Amorphous order

Jorge Kurchan

PMMH-ESPCI, Paris
jorge@pmmh.espci.fr
http://www.pmmh.espci.fr/∼jorge

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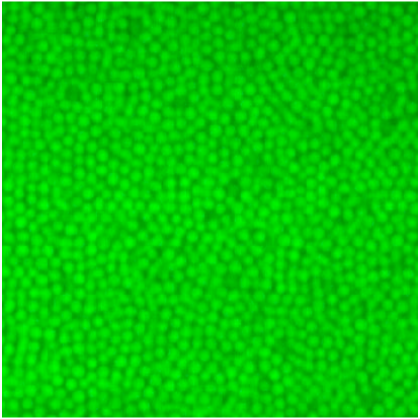
with:

Dov Levine



Liquid or Glass ?

Is there *purely geometric* order in a *solid*?

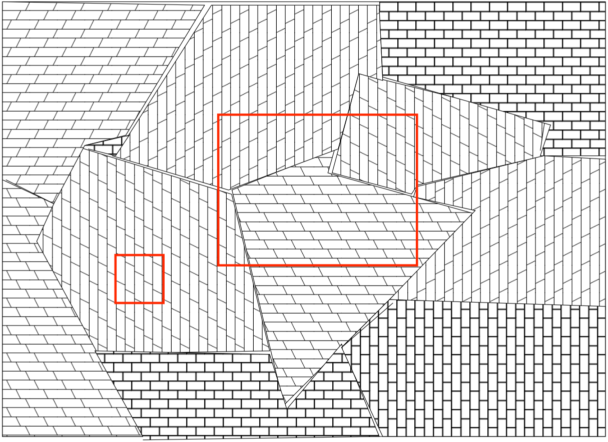
i.e. an order that can be read from a configuration, without knowing the interactions or dynamics

just as in crystals or quasicrystals

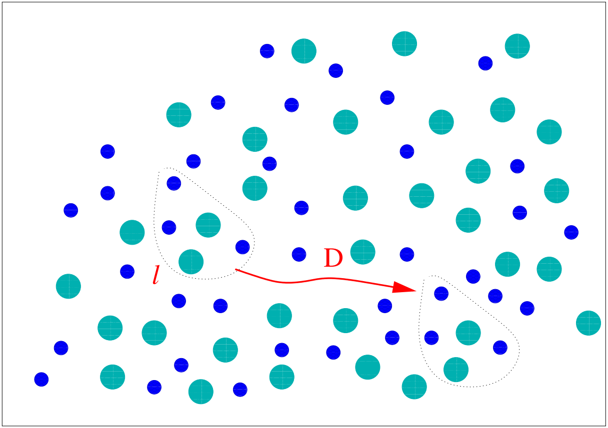
Solid: spatial density modulations not erased by thermal fluctuations

Patch - recurrence length $D(l)$ crossover l_o

detects crystallite length.



Generalize this to general systems



Perfect order (three kinds)

Three levels of order

0101010101010101010101010101

Periodic, *Fourier transform gives deltas.*

1011010110110101101011011010110110

Fibonacci sequence *Quasiperiodic, Fourier transfom → dense set of δ-functions*

01101001100101101001011001101001

Thue-Morse sequence *'Non-Pisot" Fourier transform has no δ functions.*

Patch repetition is a matter of entropy

subextensive entropy $\rightarrow$ infinite length

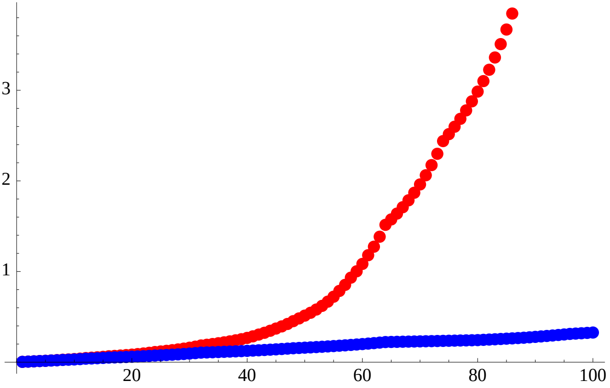
independent pieces will always yield extensive entropy

Finite correlation lengths in imperfect sequences

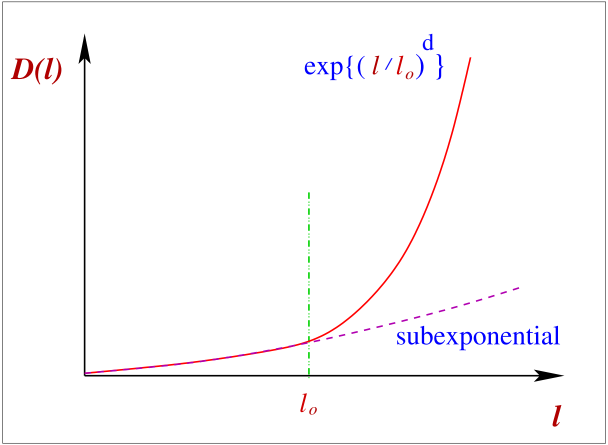
$D(l)$ vs. l

11001011010010110010110011010000101101001011101101001100

11001011010010111**00101100110100**00101101001011**101101001100**

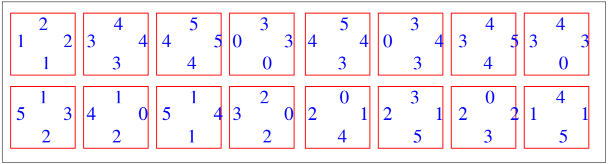


Patch - recurrence length $D(l)$ crossover l_o



More examples: higher dimensionalities

Wang Tiles

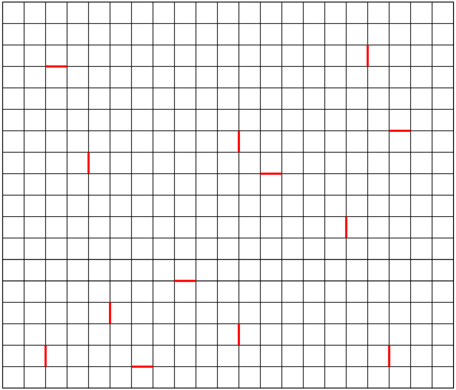


Quasiperiodic ground states

can be seen as a 12-state spin model (Leuzzi and Parisi)

Monte Carlo dynamics is slow

annealing to zero temperature leads to a system with point defects

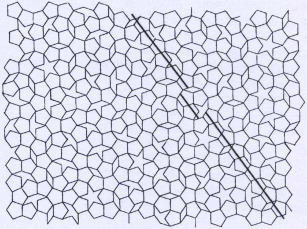


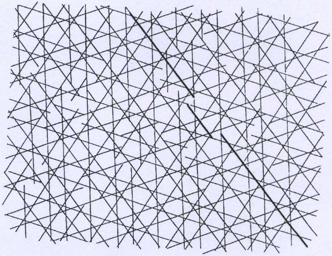
coherence length = inter-defect lenght!!!

Note that coherence is lost through energetically pointlike defects

There is a way to see this, through the Ammann lines.

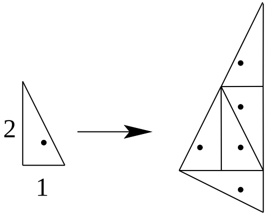
For example, in a true quasicrystal, one can see this: (Li, Zhang and Kuo):

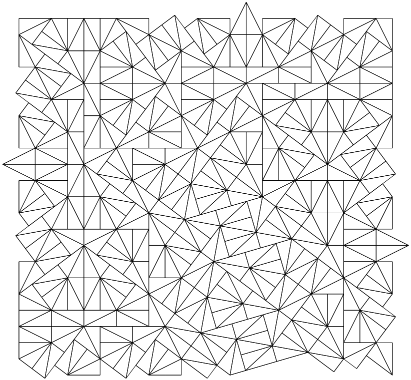

(b)


(c)

An amorphous Pinwheel tiling

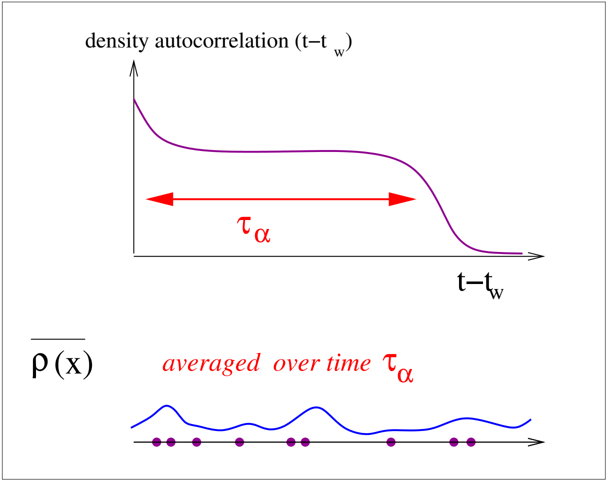
of Radin and Conway (fig: Baake, Frettlöh and Grimm)



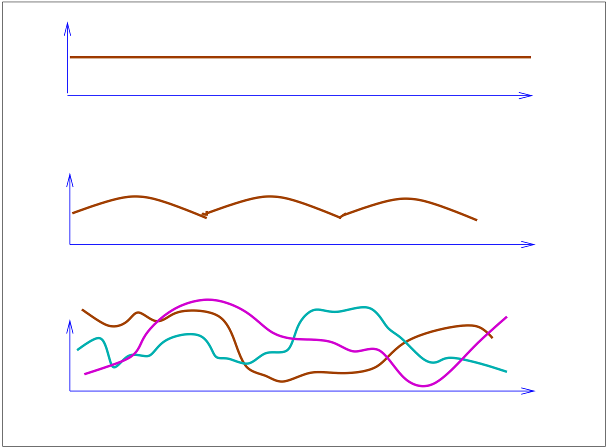


again, infinite coherence length

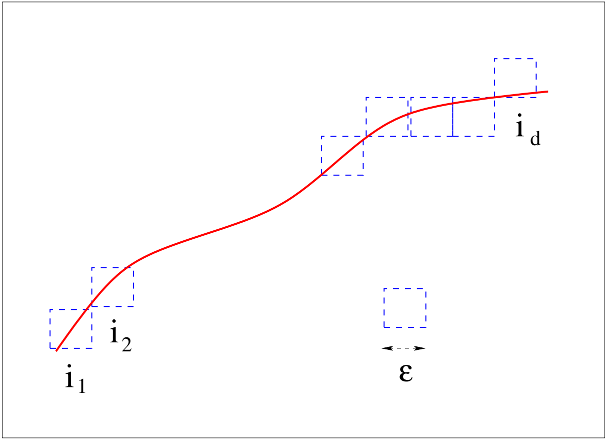
Particle systems: supercooled liquid



Average density profiles: constant, periodic, ‘chaotic’



We need to count profiles $\leftrightarrow$ identify patches



inspiration from dynamic systems

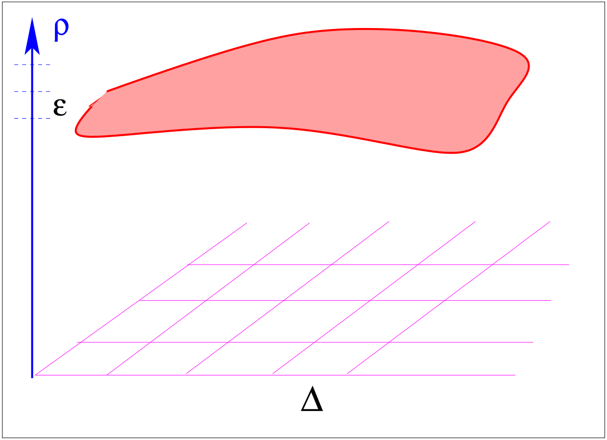
The limit is well-defined:

$$K_1 \sim -\lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau d} \sum_{i_1, \ldots, i_d} P_\epsilon(i_1, \ldots, i_d) \ln P_\epsilon(i_1, \ldots, i_d)$$

Renyi: a measure of 'rare' patches (very frequent or very unfrequent):

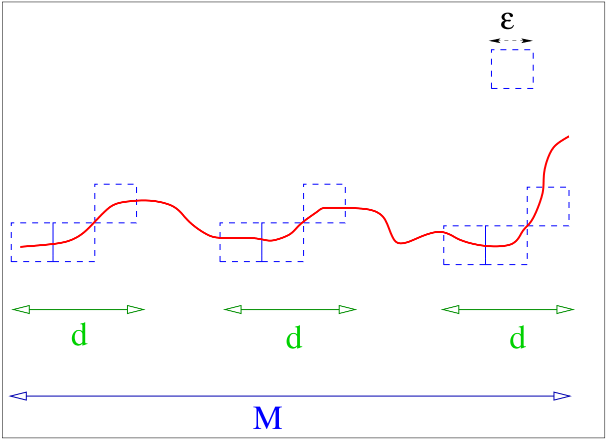
$$K_q \sim -\lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau d(q-1)} \ln \left(\sum_{i_1, \ldots, i_d} P_\epsilon(i_1, \ldots, i_d)^q \right)$$

$\ldots \to \mathcal{P}[P_\epsilon]$ **by Legendre transform.**



$$t \to \vec{r} \qquad x \to \rho$$

Grassberger-Procaccia:



count the number of repetitions n_i of a patch of size d within a large box M and average over patches

$$P_\epsilon(i_1, ..., i_d)^q \sim \frac{1}{M} \sum_i [n_i^d(\epsilon)]^{q-1} \sim \epsilon^\phi \, e^{\tau(q-1)d \, K_q}$$

So that:

$$K_d \sim \lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau(q-1)} \frac{\delta}{\delta d} \ln \left[\sum_i [n_i^d(\epsilon)]^{q-1} \right]$$

for K_1 we use $\left[\sum_i \ln[n_i^d(\epsilon)]\right]$

practical because we work at finite precision

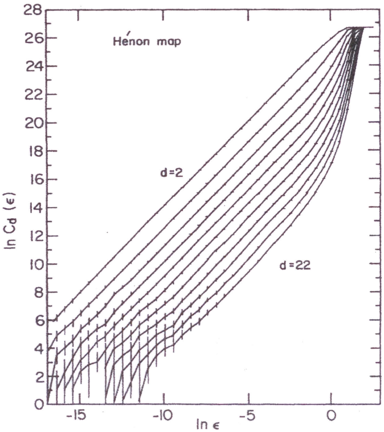


FIG. 3. Same as Fig. 1, but for the Hénon map. The values of d are $d=2$ (top curve), 4,6,8, . . . ,22 (bottom curve).

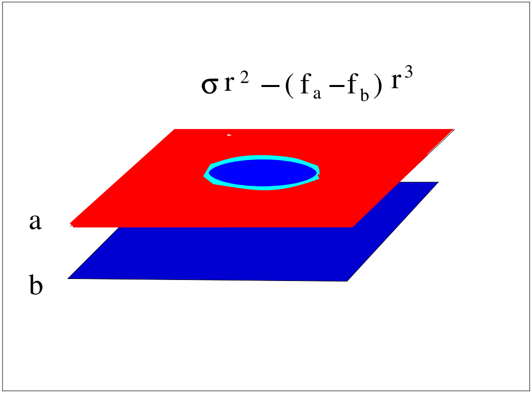
Now, let us argue that if the timescale goes to infinity

in any super-Arrhenius manner

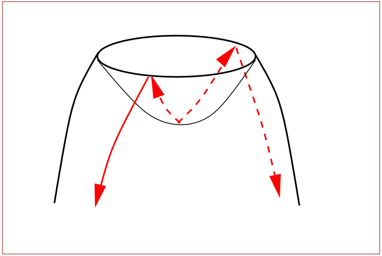
complexity is necessarily subextensive

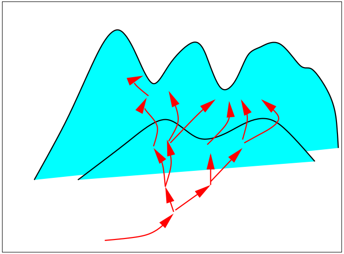
- **and lengthscale goes to infinity**

Usual nucleation argument



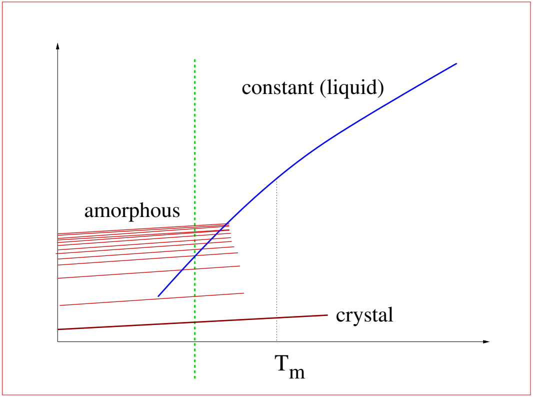
An entropic nucleation argument

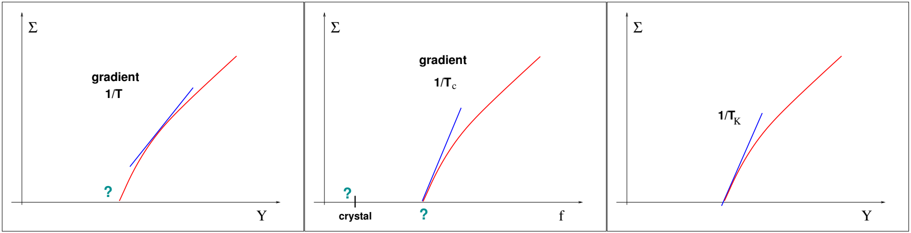




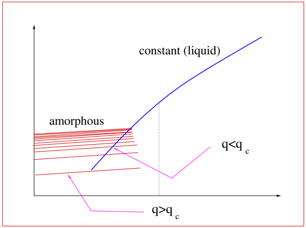
$$V_{eff} = V(r) - (d-1)T \ \ln r = V(r) - TS(r)$$

Kauzmann/ Random First Order scenario





The information of Σ is entirely contained in the Renyi entropies



Rare versus frequent patches

Renyi entropy R_q vanishes for $q > x$ where x is the Parisi parameter

We may frame the discussion in terms of well-defined, measurable quantities

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Order appears as a logical consequence of super-Arrhenius timescales