Shock formation in the periodic exclusion process with asymmetric coupling

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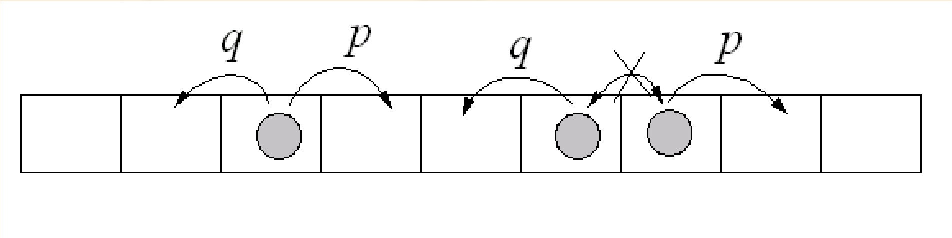
Outline

- Introduction
- Model
- Strong Coupling
- Weak Coupling
- Conclusion

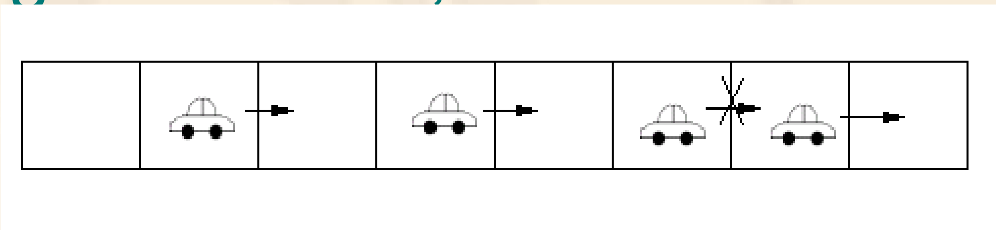
Introduction

- Asymmetric exclusion process (ASEP) describes particles hopping with hard-core repulsion along a 1D lattice.

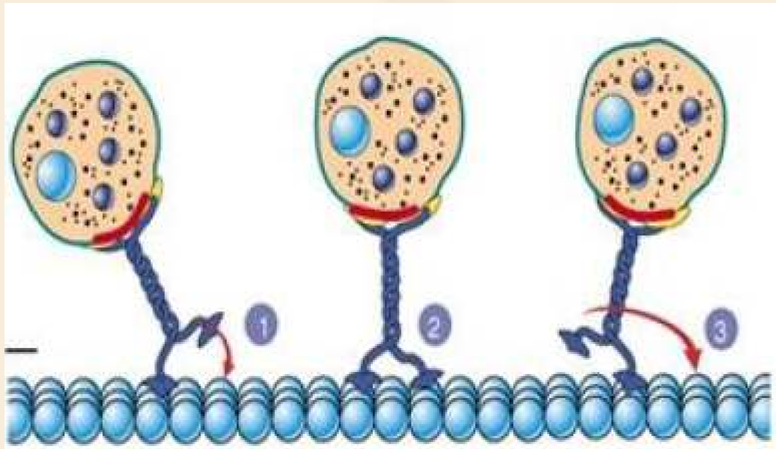


- $p = q \neq 0$: symmetric exclusion process (SEP)
- $p \neq q \neq 0$: partial asymmetric exclusion process (PASEP)
- p or q =0: totally asymmetric exclusion process (TASEP)

◆Exclusion process provides a good description of traffic flow, the kinetics of biopolymerization, polymer dynamics in dense media, diffusion through membrane channels, dynamics of motor proteins moving along rigid filaments, etc



Vehicles moving on road



Molecular motors moving on Filament

- ASEP could be solved analytically, sometimes exactly, under certain circumstances. (Mean Field, Matrix Product, Bethe Ansatz)

- Despite their simplicity, ASEP and related models show a range of nontrivial macroscopic phenomena, such as boundary induced phase transitions, spontaneous symmetry breaking, phase separation, localized shock, etc.

Boltzmann Medal

The **Boltzmann Award** is presented by the C3 Commission on Statistical Physics of the IUPAP every three years, at the Statphys Conference. The award, consisting of a gilded medal, honours outstanding achievements in Statistical Physics. The recipient is a scientist who has not received the Boltzmann Medal or Nobel Prize before.

The C3 commission is pleased to announce that the Boltzmann Medal for 2010 will be awarded during StatPhys24 to

John Cardy, *for his numerous seminal contributions to two-dimensional critical phenomena in statistical physics, including the development and application of conformal field theory, finite-size effects and percolation.*

Bernard Derrida, *for his major contributions to the understanding of disordered and of out-of-equilibrium systems, in particular through the random energy model, and through his breakthroughs in the asymmetric exclusion model.*

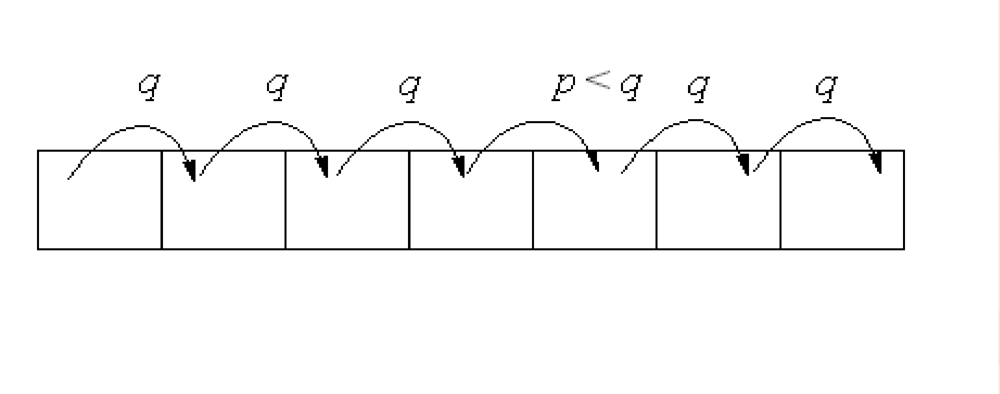

John Cardy


Bernard Derrida

- “Our interest in the ASEP lies in its having acquired the status of a fundamental model of nonequilibrium statistical physics in its own right in much the same way that the Ising model has become a paradigm for equilibrium critical phenomena ”

Blythe RA, Evans MR, Nonequilibrium steady states of matrix-product form: a solver's guide. *J.Phys.A* 40, R333-R441 (2007)

- ◆ The shock appearing in ASEP is an interesting phenomenon.
- ➢ Shock induced by sitewise or particlewise disorder (defect)



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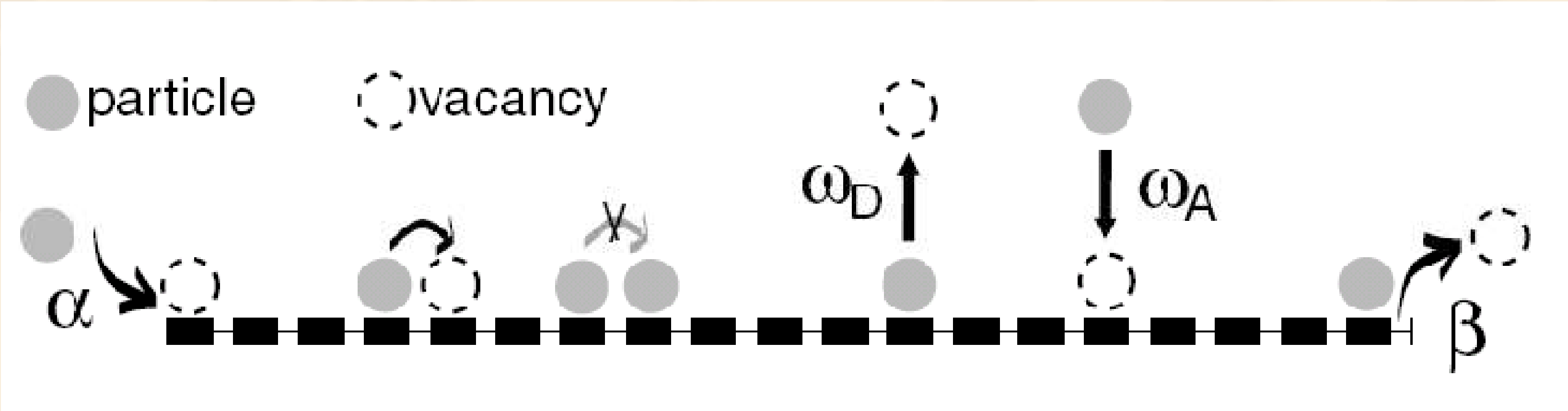
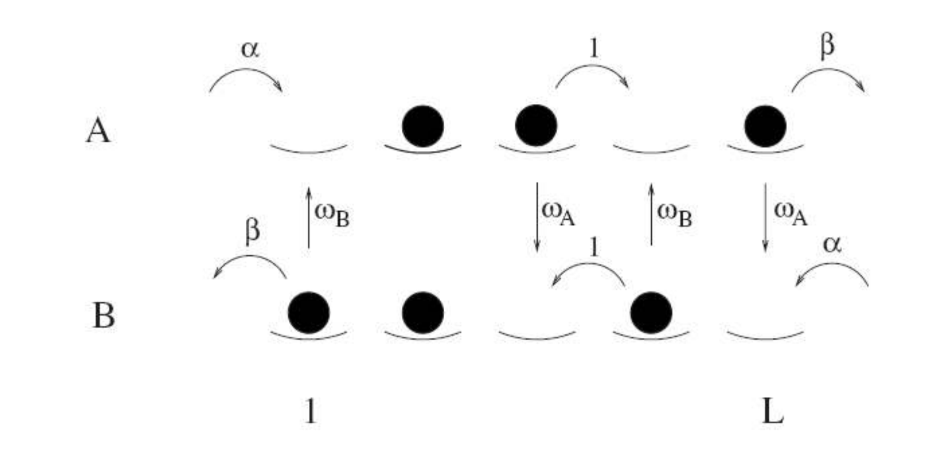
- G.Tripathy, M.Barma, Phys. Rev. lett. 78, 3039 (1997).
- T. Chou and G. Lakatos, Phys. Rev. lett. 93, 198101 (2004).
- A.B. Kolomeisky, J. Phys. A 31, 1153 (1998)

- Shock induced by particle attachment and detachment in a mesoscopic scaling

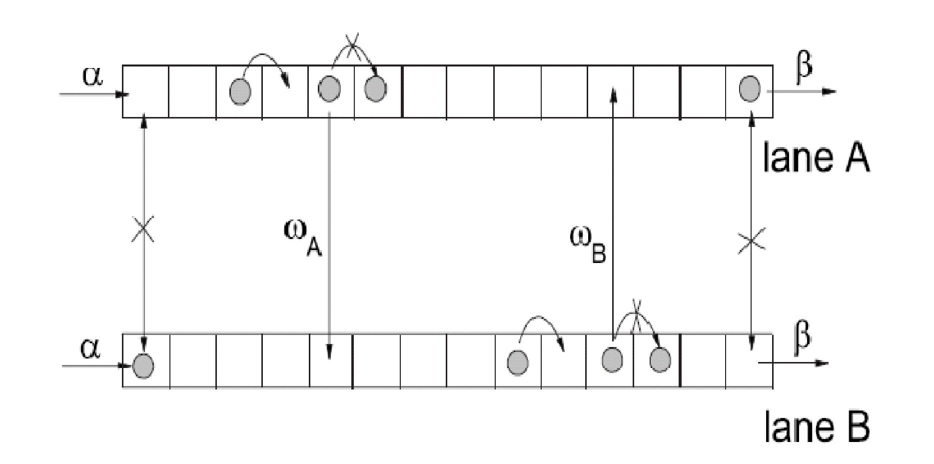


A. Parmeggiani, T. Franosch, and E. Frey, Phys. Rev. Lett. 90, 086601 (2003)

➢ Shock in two-lane and multi-lane ASEP

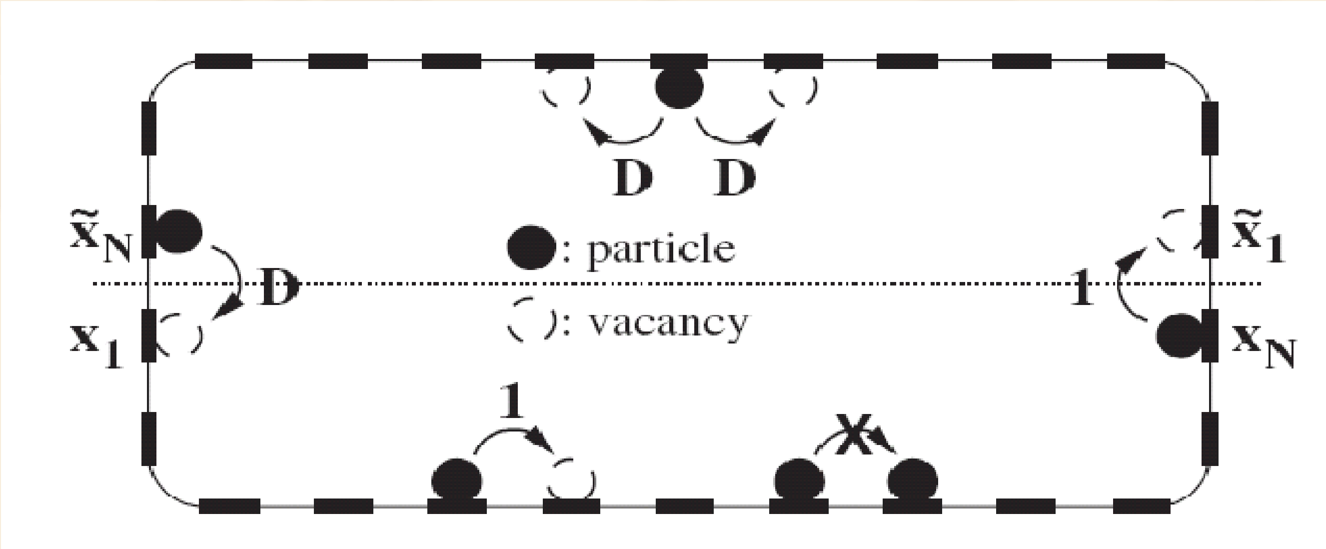


R. Juhász, Phys. Rev. E 76, 021117 (2007).



R.Jiang, M.B.Hu, Y.H.Wu et al., Phys. Rev. E 77, 041128 (2008)

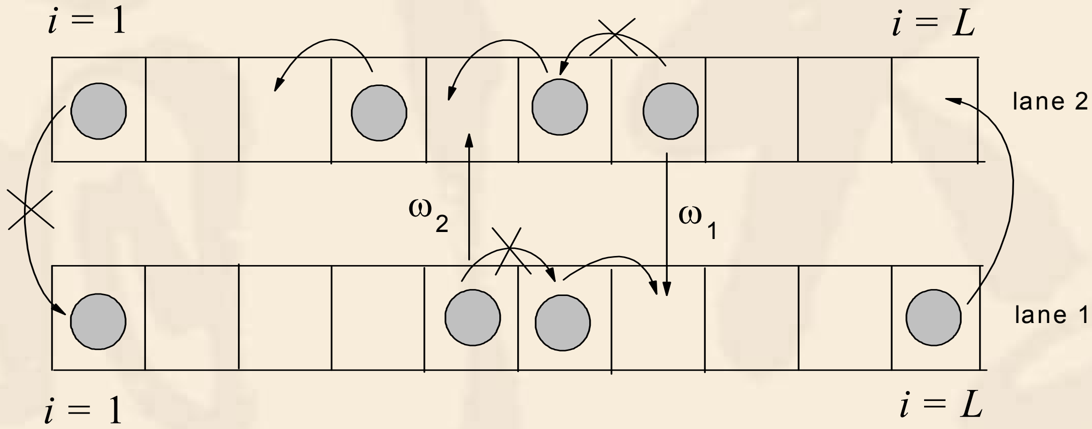
- Shock in a periodic one-dimensional exclusion process composed of a driven and a diffusive part



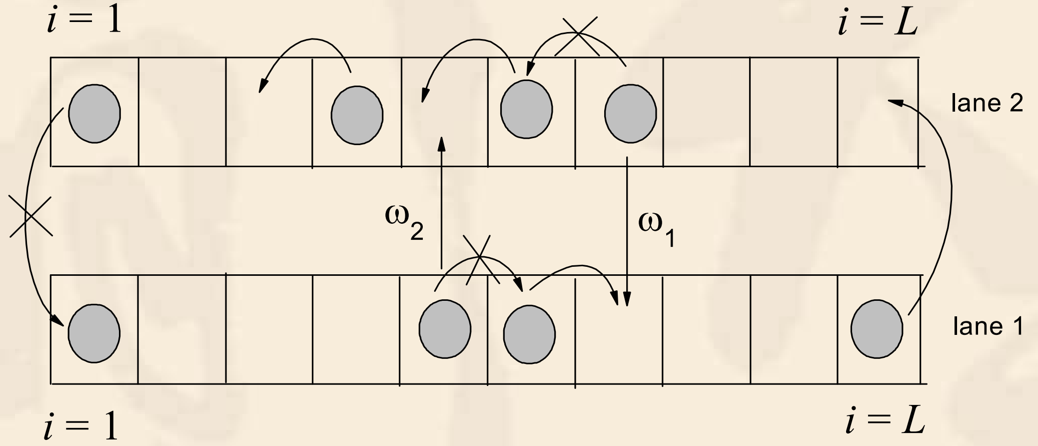
H. Hinsch and E. Frey, Phys. Rev. Lett. 97, 095701 (2006)

- Motivated by the above mentioned works, we study a periodic asymmetric exclusion process consisting of two ASEP parts, and particles are allowed to jump between the two parts.

Model



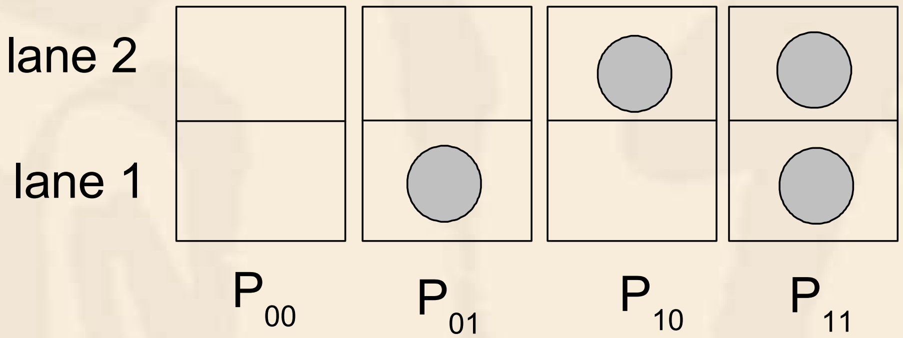
- The periodic ASEP is composed of two lanes with length *L*
- On lane 1 (2), the particles move from left (right) to right (left)
- The particles move forward with rate 1 if the site in front is empty. Otherwise,they jump to the other lane with rates $\omega 1$ and $\omega 2$, respectively, provided the target site is empty



- At the end i = 1 (i = L) particles move from lane 2 to lane 1 (lane 1 to lane 2)
- In the model, the random sequential update is adopted.
- Thus, there are 3 parameters, $\omega 1$, $\omega 2$ and density $\rho = N/(2L)$. Here N is the total number of particles in the system.

Strong Coupling

- ω_1 and ω_2 are constants independent of the system size. We only consider the case of asymmertic coupling $\omega_1 \neq \omega_2$
- Considering the lattice sites far away from the two ends of the system, the occupation of vertical clusters is independent of the position. We implement a vertical cluster mean field analysis, which was firstly proposed by Pronina and Kolomeisky.



- *P*11 as the probability of finding a vertical cluster with both lattice sites filled
- *P*01 and *P*10 as the probabilities of having a half-empty vertical cluster with a particle in lane 1 and in lane 2
- *P*00 as the probability of having no particles at both lattice sites

- Through mean field analysis, we have got the three kind of solutions

Solution A, *P11* = *P00* , we have

$$\rho_1 + \rho_2 = 1$$

Solution B: *P*10 = *P*01 = *P*00 = 0, *P*11 = 1, which means that

$$\rho_1 = \rho_2 = 1$$

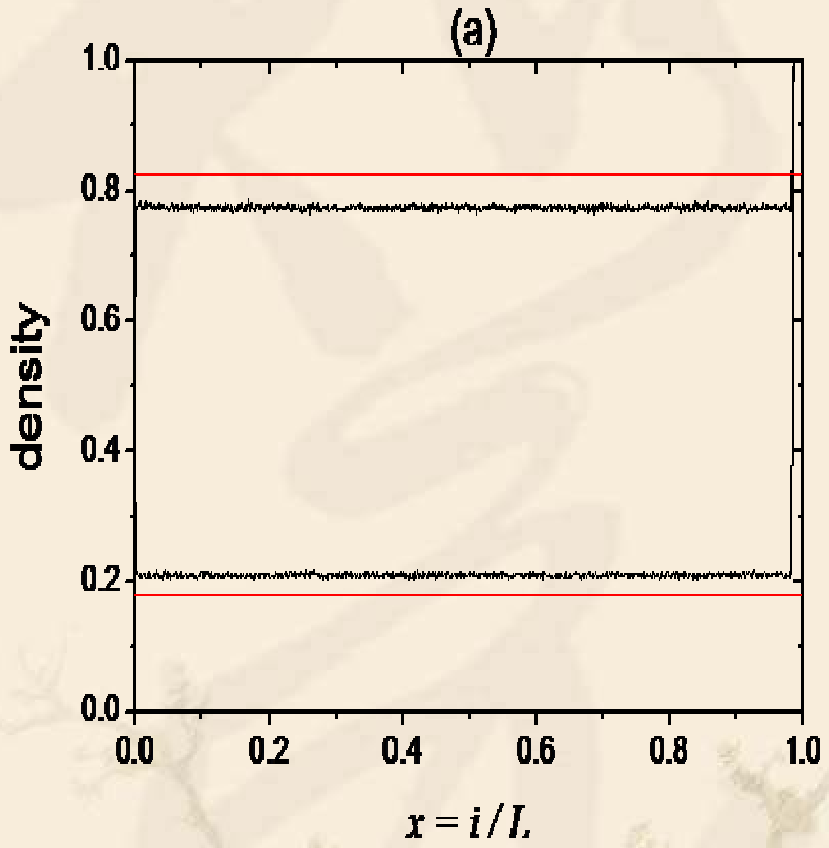
Solution C: *P*10 = *P*01 = *P*11 = 0, *P*00 = 1, which means that

$$\rho_1 = \rho_2 = 0$$

- So,Mean field analysis predicts that:

(a) When $\rho = \rho_C = 0.5$, the system is dominated by Solution A. Lanes 1 and 2 are in homogeneous state , and

$$\rho_1 + \rho_2 = 1$$

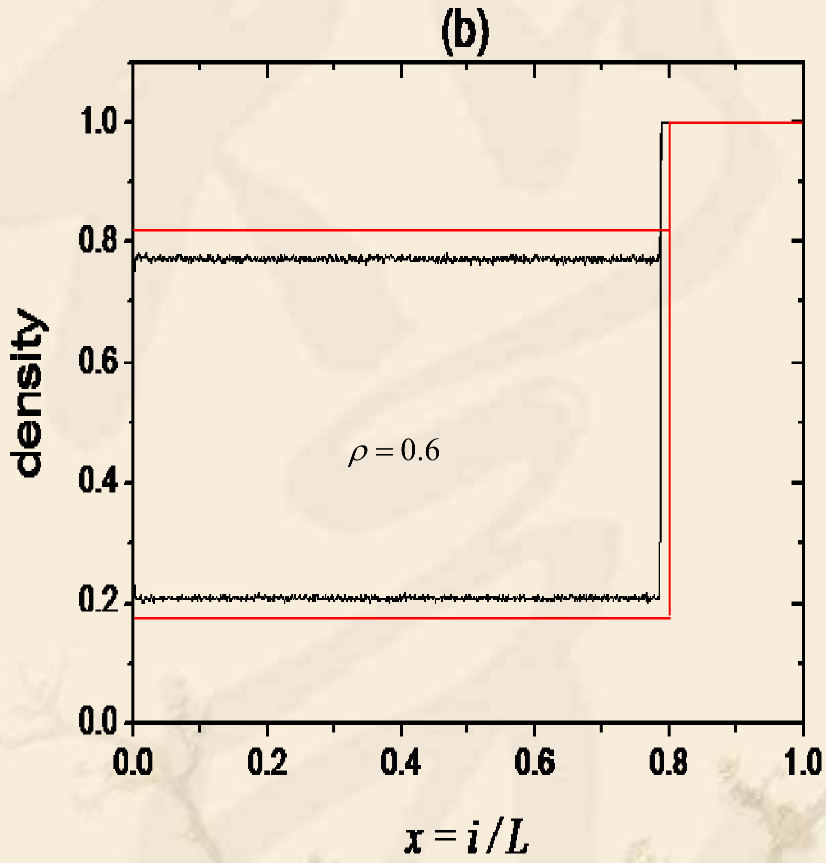


The red lines—the mean field prediction, the black lines— the simulation results.

(b) When $\rho > \rho_c$ ($\rho = 0.6$),

none of the three solutions can be maintained. So, a shock appears in the system.

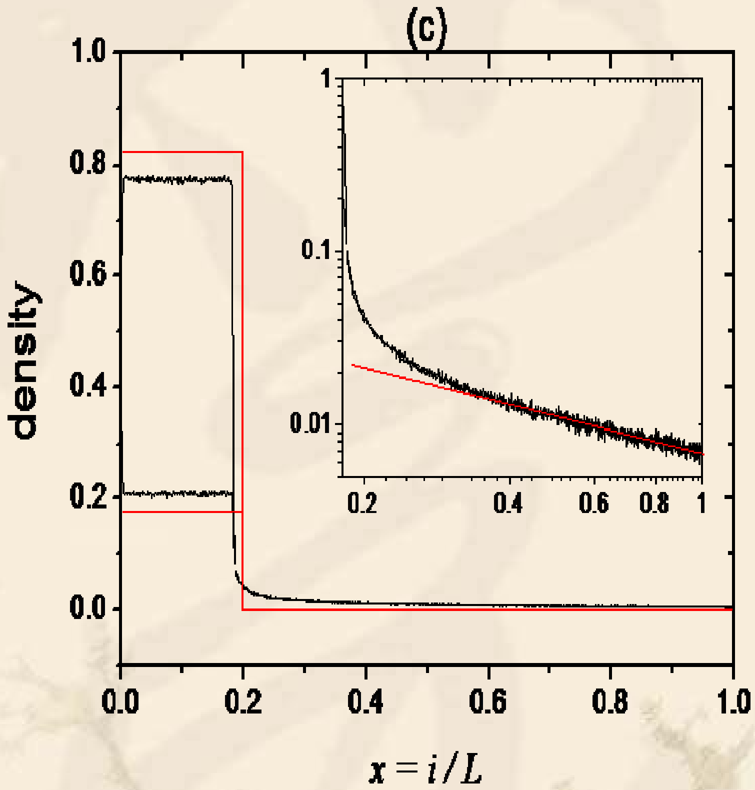
The system is dominated by Solution A left of the shock and is dominated by Solution B right of the shock.



The red lines—the mean field prediction, the black lines— the simulation results.

(c) When $\rho < \rho_c$($\rho = 0.1$),
the system is dominated by Solution A left of the shock and is dominated by Solution C right of the shock

With the decrease of density, the shock moves left



The red lines—the mean field prediction,
the black lines— the simulation results

Weak Coupling

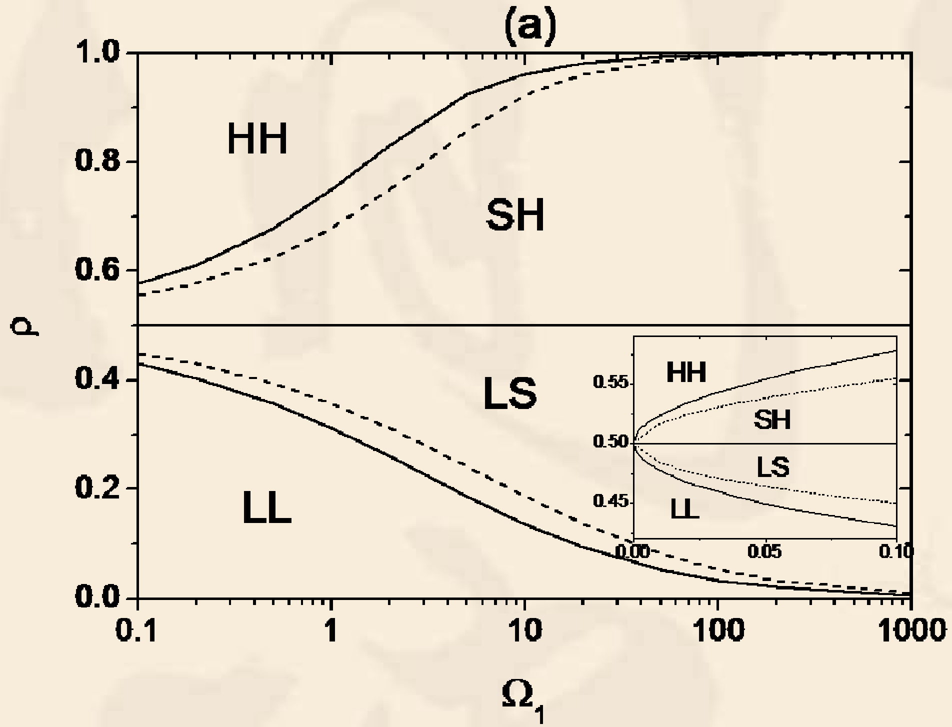
- ω1 and ω2 are inversely proportional to the system size *L*. To define rescaled jump rates $\Omega_1 = \omega_1 L$, $\Omega_2 = \omega_2 L$, $K = \Omega_2 / \Omega_1$ as proposed in previous works -------

Róbert, Juhász, PHYSICAL REVIEW E **76**, 021117 2007, Weakly coupled, antiparallel, totally asymmetric simple exclusion processes (with open boundaries, where the particles move in the two lanes in opposite directions and are allowed to jump to the other lane with rates inversely proportional to the length of the system)

- By using Monte Carlo simulations and Mean field analysis, the density profiles and phase structure of the model are analyzed.

- As shown by Róbert, Juhász under open boundary conditions, five phases could be identified, i.e., LL, LS, LH, SH, HH, but for our period boundary conditions there are some differences.

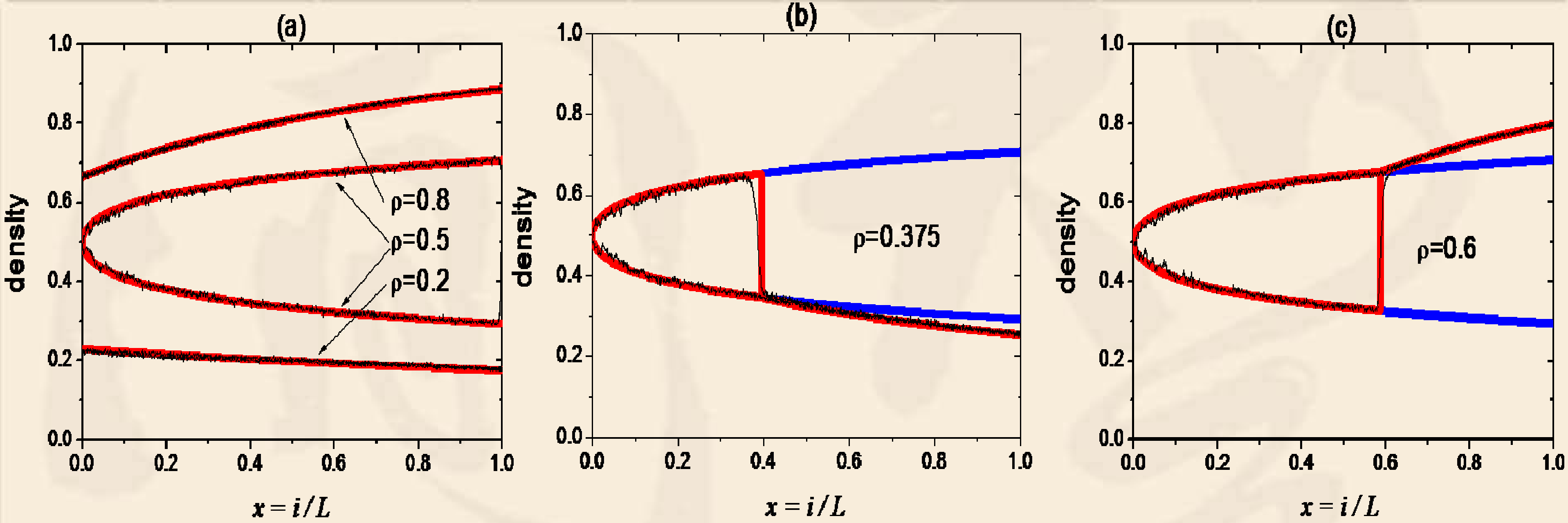
- LL: both lanes are in low density (LD);
- LS: one lane is in LD and shock appears on the other lane;
- LH: one lane is in LD, the other lane is in high density (HD)
- SH: one lane is in HD and shock appears on the other lane;
- HH: both lanes are in HD



Phase diagram of the system

***K* = 0 (solid lines), *K* = 0.5 (dashed lines)**

- For open B-C, each phase of LL, LS, LH, SH, HH occupies a 2D region
- For period B-C, the LH phase becomes a boundary line separating the LS phase and SH phase instead of occupying a 2D region.



Density profiles along lanes :

(a) LH (ρ = 0.5), LL (ρ = 0.2) and HH (ρ = 0.8); (b) LS; (c) SH.

The black lines— simulation results； The red lines—analytical results;

The blue lines— analytical results for LH for comparison.

The parameters—Ω1 = 1； Ω2 = 0； system size L = 10 000.

- Mean field analysis

$$(1-2\rho_1)\frac{\partial \rho_1}{\partial x}+\Omega_1\rho_1^2(1-\rho_2)-\Omega_2\rho_2^2(1-\rho_1)=0$$

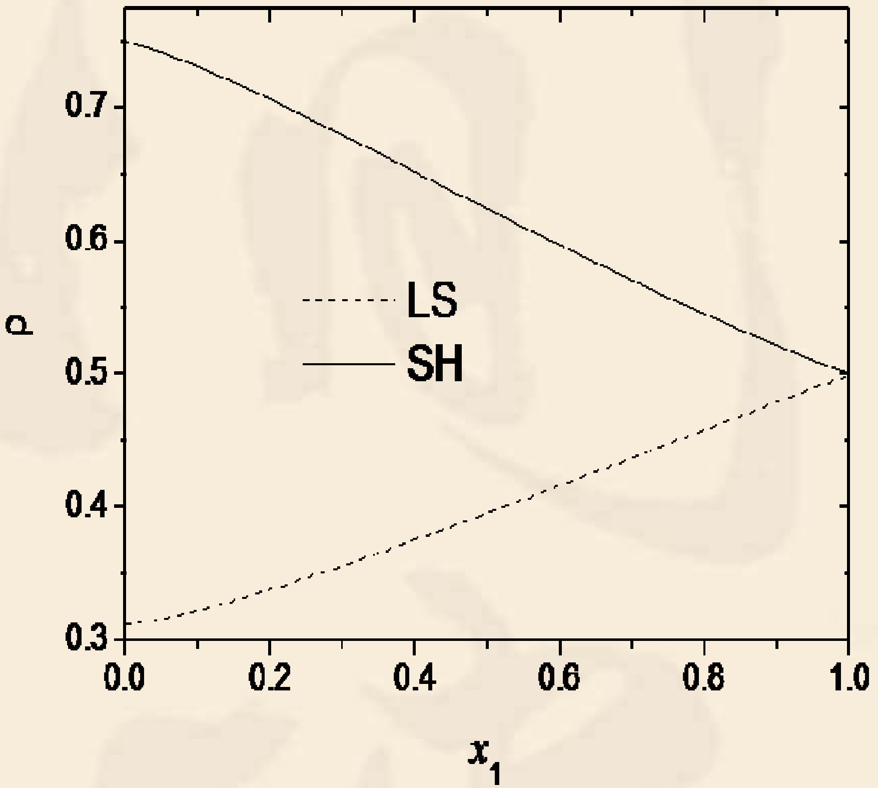
$$(2\rho_2-1)\frac{\partial \rho_2}{\partial x}-\Omega_1\rho_1^2(1-\rho_2)+\Omega_2\rho_2^2(1-\rho_1)=0$$

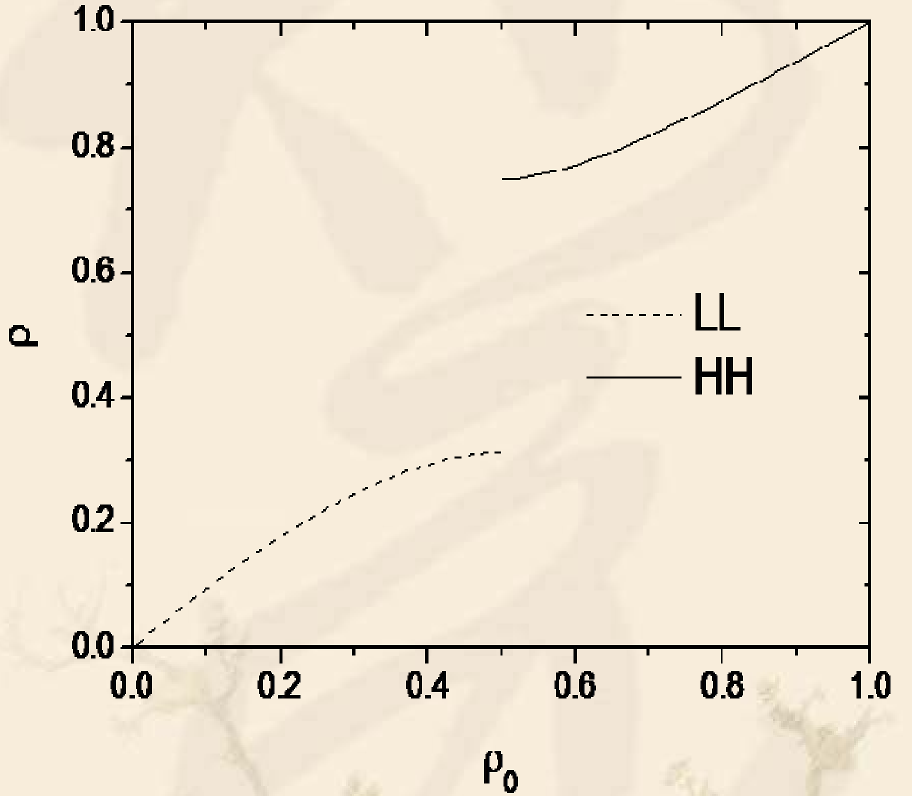
When $\rho_1=\rho_2$ ====> $(1-2\rho_1)\frac{\partial \rho_1}{\partial x}+(\Omega_1-\Omega_2)\rho_1^2(1-\rho_1)=0$

Integration leads to $\left(-\ln\frac{\rho_1}{1-\rho_1}-\frac{1}{\rho_1}\right)\Bigg|_{x_1}^{x_2}=(\Omega_2-\Omega_1)(x_2-x_1)$

When $\rho_1+\rho_2=1$ ====> $(1-2\rho_1)\frac{\partial \rho_1}{\partial x}+\Omega_1\rho_1^3-\Omega_2(1-\rho_1)^3=0$

we integrate the equation numerically





The system density versus shock location in LS and SH phases

The system density versus ρ_0 in LL and HH phases

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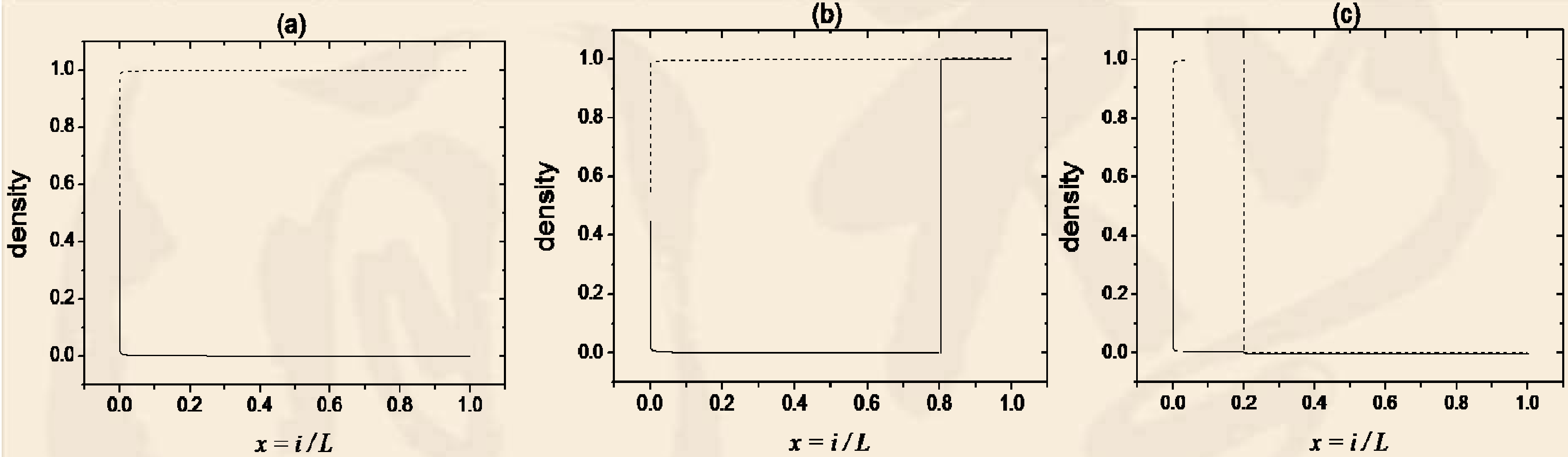

Density profiles along lanes: (a) LH (ρ = 0.5); (b) SH (ρ = 0.6); (c) LS (ρ = 0.1). The parameters : $\Omega_1 = 10^6$, $\Omega_2 = 0$

The main differences from the strong coupling (i) ρc = 0.5 in the W-C case and $\rho c \neq 0.5$ in the S-C case. (ii) the densities $\rho 1$ = 1, $\rho 2$ = 0 are independent of K and Ω1 in the W-C case. In contrast, $\rho 1$ and $\rho 2$ depend on $\omega 1$ and $\omega 2$ in the S-C case. (iii) when $\rho = \rho c$, the shock either appears on lane 1 or on lane 2 in the W-C case,but shock appears on both lanes in the S-C case.

Conclusion

- The shock in an ASEP induced by particle detachment and attachment is an interesting nonequilibrium phenomenon. While previous works focus on open boundary conditions, we study shock formation in a periodic ASEP, which is composed of two equal parts with particles allowed to jump between the two parts.

Conclusion

- We have investigated the effects of asymmetric strong and weak coupling
- Under strong coupling
- When $\rho = \rho c$, both lanes are in a homogeneous state
- When $\rho > \rho c$, a shock separating the fully occupied state from state H appears;
- When $\rho < \rho c$ a shock separating the zero-density state from state H appears.

Conclusion

- Under weak coupling, four phases (i.e., LL, LS, SH, HH) are identified as arising from open boundaries
- LH appears as a boundary line instead of occupying a 2D region.
- These are different from the case for the open boundary condition because under periodic boundary conditions:
- (i) the total current is always zero

  (ii) $\rho 1 + \rho 2 = 1$ or $\rho 1 = \rho 2$ everywhere due to flow conservation.

Conclusion

- We also carried out mean field analysis for the model
  - In the weak coupling situation, it is in good agreement with the simulations
  - In the strong coupling situation, the vertical cluster mean field results slightly deviate from the simulations due to the correlations being neglected

Thanks for your attention