[P7] Ensemble Inequivalence in the Spherical Spin Glass Model with Nonlinear Interactions

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Ensemble inequivalence is one of the very interesting phenomena concerning the foundation of statistical mechanics. In the conventional understanding, when we take the thermodynamic limit, the physical properties of a systems do not depend on the choice of statistical ensemble. However, in systems with long-range interactions with first-order phase transitions, the microcanonical ensemble gives different results from the corresponding canonical ones [1]. Examples of such striking features include the negative specific heat and the entropy of nonconcaved shape under the microcanonical ensemble. Such ensemble inequivalence has traditionally been discussed mainly in astrophysics [2].

Recently, ensemble inequivalence is investigated in spin systems with long-range interactions with first-order phase transitions. Previous studies of the absence of disorder case [1, 3] and the disorder case [4] give many significant results. However, it is an interesting problem whether or not other systems of disordered spin also show ensemble inequivalence.

In this study, we investigate the ensemble inequivalence of the spherical spin glass model [5] with nonlinear interactions,

$$H = -N\,JV\left(\frac{1}{N}\sum_{i<j}^{N}\varepsilon_{ij}\,S_iS_j\right), \tag{1}$$

$$V(x) = \frac{1}{2^{p-1}p}\left[(1+x)^p - 1\right], \tag{2}$$

where ε_{ij} is a dimensionless random parameter, whose probability distribution obeys the Gaussian distribution with average ε_0 and variance $1/N$. We solve this model exactly for any p and show that first-order phase transitions exist between the paramagnetic and spin glass or ferromagnetic phases for $p \geq 5$. In such a case, we observe ensemble inequivalence as the difference in thermodynamic functions and phase diagrams.

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