)_k$ fusion rules

$$\frac{1}{2} \times \frac{1}{2} = 0 + 1$$

energetically split
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Anyonic Heisenberg model

SU(2)$_k$ fusion rules

$$\frac{1}{2} \times \frac{1}{2} = 0 + 1$$

→

energetically split
multiple fusion outcomes

"Heisenberg" Hamiltonian

$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

Which fusion channel is favored? – Non-universal

p-wave superconductor
M. Cheng *et al.*, PRL (2009)

$1/2 \times 1/2 \rightarrow \mathbf{0}$
short distances, then oscillates

Moore-Read state
M. Baraban *et al.*, PRL (2009)

$1/2 \times 1/2 \rightarrow \mathbf{1}$
short distances, then oscillates

Kitaev's honeycomb model
V. Lahtinen *et al.*, Ann. Phys. (2008)

$1/2 \times 1/2 \rightarrow \mathbf{0}$

Connection to topological charge tunneling: P. Bonderson, PRL (2009)

Anyonic Heisenberg model

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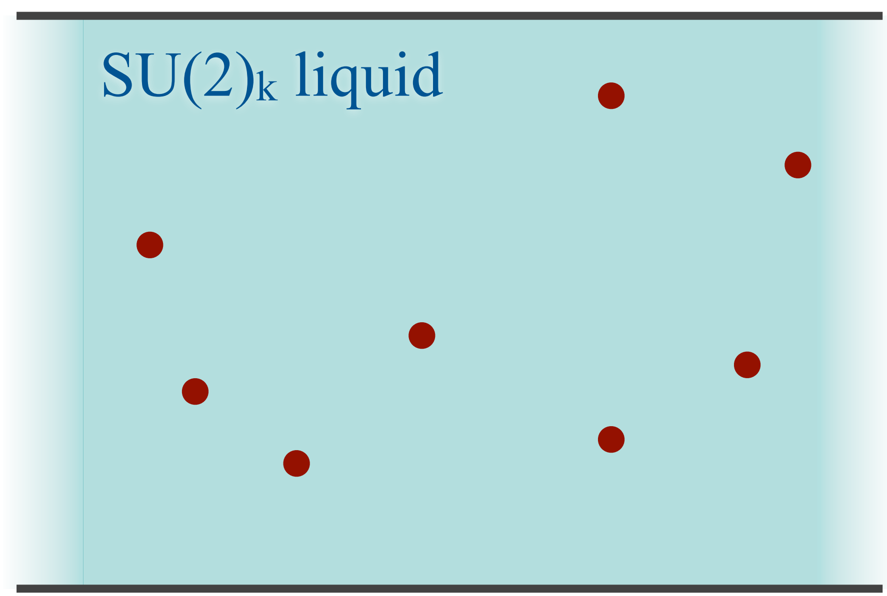

Anyonic Heisenberg model

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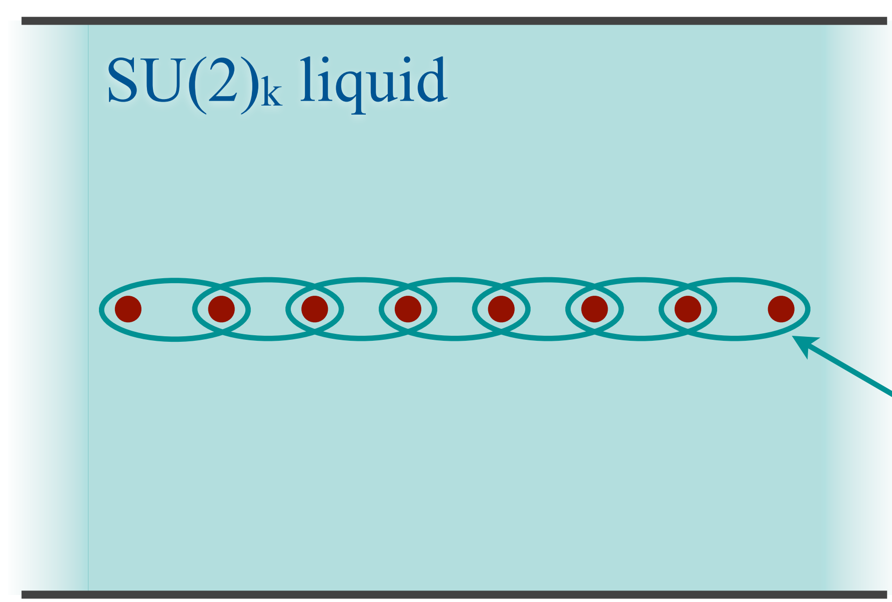


chain of anyons

‘golden chain’ for $SU(2)_3$

Anyonic Heisenberg model

Prog. Theor. Phys. Suppl. **176**, 384 (2008).

SU(2)$_k$ fusion rules

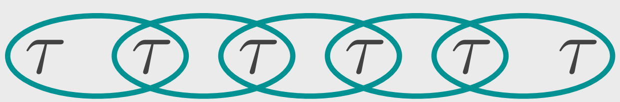
$$\frac{1}{2} \times \frac{1}{2} = 0 + 1$$

energetically split
multiple fusion outcomes

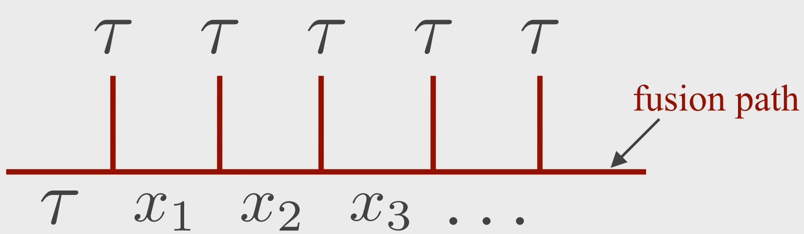
"Heisenberg" Hamiltonian

$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

Example: chains of anyons



$(\tau = 1/2)$



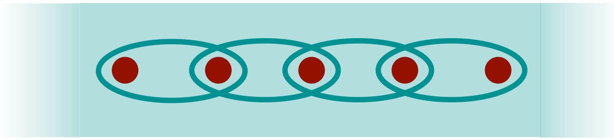
Hilbert space

$$|x_1, x_2, x_3, \ldots\rangle$$

Hamiltonian

$$H = \sum_i F_i \, \Pi_i^0 \, F_i$$

F-matrix = 6j-symbol



Critical ground state

Finite-size gap

$$\Delta(L) \propto (1/L)^{z=1}$$

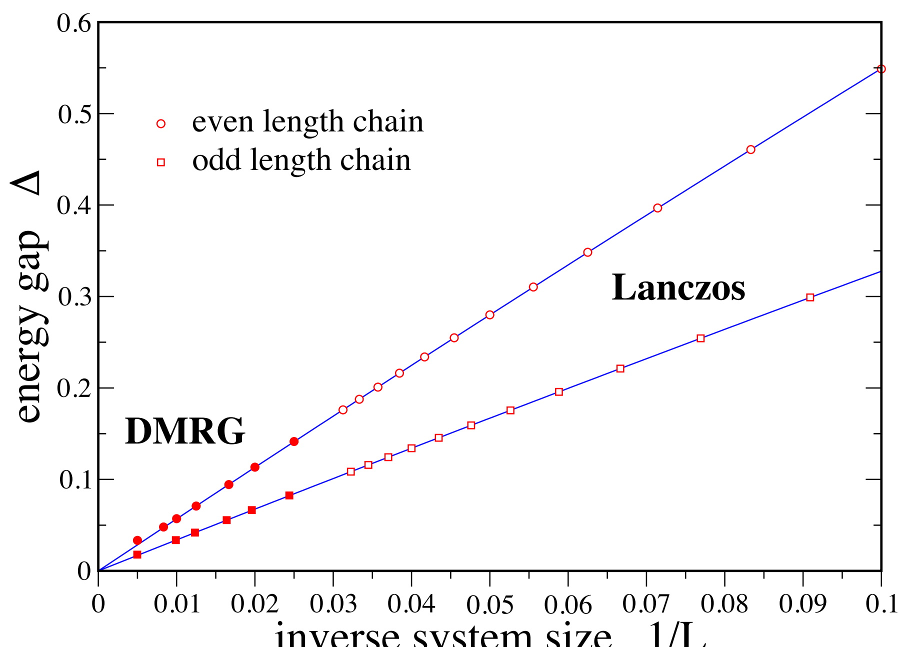
conformal field theory description

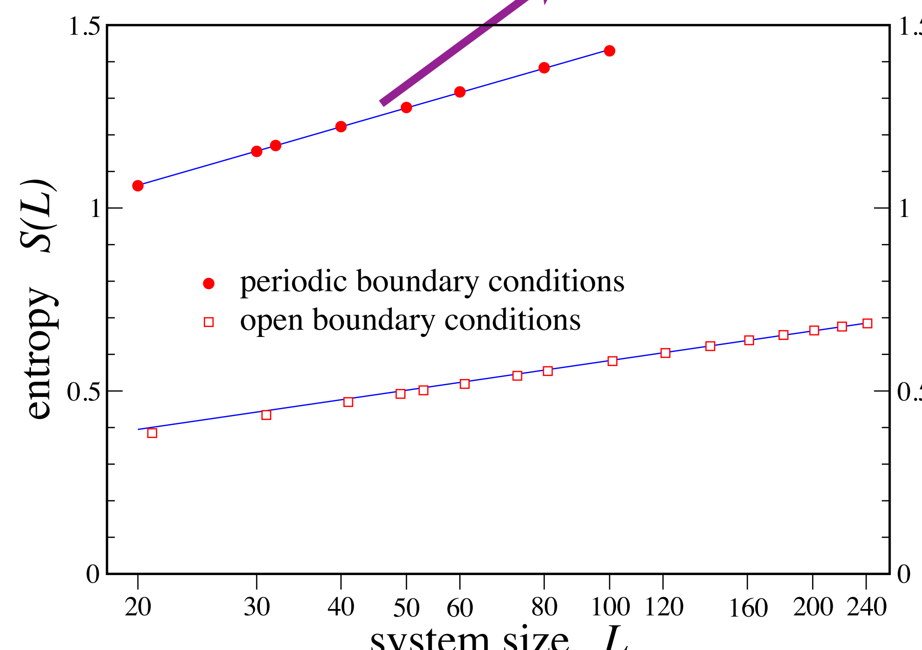
Entanglement entropy

$$S_{\mathrm{PBC}}(L) \propto \frac{c}{3} \log L$$

central charge

$c = 7/10$





Mapping & exact solution

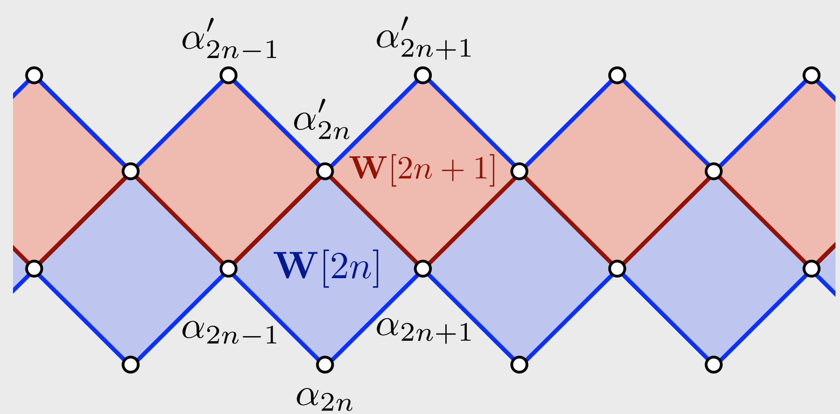
The operators $X_i = -d\,H_i$ form a representation of the **Temperley-Lieb algebra**

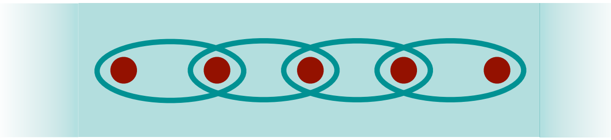
$$(X_i)^2 = d \cdot X_i \qquad X_i X_{i\pm 1} X_i = X_i \qquad [X_i, X_j] = 0$$

for $|i-j| \geq 2$

$$d = 2\cos\left(\frac{\pi}{k+2}\right)$$

The transfer matrix
is an integrable representation
of the RSOS model.



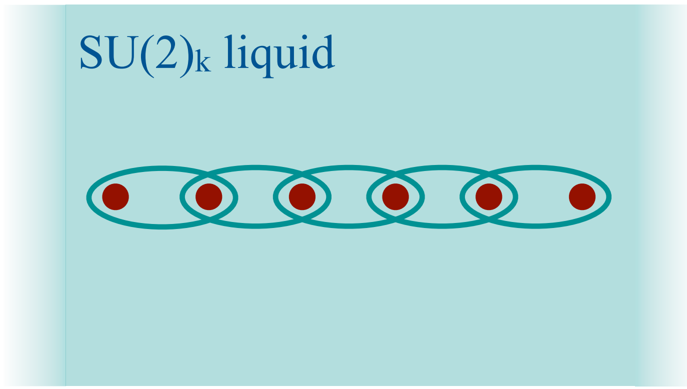


Deformed spin-1/2 chains

level k	$1/2 \times 1/2 \rightarrow \mathbf{0}$ 'antiferromagnetic'	$1/2 \times 1/2 \rightarrow \mathbf{1}$ 'ferromagnetic'
2	Ising c = 1/2	Ising c = 1/2
3	tricritical Ising c = 7/10	3-state Potts c = 4/5
4	$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$	$\frac{SU(2)_k}{U(1)}$
5		
k	**k-critical Ising** c = 1-6/(k+1)(k+2)	**Z_k-parafermions** c = 2(k-1)/(k+2)
∞	Heisenberg AFM c = 1	Heisenberg FM c = 2

Gapless modes & edge states

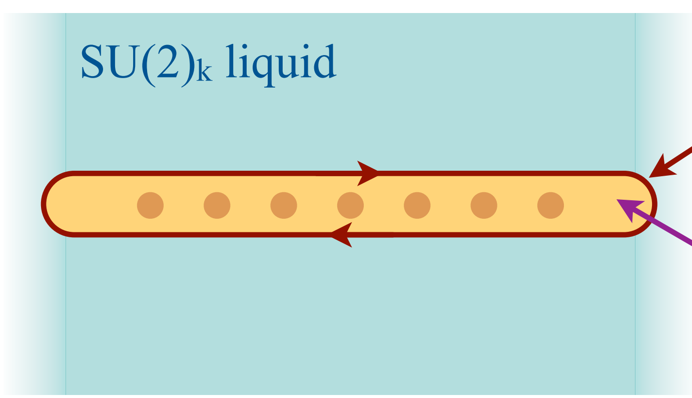
Phys. Rev. Lett. **103**, 070401 (2009).



critical theory
(AFM couplings)

$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

finite density
interactions



gapless modes = edge states

$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

nucleated liquid

$$SU(2)_{k-1} \times SU(2)_1$$

Gapless modes & edge states

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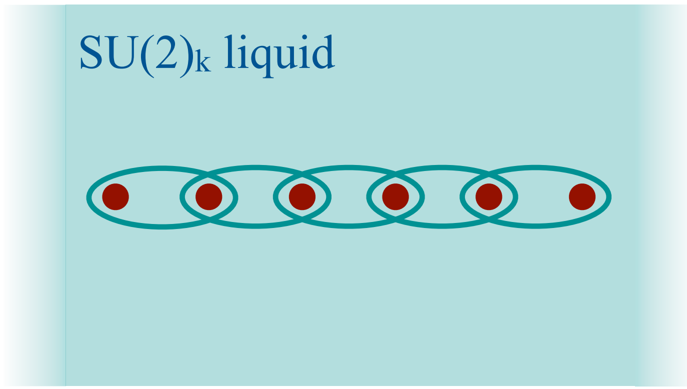
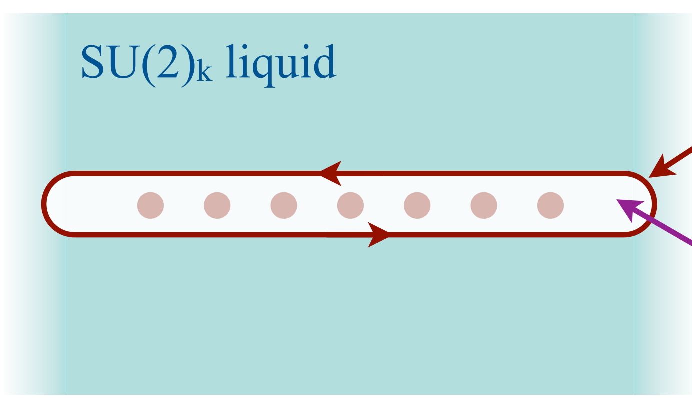


critical theory
(FM couplings)

$$\frac{SU(2)_k}{U(1)}$$

finite density
interactions

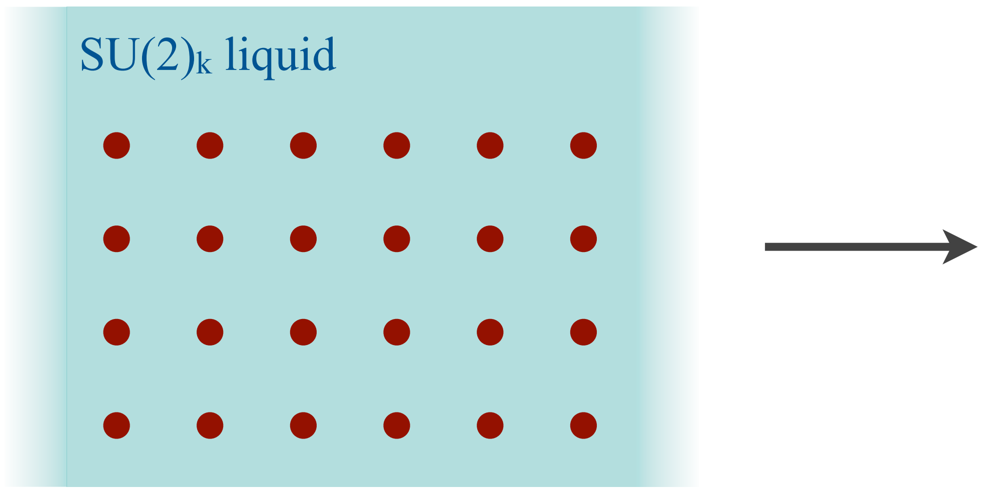


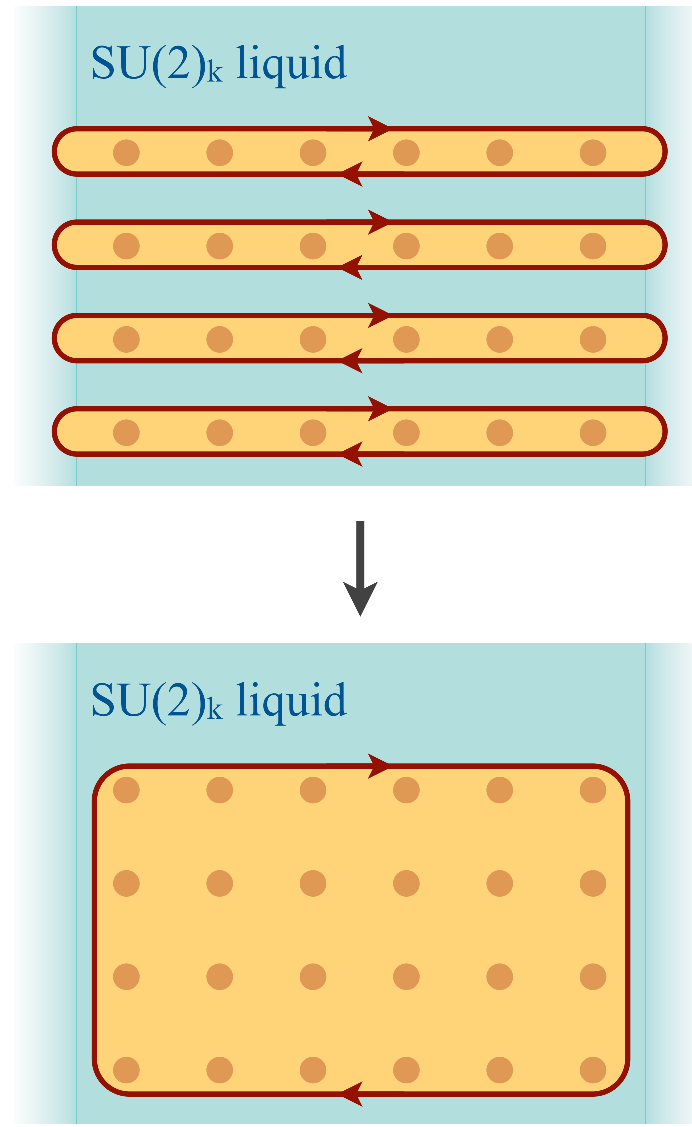
gapless modes = edge states

$$\frac{SU(2)_k}{U(1)}$$

nucleated liquid $U(1)$
(Abelian)

Approaching two dimensions



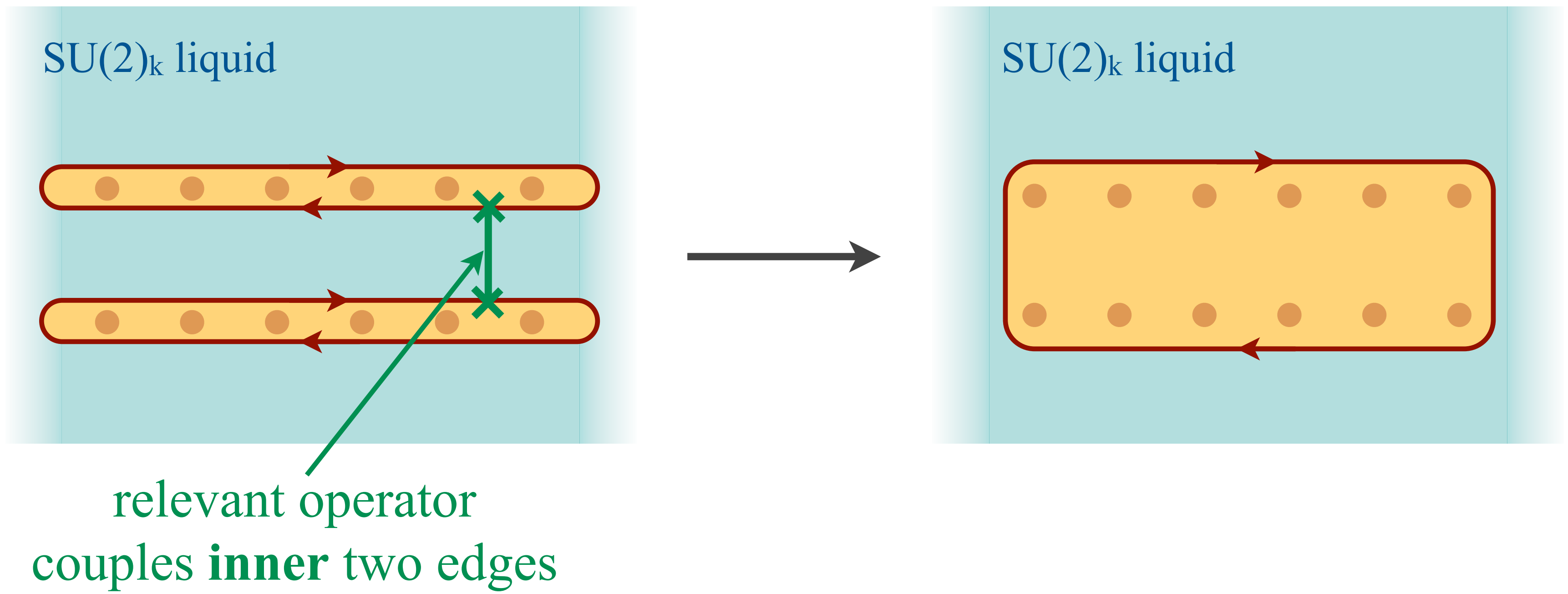


The 2D collective state

A **gapped topological liquid**
that is distinct from the parent liquid.

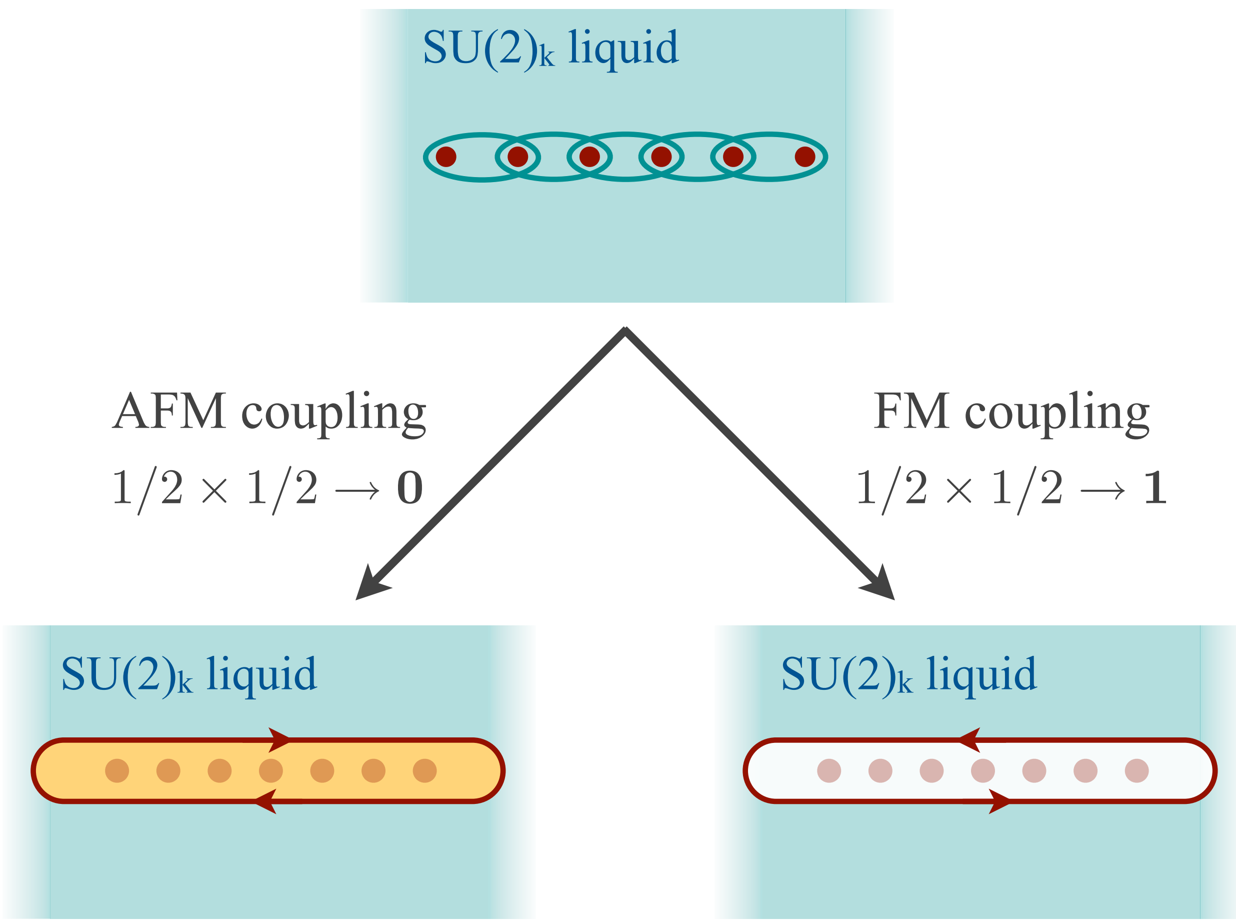
Supported by results for
N-leg ladders.

Coupling two chains



A powerful correspondence

Phys. Rev. Lett. **103**, 070401 (2009).



nucleated liquid
$SU(2)_{k-1} \times SU(2)_1$

nucleated liquid
$U(1)$

collective states
of anyonic spin chains

↓

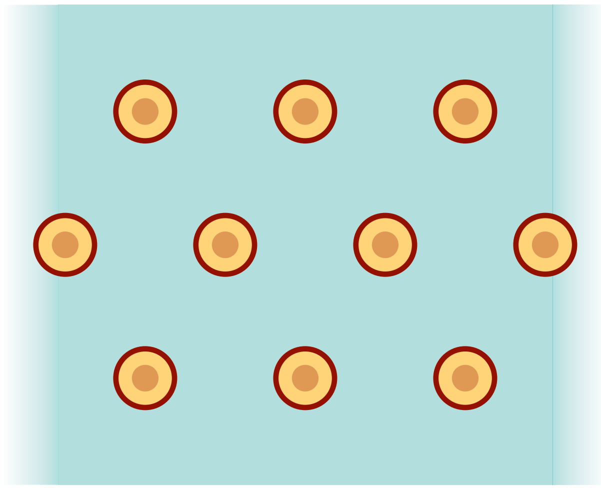
edge states
of topological liquids

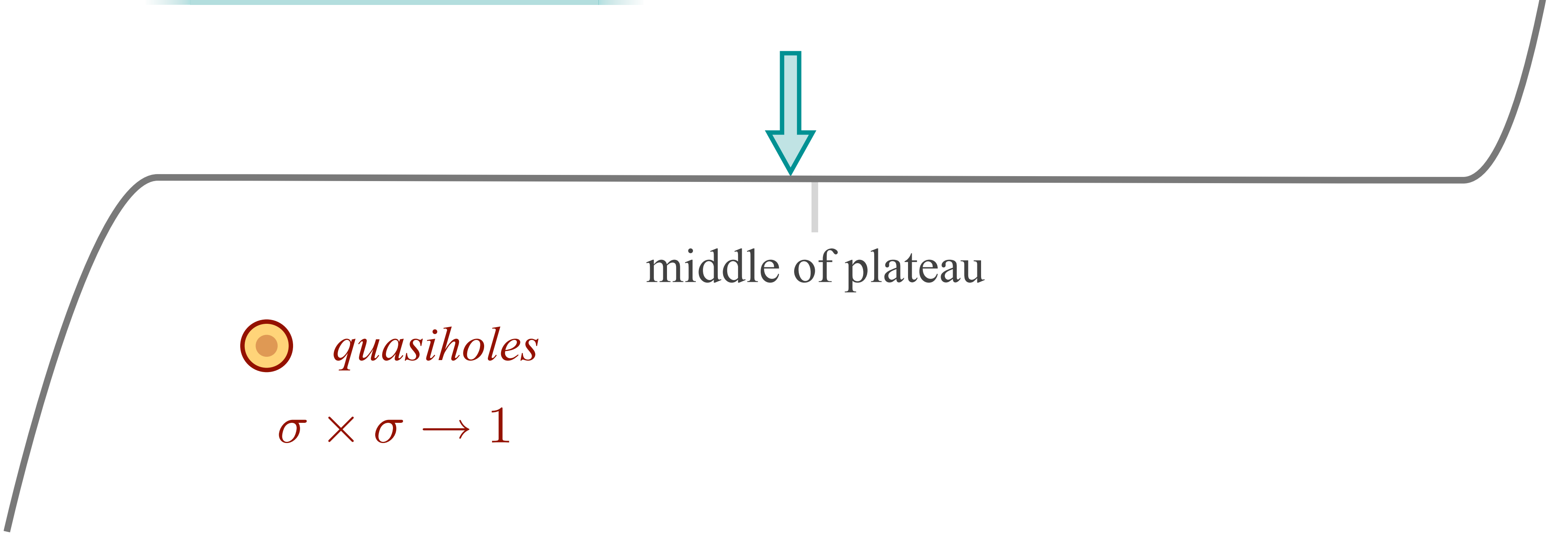
↓

nucleation of novel
topological liquids

Quantum Hall plateaus

$a \gg \xi_m$



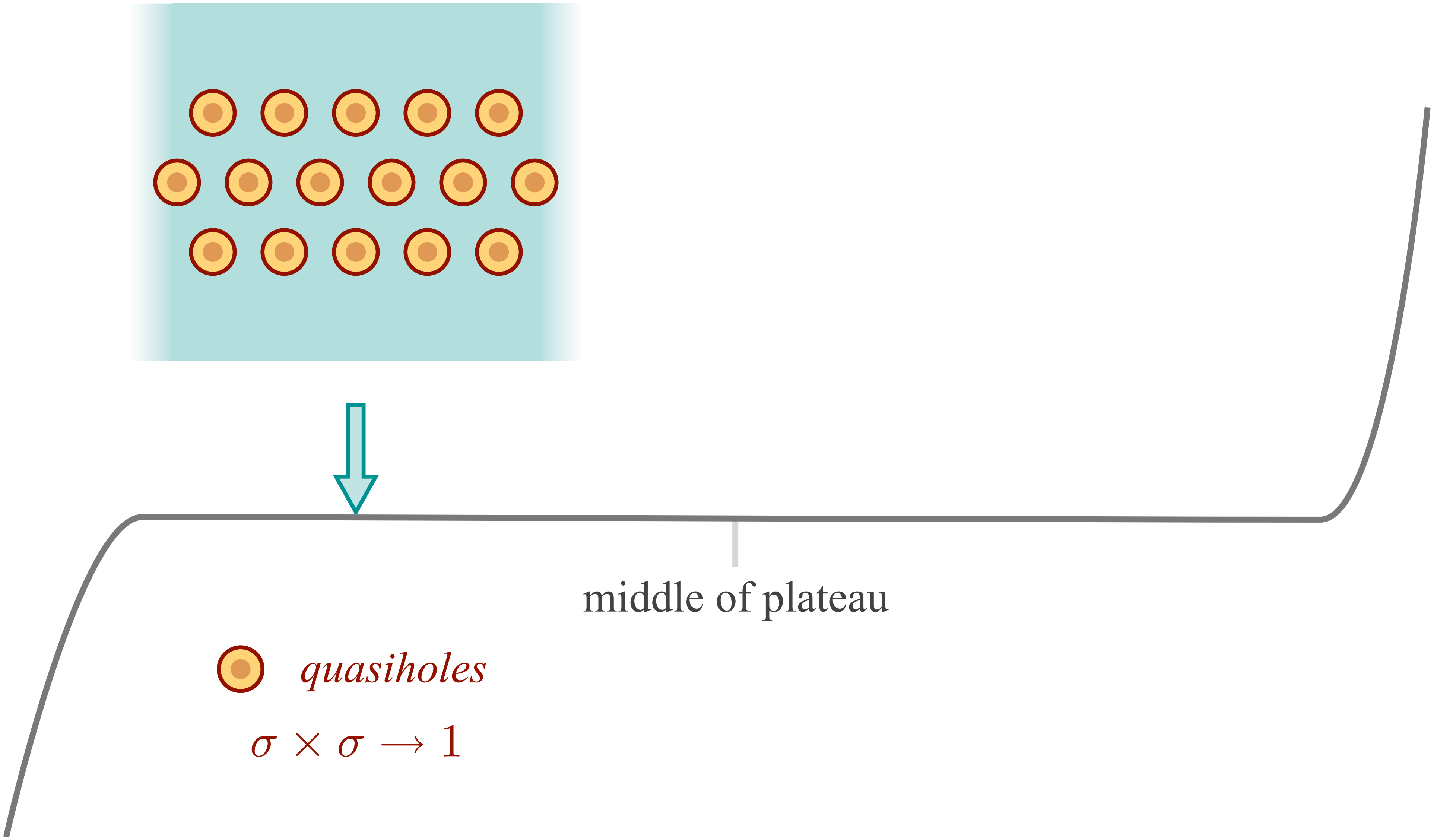


quasiholes

$\sigma \times \sigma \to 1$

Quantum Hall plateaus

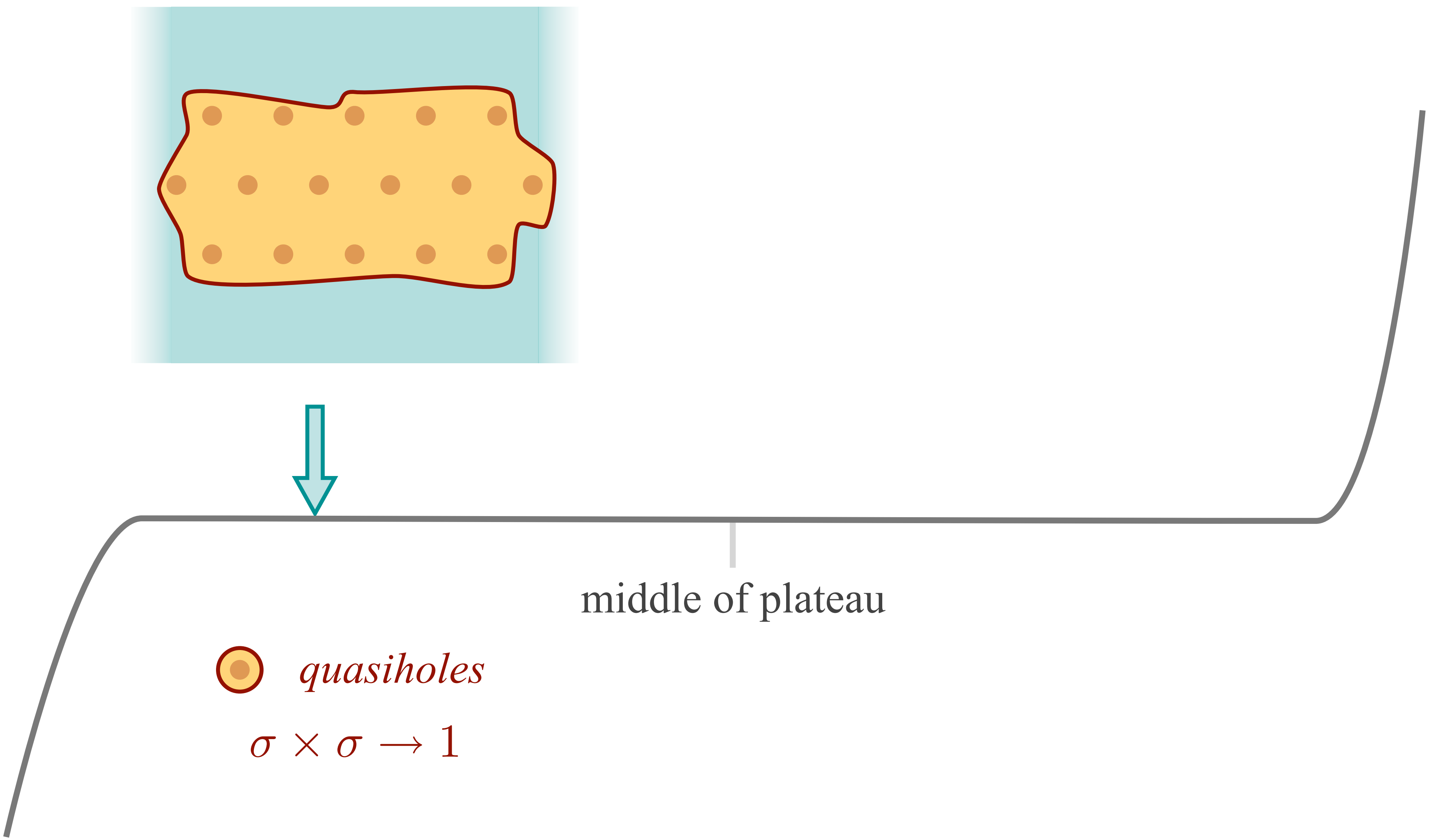
$a \approx \xi_m$



$\sigma \times \sigma \to 1$

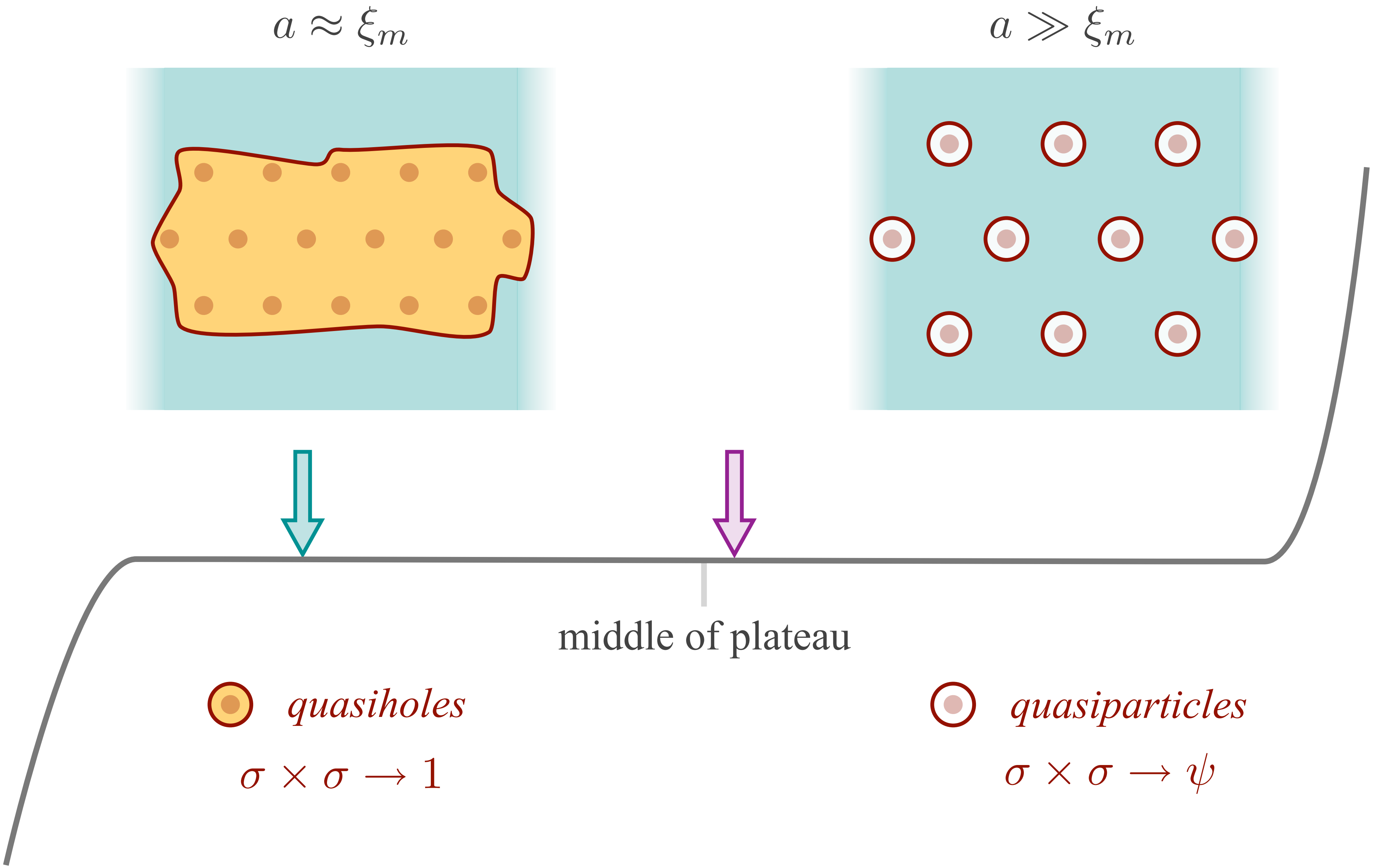
Quantum Hall plateaus

$a \approx \xi_m$



$\sigma \times \sigma \rightarrow 1$

Quantum Hall plateaus



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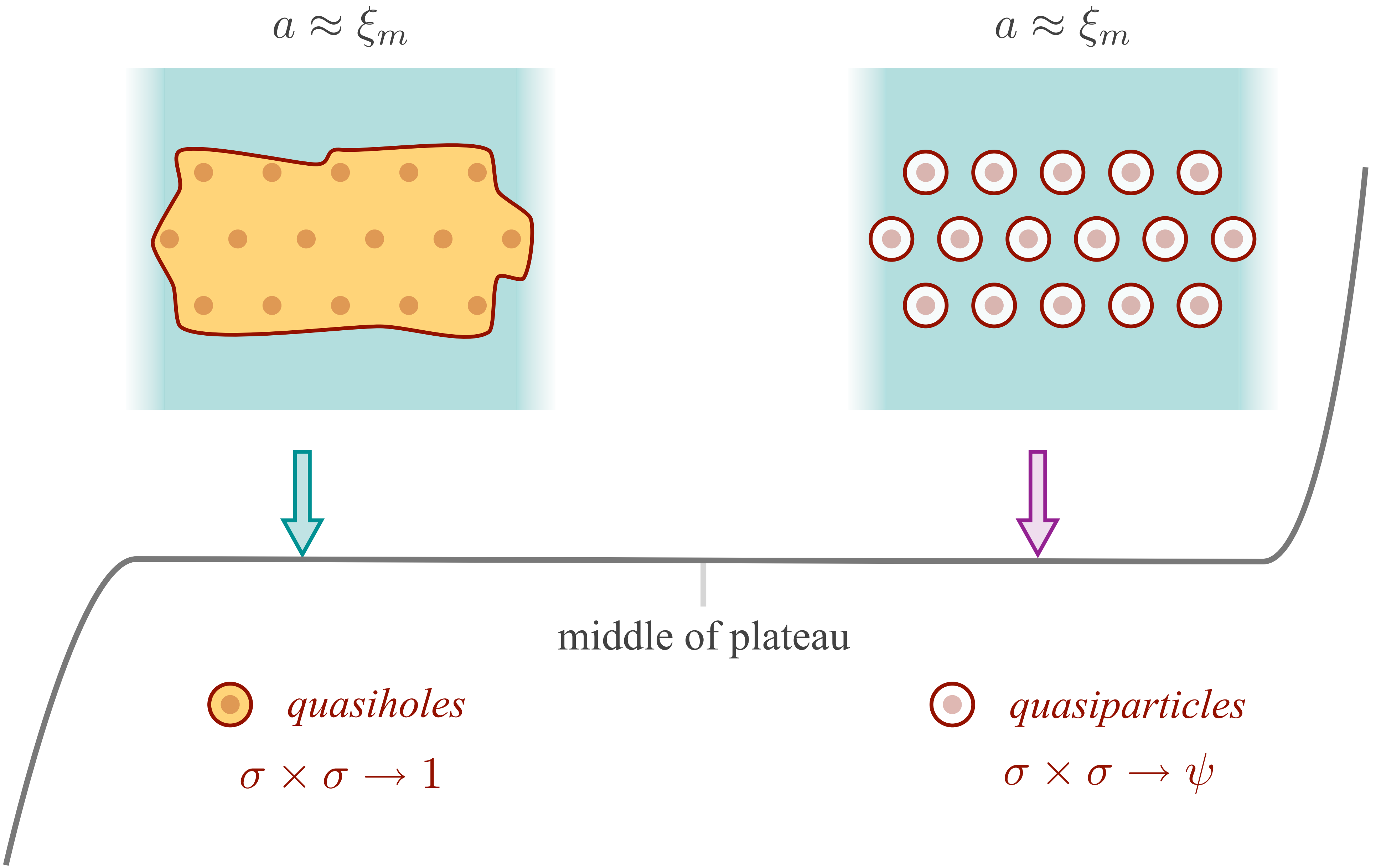
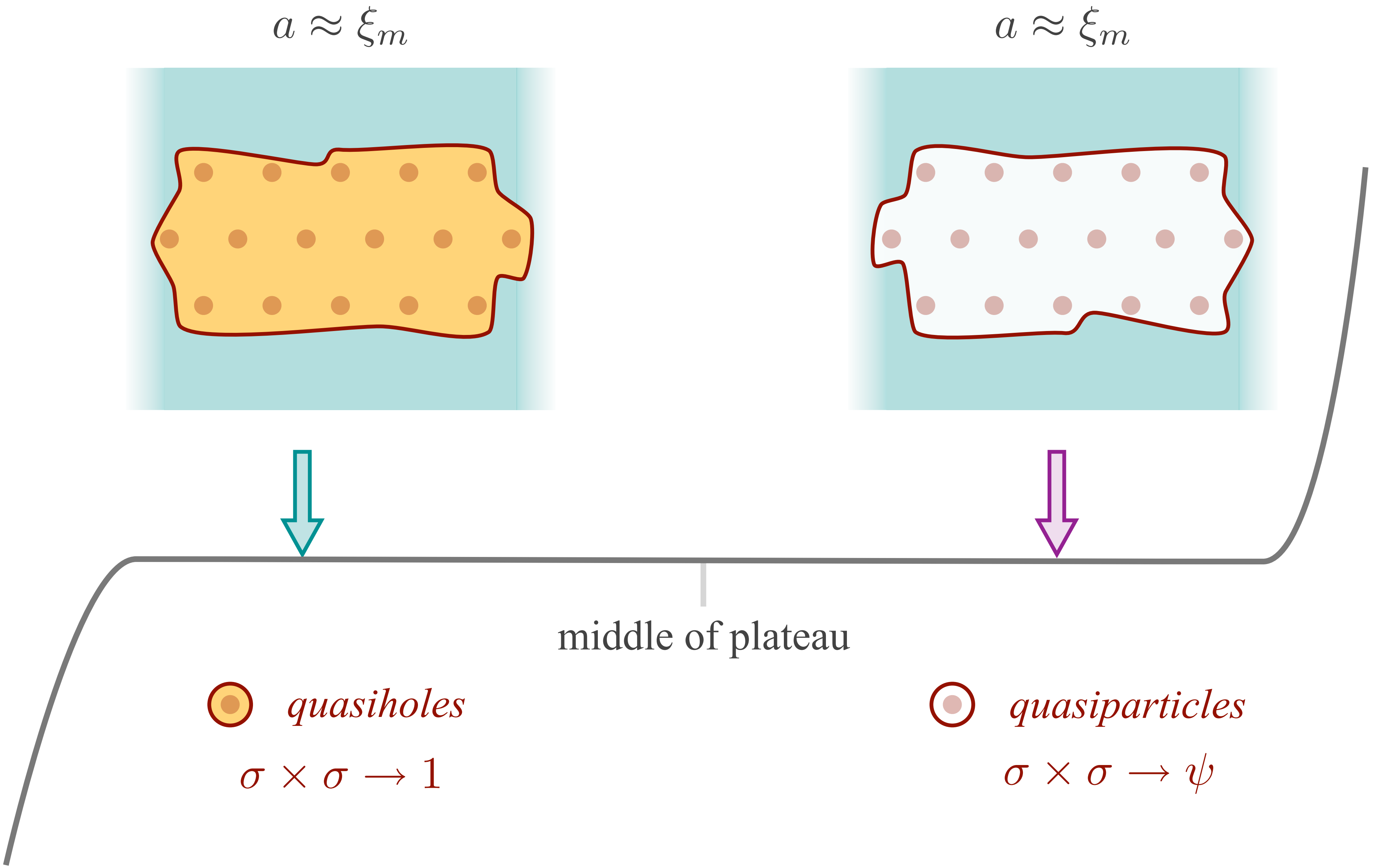
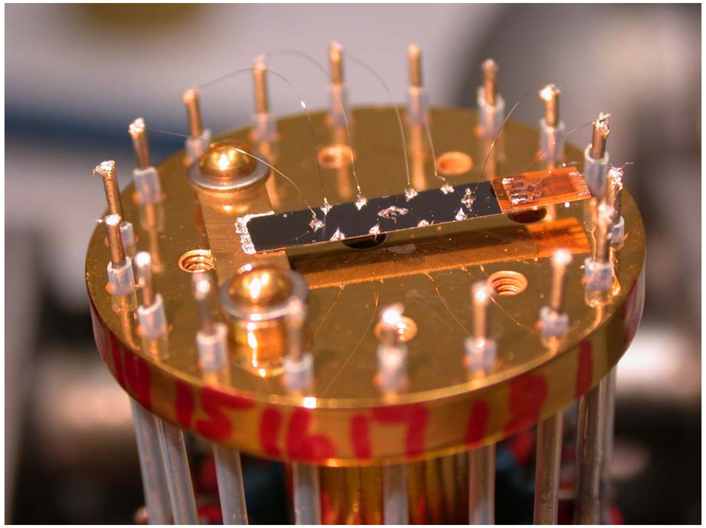


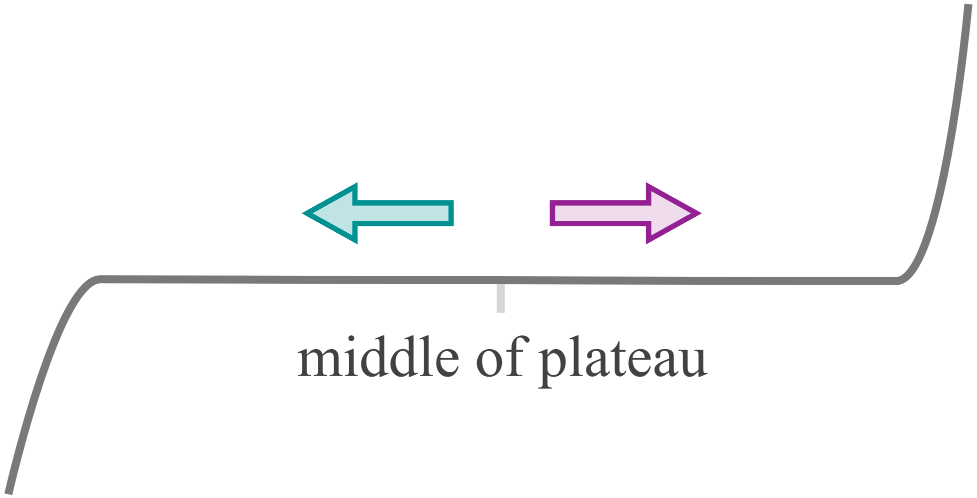
Quantum Hall plateaus



Experimental consequences



Caltech thermopower experiment

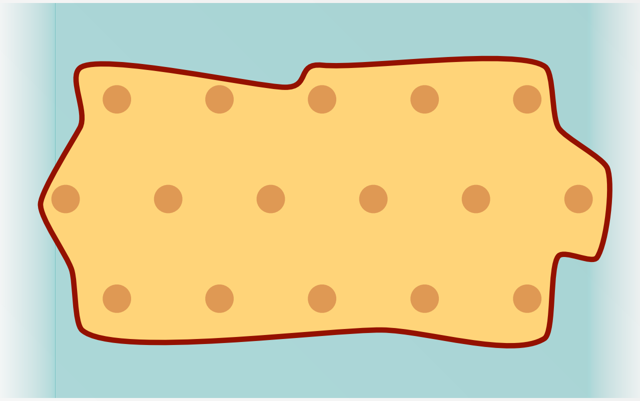


What changes (experimentally) as we move on the plateau?

electrical transport	**unchanged** – remain on the plateau
heat transport (neutral modes)	**changes** – evidence of the new liquid

Conclusions

- **Interactions split the degeneracy** of a set of localized, non-Abelian anyons.
- For a given topological liquid **a finite density of interacting anyons nucleates a new topological liquid**.
- The nucleated liquid is separated from the parent liquid by a **neutral, chiral edge state**.
- Relevant physics when moving **off the center of quantum Hall plateau**.



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