Ageing and dynamical symmetries in non-equilibrium systems without detailed balance

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MH, J.D. Noh and M. Pleimling, Phys. Rev. **E85**, 030102(R) (2012)
MH, arXiv:1009.4139 and arXiv:1205.5901

Overview :

1. Ageing phenomena
2. Interface growth (KPZ universality class)
3. Critical contact process (DP universality class)
4. Form of the scaling functions
5. Logarithmic conformal & ageing invariance
6. Numerical experiments (KPZ and DP classes in $1D$)
7. Conclusions

1. Ageing phenomena

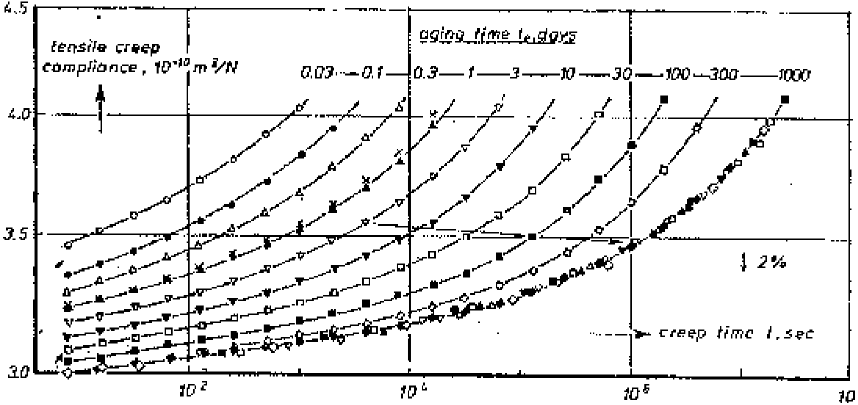
known & practically used since prehistoric times (metals, glasses)
systematically studied in physics since the 1970s — Struik '78
occur in widely different systems
(structural glasses, spin glasses, polymers, simple magnets, ...)

Three defining properties of **ageing** :

1. slow relaxation (non-exponential !)
2. **no** time-translation-invariance (TTI)
3. dynamical scaling — without fine-tuning of parameters

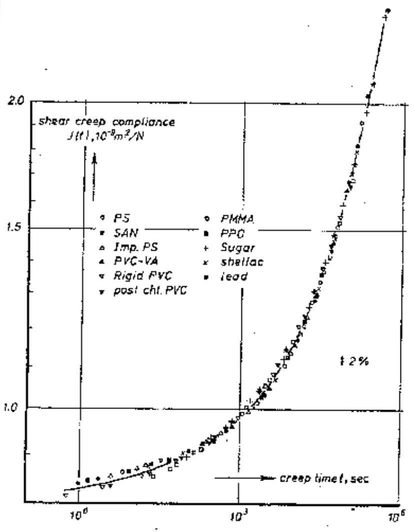
Most existing studies on 'magnets' : relaxation towards equilibrium

Question : what can be learned about intrisically **irreversible** systems by studying their ageing behaviour ?



1. observe slow relaxation after quenching PVC from melt to low T
2. creep curves depend on **waiting time** t_e and **creep time** t
3. find master curve for all $(t, t_e) \longrightarrow$ dynamical scaling

$\rightarrow$ three defining properties of **physical ageing**



master curves of distinct materials are **identical**

independent of 'details'

⟶ Universality !

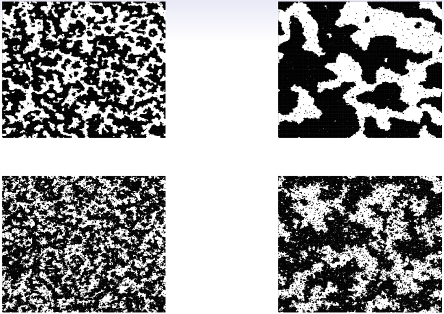
good for theorists . . .

hint for hidden symmetry ?

Struik 78

conceptual confirmation in phase-ordering : Allen-Cahn equation

$t = t_1$ $\quad$ $t = t_2 > t_1$

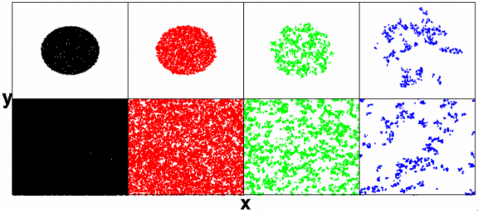


magnet $T < T_c$

$\longrightarrow$ ordered cluster

magnet $T = T_c$

$\longrightarrow$ correlated cluster



critical contact process

$\Longrightarrow$ **cluster dilution**

voter model, contact process,. . .

$$L(t) \sim t^{1/z}$$

common feature : growing length scale

z : dynamical exponent

Two-time observables : analogy with 'magnets'

time-dependent order-parameter $\phi(t, \mathbf{r})$

two-time **correlator** $\quad C(t,s) := \langle \phi(t,\mathbf{r})\phi(s,\mathbf{r})\rangle - \langle \phi(t,\mathbf{r})\rangle \langle \phi(s,\mathbf{r})\rangle$

two-time **response** $\quad R(t,s) := \left.\frac{\delta \langle \phi(t,\mathbf{r})\rangle}{\delta h(s,\mathbf{r})}\right|_{h=0} = \left\langle \phi(t,\mathbf{r})\widetilde{\phi}(s,\mathbf{r})\right\rangle$

t : observation time, s : waiting time

a) system **at** equilibrium : **fluctuation-dissipation theorem**

$$R(t-s) = \frac{1}{T}\frac{\partial C(t-s)}{\partial s} \quad , \qquad T \text{ : temperature}$$

b) **far from equilibrium** : C and R **independent** !

The **fluctuation-dissipation ratio** (FDR) $\quad$ Cugliandolo, Kurchan, Parisi '94

$$X(t,s) := \frac{TR(t,s)}{\partial C(t,s)/\partial s}$$

measures the distance with respect to equilibrium :

$$X_{\rm eq} = X(t-s) = 1$$

Scaling regime : $t, s \gg \tau_{\rm micro}$ **and** $t - s \gg \tau_{\rm micro}$

$$C(t,s) = s^{-b} f_C\left(\frac{t}{s}\right) \ , \ R(t,s) = s^{-1-a} f_R\left(\frac{t}{s}\right)$$

asymptotics : $f_{C,R}(y) \sim y^{-\lambda_{C,R}/z}$ for $y \gg 1$

λ_C : autocorrelation exponent, λ_R : autoresponse exponent,
z : dynamical exponent, a, b : ageing exponents

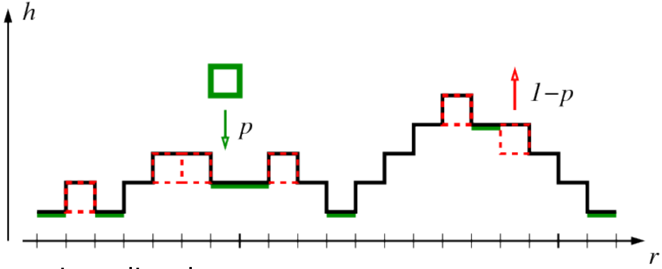
ex. : critical particle-reaction model (contact process),
initial particle density > 0 BAUMANN & GAMBASSI 07

$$\lambda_C = \lambda_R = d + z + \frac{\beta}{\nu_\perp} \ , \ \ b = \frac{2\beta'}{\nu_\|}$$

$\longrightarrow$ stationary-state critical exponents $\beta, \beta', \nu_\perp, \nu_\| = z\nu_\perp$

2. Interface growth

deposition (evaporation) of particles on a substrate $\to$ height profile $h(t, \mathbf{r})$
generic situation : RSOS (**r**estricted **s**olid-**o**n-**s**olid) model KIM & KOSTERLITZ 89



$p =$ deposition prob.
$1 - p =$ evap. prob.

here $p = 0.98$

some universality classes :

(a) KPZ $\partial_t h = \nu \nabla^2 h + \frac{\mu}{2} (\nabla h)^2 + \eta$ KARDAR, PARISI, ZHANG 86

(b) EW $\partial_t h = \nu \nabla^2 h + \eta$ EDWARDS, WILKINSON 82

(c) MH $\partial_t h = -\nu \nabla^4 h + \eta$ MULLINS, HERRING 63 ; WOLF, VILLAIN 80

η is a gaussian white noise with $\langle \eta(t, \mathbf{r}) \eta(t', \mathbf{r}') \rangle = 2\nu T \delta(t - t') \delta(\mathbf{r} - \mathbf{r}')$

Family-Viscek scaling on a spatial lattice of extent L^d : $\overline{h}(t) = L^{-d} \sum_j h_j(t)$

$$w^2(t;L) = \frac{1}{L^d} \sum_{j=1}^{L^d} \left\langle \left(h_j(t) - \overline{h}(t) \right)^2 \right\rangle = L^{2\zeta} f\left(tL^{-z} \right) \sim \begin{cases} L^{2\zeta} & ; \text{ if } tL^{-z} \gg 1 \\ t^{2\beta} & ; \text{ if } tL^{-z} \ll 1 \end{cases}$$

β : growth exponent, ζ : roughness exponent, $\boxed{\zeta = \beta z}$

two-time correlator :

limit $L \to \infty$

$$C(t,s;\mathbf{r}) = \langle h(t,\mathbf{r}) h(s,\mathbf{0}) \rangle - \left\langle \overline{h}(t) \right\rangle \left\langle \overline{h}(s) \right\rangle = s^{-b} F_C \left(\frac{t}{s}, \frac{\mathbf{r}}{s^{1/z}} \right)$$

with ageing exponent : $\boxed{b = -2\beta}$

KALLABIS & KRUG 96

two-time integrated response :

* sample **A** with deposition rates $p_i = p \pm \epsilon_i$, up to time s,
* sample **B** with $p_i = p$ up to time s ;

then switch to common dynamics $p_i = p$ for all times $t > s$

$$\chi(t,s;\mathbf{r}) = \int_0^s \mathrm{d}u \, R(t,u;\mathbf{r}) = \frac{1}{L} \sum_{j=1}^{L} \left\langle \frac{h_{j+r}^{(\mathbf{A})}(t;s) - h_{j+r}^{(\mathbf{B})}(t)}{\epsilon_j} \right\rangle = s^{-a} F_\chi \left(\frac{t}{s}, \frac{|\mathbf{r}|^z}{s} \right)$$

Effective action of the KPZ equation :

$$\mathcal{J}[\phi,\widetilde{\phi}] = \int \mathrm{d}t\mathrm{d}\mathbf{r}\ \left[\widetilde{\phi}\left(\partial_t\phi - \nu\boldsymbol{\nabla}^2\phi - \frac{\mu}{2}(\boldsymbol{\nabla}\phi)^2\right) - \nu T\widetilde{\phi}^2\right]$$

$\Longrightarrow$ Very special properties of KPZ in $d = 1$ spatial dimension !

Exact critical exponents $\beta = 1/3$, $\zeta = 1/2$, $z = 3/2$, $\lambda_C = 1$ — KPZ 86 ; Krech 97

related to precise symmetry properties :

A) tilt-invariance (Galilei-invariance) — Forster, Nelson, Stephen 77

kept under renormalisation **!** — Medina, Hwa, Kardar, Zhang 89

B) time-reversal invariance — Lvov, Lebedev, Paton, Procaccia 93; Frey, Täuber, Hwa 96

A) tilt-invariance (holds for any dimension d)

$$t \mapsto t \;\;,\;\; \mathbf{r} \mapsto \mathbf{r} - \boldsymbol{\epsilon} t \;\;,\;\; h(t,\mathbf{r}) \;\mapsto\; h(t,\mathbf{r}-\boldsymbol{\epsilon} t) - \frac{1}{\mu}\boldsymbol{\epsilon}\cdot\mathbf{r} + \frac{t}{2\mu}\boldsymbol{\epsilon}^2$$

$$\eta(t,\mathbf{r}) \;\mapsto\; \eta(t,\mathbf{r}-\boldsymbol{\epsilon} t)$$

combination with dynamical scaling gives the exponent relation

$$\boxed{\zeta + z = 2}$$

this is preserved in the loop expansion **!**

N.B. in $1D$, gaussian stationary distribution fixes $\zeta = \frac{1}{2}$.

B) Special KPZ symmetry in $1D$: let $v = \frac{\partial \phi}{\partial r}$, $\widetilde{\phi} = \frac{\partial}{\partial r}\left(\widetilde{p} + \frac{v}{2T}\right)$

$$\mathcal{J} = \int \mathrm{d}t\mathrm{d}r \left[\widetilde{p}\partial_t v - \frac{\nu}{4T}(\partial_r v)^2 - \frac{\mu}{2}v^2\partial_r\widetilde{p} + \nu T(\partial_r\widetilde{p})^2\right]$$

is invariant under **time-reversal**

$$t \mapsto -t \ , \ v(t,r) \mapsto -v(-t,r) \ , \ \widetilde{p} \mapsto +\widetilde{p}(-t,r)$$

$\Rightarrow$ **fluctuation-dissipation relation** for $t \gg s$

$$TR(t,s;r) = -\partial_r^2 C(t,s;r)$$

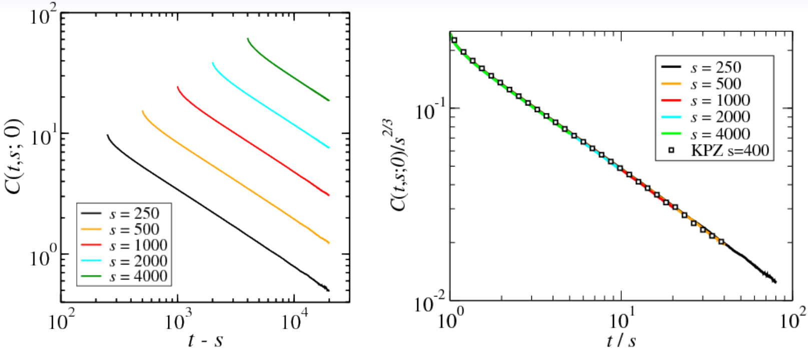
distinct from the equilibrium FDT $TR(t-s) = \partial_s C(t-s)$

Combination with ageing scaling, gives the ageing exponents :

$\lambda_R = \lambda_C = 1$ and $1 + a = b + \frac{2}{z}$

Kallabis, Krug 96 mh, Noh, Pleimling '12

1D relaxation dynamics, starting from an initially flat interface

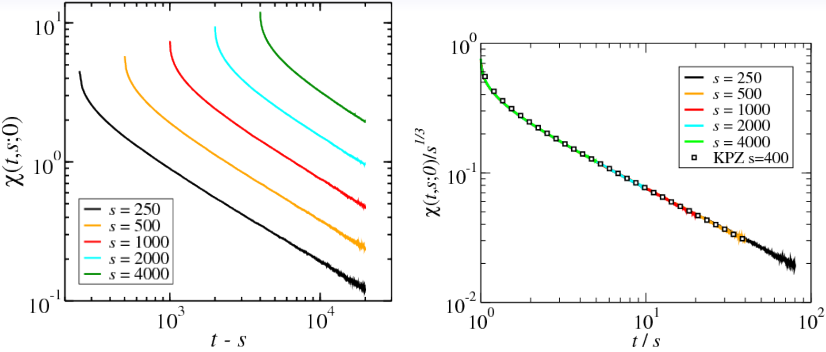


observe all **3** properties of **ageing** : { slow dynamics; no TTI; dynamical scaling }

confirm expected exponents $b = -2/3$, $\lambda_C/z = 2/3$

N.B. : this confirmation is out of the stationary state

Kallabis & Krug 96 ; Krech 97 ; Bustingorry *et al.* 07-10 ; Chou & Pleimling 10 ; D'Aquila & Täuber 11

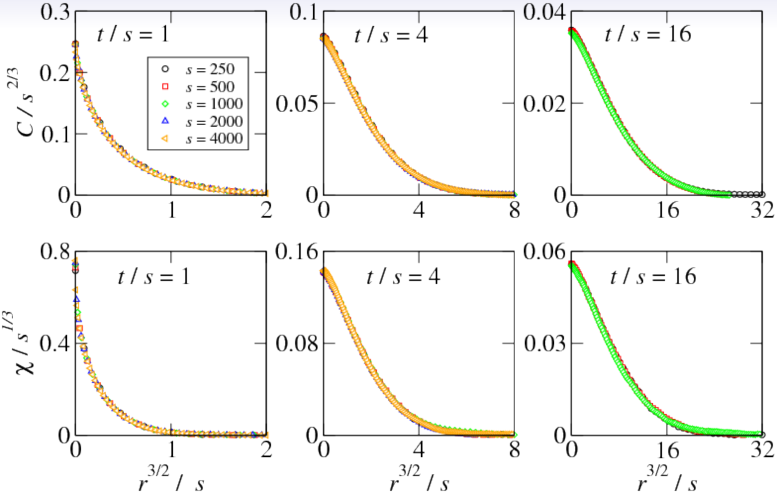


observe all **3** properties of **ageing** : { slow dynamics; no TTI; dynamical scaling }

exponents $a = -1/3$, $\lambda_R/z = 2/3$, as expected from FDR

N.B. : numerical tests for 2 models in KPZ class

Simple ageing is also seen in space-time observables



correlator $C(t,s;r) = s^{2/3} F_C\left(\frac{t}{s}, \frac{r^{3/2}}{s}\right)$

integrated response $\chi(t,s;r) = s^{1/3} F_\chi\left(\frac{t}{s}, \frac{r^{3/2}}{s}\right)$

} confirm $z = 3/2$

Values of some growth and ageing exponents in $1D$

model	z	a	b	$\lambda_R = \lambda_C$	β	ζ
KPZ	$3/2$	$-1/3$	$-2/3$	1	$1/3$	$1/2$
exp			$\approx -2/3^{\dagger}$	$\approx 1^{\dagger}$	0.336(11)	0.50(5)
EW	2	$-1/2$	$-1/2$	1	$1/4$	$1/2$
MH	4	$-3/4$	$-3/4$	1	$3/8$	$3/2$

Takeuchi, Sano, Sasamoto, Spohn 10/11/12

† scaling holds only for flat interface

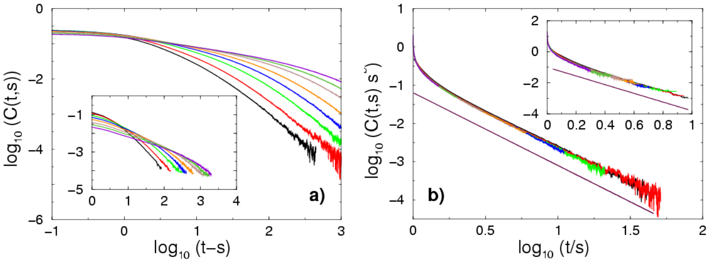
Two-time space-time responses and correlators consistent with simple ageing for $1D$ KPZ

Similar results known for EW and MH universality classes

Roethlein, Baumann, Pleimling 06

3. Critical contact process $A \xrightarrow{p} 2A,\ A \xrightarrow{1} \emptyset$

ageing and scaling for $C(t,s)$: **critical** contact process



main figures : $1D$, insets : $2D$

observe all **3** properties of **ageing** : $\left\{ \begin{array}{l} \text{slow dynamics} \\ \text{no TTI} \\ \text{dynamical scaling} \end{array} \right.$

contrast to critical magnets : $\boxed{a \neq b}$ $\Longrightarrow$ **no** finite FDR !

Ramasco, mh, Santos, da Silva Santos 04 ; Enss, mh, Schollwöck 04

Effective action at criticality

Janssen, de Dominicis 70s-80s

$$\mathcal{J}[\widetilde{\phi},\phi] = \int \mathrm{d}t\mathrm{d}\mathbf{r}\, \left[\widetilde{\phi}\left(D\partial_t\phi - \nabla^2\phi\right) - \kappa\widetilde{\phi}\left(\widetilde{\phi}-\phi\right)\phi\right]$$

rapidity-reversal symmetry : $\mathcal{J}$ is invariant under Grassberger 79, Janssen 81

$$t \mapsto -t \;\;,\;\; \phi(t,\mathbf{r}) \mapsto -\widetilde{\phi}(-t,\mathbf{r}) \;\;,\;\; \widetilde{\phi}(t,\mathbf{r}) \mapsto -\phi(-t,\mathbf{r})$$

$\Rightarrow$ ϕ and $\widetilde{\phi}$ must have equal scaling dimensions $\Rightarrow$ $\boxed{1+a=b}$

new form of FDR, $\frac{1}{\Xi}$ measures distance from stationarity Enss *et. al.* 04, Baumann & Gambassi 07

$$\Xi(t,s) := \frac{R(t,s)}{C(t,s)} = \frac{f_R(t/s)}{f_C(t/s)} \;\;,\;\; \Xi_\infty := \lim_{s\to\infty}\left(\lim_{t\to\infty}\Xi(t,s)\right)$$

for $d=4-\varepsilon$ (one-loop calculation) & $1D$ num. TMRG estimate B & G 07, Enss *et al.* 04

$$\Xi_\infty = 2\left[1-\varepsilon\left(\frac{119}{480}-\frac{\pi^2}{120}\right)\right] + \mathrm{O}(\varepsilon^2) \;\;,\;\; \Xi_\infty = 1.15(5)$$

NB : $1+a=b$ **invalid** in other non-equilibrium universality classes

numerical values of some non-equilibrium exponents

contact process (CP) $A \to 2A, A \to \emptyset$, parity-conserved model (PC) $A \leftrightarrow 3A, 2A \to \emptyset$, diffusion-coagulation (DC) $2A \to A$

	d	a	b	λ_C/z	λ_R/z		
CP	1	$-0.68(5)$	0.32(5)	1.85(10)	1.85(10)	TMRG	[1]
		$-0.57(10)$	0.3189	1.9(1)	1.9(1)	MC	[2]
		-0.6810			1.76(5)	MC	[3]
		-0.6810	0.3189	1.7921	1.7921	scal	[5]
	2	0.3(1)	0.901(2)	2.8(3)	2.75(10)	MC	[2]
		$-0.198(2)$	0.901(2)	2.58(2)	2.58(2)	scal	[5]
			0.9(1)	2.5(1)		exp	[6]
	> 4	$d/2-1$	$d/2$		$d/2+2$	MF	[2]
PC	1	$-0.430(4)$	0.570(4)	1.9(1)	1.9(2)	MC	[4]
		$-0.430(4)$	0.570(4)	1.86(1)	1.86(1)	scal	
DC	1	$-1/2$	1	2	2	exact	[7]

[1] ENSS *et. al.* 04 ; [2] RAMASCO *et. al.* 04 ; [3] HINRICHSEN 06 ; [4] ÓDOR 06 ;

[5] BAUMANN & GAMBASSI 07 ; [6] TAKEUCHI *et. al.* 09 ; [7] DURANG, FORTIN, MH 11

4. Form of the scaling functions

Question : **?** Are there model-independent results on the form of universal scaling functions **?**

'Natural' starting point : try to draw analogies with conformal invariance at equilibrium

* Equilibrium critical phenomena : **scale-invariance**
* For sufficiently **local** interactions : extend to conformal invariance
space-dependent re-scaling (angles conserved) $\mathbf{r} \mapsto \mathbf{r}/b(\mathbf{r})$

BATEMAN & CUNNINGHAM 1909/10, POLYAKOV 70

In **two** dimensions : ∞ many conformal transformations
($w \mapsto \beta(w)$ complex analytic)
$\Rightarrow$ exact predictions for critical exponents, correlators, ... BPZ 84

Hidden assumptions :

1) extension scale-invariance $\longrightarrow$ conformal invariance **?**

formally : energy-momentum tensor symmetric & traceless — Callan, Coleman, Jackiw '70

but counterexamples :
- lattice animals — Miller & De Bell 93
- hydrodynamics — Riva & Cardy 05
- renormalised FT — Fortin, Grinstein, Stergiou 12

2) choice of so-called 'primary' scaling operators

not all physical models are unitary minimal CFTs $\longrightarrow$ SLE

3) how do primary operators transform ?

usual form

$$\phi'(w) = \beta'(w)^{\Delta}\phi(\beta(w))$$

alternative : logarithmic partner ψ — Gurarie 93, Khorrami *et al.* 97,. . .

$$\psi'(w) = \beta'(w)^{\Delta}\left[\psi(\beta(w)) + \ln\beta'(w)\cdot\phi(\beta(w))\right]$$

Logarithmic conformal invariance has been found in, e.g.

- critical $2D$ percolation — Cardy 92, Watts 96, Mathieu & Ridout 07/08
- disordered systems — Caux *et al.* 96
- sand-pile models — Ruelle *et al.* 08-10

What about **time**-dependent critical phenomena **?** CARDY 85

Characterised by **dynamical exponent** z : $t \mapsto tb^{-z}$, $\mathbf{r} \mapsto \mathbf{r}b^{-1}$

Can one extend to **local** dynamical scaling, with $z \neq 1$?

If $z = 2$, the **Schrödinger group** is an example : JACOBI 1842, LIE 1881

$$t \mapsto \frac{\alpha t + \beta}{\gamma t + \delta} \ , \ \mathbf{r} \mapsto \frac{\mathcal{D}\mathbf{r} + \mathbf{v}t + \mathbf{a}}{\gamma t + \delta} \ ; \ \alpha\delta - \beta\gamma = 1$$

$\Rightarrow$ study **ageing** phenomena as paradigmatic example

essential : (i) **absence** of TTI & (ii) **Galilei**-invariance

Transformation $t \mapsto t'$ with $\beta(0) = 0$ and $\dot{\beta}(t') \geq 0$ and

$$t = \beta(t') \ , \ \phi(t) = \left(\frac{\mathrm{d}\beta(t')}{\mathrm{d}t'}\right)^{-x/z} \left(\frac{\mathrm{d}\ln\beta(t')}{\mathrm{d}t'}\right)^{-2\xi/z} \phi'(t')$$

out of equilibrium, have **2 distinct** scaling dimensions, x and ξ.

mean-field for **magnets** : expect $\begin{cases} \xi = 0 \text{ in ordered phase } T < T_c \\ \xi \neq 0 \text{ at criticality } T = T_c \end{cases}$

NB : if TTI (equilibrium criticality), then $\xi = 0$.

physical requirement :

co-variance of **response functions** under local scaling !

why : certain extended scaling symmetries **predict causality** for co-variant n-point functions !

$\Rightarrow$ set of linear differential equations for $R(t,s)$ **most simple case** !

$$R(t,s) = \left\langle \phi(t)\widetilde{\phi}(s) \right\rangle = s^{-1-a} f_R\left(\frac{t}{s}\right)$$

$$f_R(y) = f_0 y^{1+a'-\lambda_R/z} (y-1)^{-1-a'} \underbrace{\Theta(y-1)}_{\text{causality}}$$

$$a = \frac{1}{z}(x+\widetilde{x}) - 1 \ , \ a' - a = \frac{2}{z}\left(\xi + \widetilde{\xi}\right) \ , \ \frac{\lambda_R}{z} = x + \xi$$

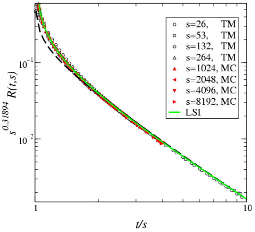
magnetic example : $1D$ Glauber-Ising model at $T = T_c = 0$:

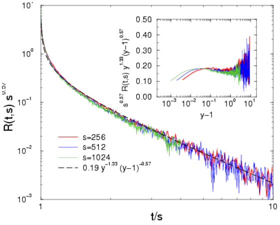
$$a = 0 \ , \ a' - a = -\frac{1}{2} \ , \ \lambda_R = 1 \ , \ z = 2$$

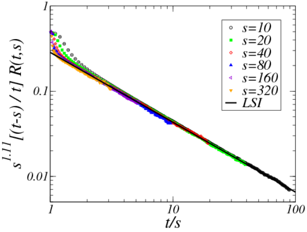
Picone, mh 04
mh, Enss, Pleimling 06

Particle models : comparison of $R(t,s)$ with LSI-prediction :

contact process (CP)	nonequil. kinetic Ising (PC)	voter Potts-3 (VP3)







CP : $a' - a \simeq 0.27$ PC : $a' - a \simeq 0.00(1)$ VP3 : $a' - a \simeq -0.1$

MH, ENSS, PLEIMLING 06
ENSS 06 ; HINRICHSEN 06

ÓDOR 06

CHATELAIN, TOMÉ, DE OLIVEIRA 11

? is this good general agreement already conclusive ?

Observation : the **hidden assumption** $a = a'$, uncritically taken over from equilibrium, is often **invalid** out of equilibrium.
Observables **cannot** always be identified with scaling operators.

5. Logarithmic conformal & ageing invariance

generalise conformal invariance → doubletts $\Psi = \begin{pmatrix} \psi \\ \phi \end{pmatrix}$ ROZANSKY & SALEUR 92 GURARIE 93

generators : $\ell_n = -w^{n+1}\partial_w - (n+1)w^n \begin{pmatrix} \Delta & 1 \\ 0 & \Delta \end{pmatrix}$

two-point functions : have $\Delta_1 = \Delta_2$ GURARIE 93, RAHIMI TABAR *et al.* 97...

$$
\begin{aligned}
F &= \langle \phi_1(w_1)\phi_2(w_2)\rangle = 0 \\
G &= \langle \phi_1(w_1)\psi_2(w_2)\rangle = G_0 |w|^{-2\Delta_1} \\
H &= \langle \psi_1(w_1)\psi_2(w_2)\rangle = (H_0 - 2G_0 \ln|w|)\,|w|^{-2\Delta_1} \\
&= w_2^{-2\Delta_1}(H_0 - 2G_0 \ln|y-1| - 2G_0 \ln|w_2|)\,|y-1|^{-2\Delta_1}
\end{aligned}
$$

with $w = w_1 - w_2$ and $y = w_1/w_2$.

Simultaneous log corrections to scaling **and** modified scaling function

Logarithmic conformal invariance has been found in, e.g.

- critical $2D$ percolation CARDY 92, WATTS 96, MATHIEU & RIDOUT 07/08
- disordered systems CAUX *et al.* 96
- sand-pile models RUELLE *et al.* 08-10

construct **logarithmic ageing-invariance** by the formal changes (generic case ; $x' = 0$ or $x' = 1$) :

$$x \mapsto \hat{x} = \begin{pmatrix} x & x' \\ 0 & x \end{pmatrix} \;,\; \xi \mapsto \hat{\xi} = \begin{pmatrix} \xi & \xi' \\ \mathbf{0} & \xi \end{pmatrix}$$

(**must** show : both dimension matrices $\hat{x}, \hat{\xi}$ are **simultaneously** Jordan !)

we find the co-variant two-point functions (with $y = t/s$) :

$$\begin{aligned}
\left\langle \phi(t)\widetilde{\phi}(s)\right\rangle &= s^{-(x+\widetilde{x})/2} f(y) \\
\left\langle \phi(t)\widetilde{\psi}(s)\right\rangle &= s^{-(x+\widetilde{x})/2} \left(g_{12}(y) + \ln s \cdot \gamma_{12}(y)\right) \\
\left\langle \psi(t)\widetilde{\phi}(s)\right\rangle &= s^{-(x+\widetilde{x})/2} \left(g_{21}(y) + \ln s \cdot \gamma_{21}(y)\right) \\
\left\langle \psi(t)\widetilde{\psi}(s)\right\rangle &= s^{-(x+\widetilde{x})/2} \left(h_0(y) + \ln s \cdot h_1(y) + \ln^2 s \cdot h_2(y)\right)
\end{aligned}$$

all scaling functions explicitly known

Question : **interesting models described by logarithmic LSI ?**

6. Numerical experiments

(A) Kardar-Parisi-Zhang (**KPZ**)

(B) directed percolation (**DP**)

(C) majority voter/Glauber models

simple ageing of the correlators and responses, especially

$$C(t,s) = s^{-b} f_C\left(\frac{t}{s}\right) \quad , \quad R(t,s) = s^{-1-a} f_R\left(\frac{t}{s}\right)$$
$$f_C(y) \sim y^{-\lambda_C/z} \quad , \quad f_R(y) \sim y^{-\lambda_R/z} \qquad y \gg 1$$

values of the non-equilibrium exponents & scaling relations

KPZ in $1D$: $\lambda_C = \lambda_R = 1$, $1 + a = b + \frac{2}{z}$, $b = -2\beta = -\frac{2}{3}$, $z = \frac{3}{2}$

DP : $\lambda_C = \lambda_R = d + z + \frac{\beta}{\nu_\perp}$, $1 + a = b = \frac{2\beta}{\nu_\parallel}$

what can be said on the form of the scaling function of the auto-response ?

N.B. : Galilei-invariance for KPZ is kept under renormalisation, unusual form

(A) **assumption :** $R(t,s) = \left\langle \psi(t)\widetilde{\psi}(s)\right\rangle$ $1D$ **KPZ equation/RSOS model**

good collapse $\Rightarrow$ **no** logarithmic corrections $\Rightarrow$ $\boxed{x' = \widetilde{x}' = 0}$

no logarithmic factors for $y \gg 1 \Rightarrow$ $\boxed{\xi' = 0}$

$\Rightarrow$ only $\widetilde{\xi}' = 1$ remains

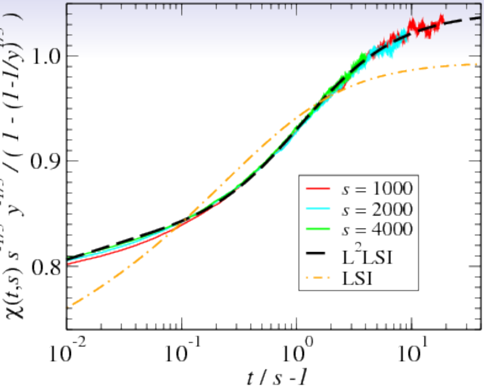
$$f_R(y) = y^{-\lambda_R/z}\left(1-\frac{1}{y}\right)^{-1-a'}\left[h_0 - g_0 \ln\left(1-\frac{1}{y}\right) - \frac{1}{2} f_0 \ln^2\left(1-\frac{1}{y}\right)\right]$$

use specific values of $1D$ KPZ class $\frac{\lambda_R}{z} - a = 1$

find integrated autoresponse $\chi(t,s) = \int_0^s \mathrm{d}u\, R(t,u) = s^{1/3} f_\chi(t/s)$

$$\begin{aligned} f_\chi(y) &= y^{1/3}\left\{A_0\left[1-\left(1-\frac{1}{y}\right)^{-a'}\right]\right. \\ &\quad \left. + \left(1-\frac{1}{y}\right)^{-a'}\left[A_1 \ln\left(1-\frac{1}{y}\right) + A_2 \ln^2\left(1-\frac{1}{y}\right)\right]\right\} \end{aligned}$$

with free parameters A_0, A_1, A_2 and a'



non-log LSI with $a = a'$:
deviations $\approx$ 20%

non-log LSI with $a \neq a'$:
works up to $\approx$ 5%

log LSI : works **better than** $\approx$ 0.1%

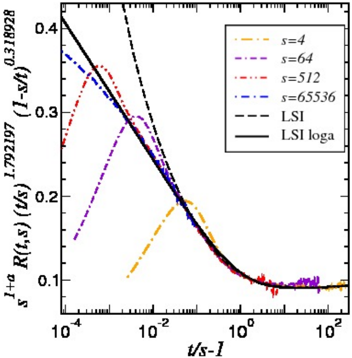
R	a'	A_0	A_1	A_2
$\langle\phi\widetilde{\phi}\rangle$ – LSI	-0.500	0.662	0	0
$\langle\phi\widetilde{\psi}\rangle$ – L^1LSI	-0.500	0.663	$-6\cdot 10^{-4}$	0
$\langle\psi\widetilde{\psi}\rangle$ – L^2LSI	-0.8206	0.7187	0.2424	-0.09087

logarithmic LSI fits data at least down to $y \simeq 1.01$, with
$a' - a \approx -0.4873$ (can we make a conjecture ?)

(B) **assumption :** $R(t,s) = \left\langle \psi(t)\widetilde{\psi}(s) \right\rangle$ **1D critical contact process**

good collapse $\Rightarrow$ **no** logarithmic corrections $\Rightarrow$ $\boxed{x' = \widetilde{x}' = 0}$

$$h_R(y) = \left(1 - \frac{1}{y}\right)^{a-a'} \left[h_0 - g_{12,0}\widetilde{\xi}' \ln(1 - 1/y) - g_{21,0}\xi' \ln(y-1) - \frac{1}{2} f_0 \widetilde{\xi}'^2 \ln^2(1 - 1/y) + \frac{1}{2} f_0 \xi'^2 \ln^2(y-1) \right]$$



find empirically :
very small amplitude of
$\ln^2$-terms

$\Rightarrow$ $\boxed{f_0 = 0}$

require both $\xi \neq 0$, $\widetilde{\xi}' \neq 0$

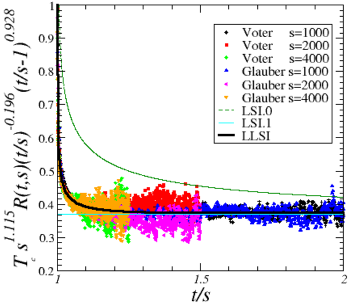
BUT : logarithmic factor for $y \gg 1$ **?**

logar. LSI fit data, at least down to $y \simeq 1.002$; with $a' - a \simeq -0.002$.

(C) <u>**assumption :**</u> $R(t,s) = \left\langle \psi(t)\widetilde{\psi}(s) \right\rangle$ $2D$ **majority voter/Glauber model** (triangular lattice)

good collapse $\Rightarrow$ **no** logarithmic corrections $\Rightarrow$ $\boxed{x' = \widetilde{x}' = 0}$

$$h_R(y) \;=\; \left(1 - \frac{1}{y}\right)^{a-a'} \left[h_0 - g_{12,0} \ln(1 - 1/y) - \frac{1}{2} f_0 \ln^2(1 - 1/y) \right]$$



no logarithmic terms for $y \gg 1$

$\Rightarrow$ $\boxed{\xi' = 0}$

can normalise $\widetilde{\xi}' = 1$

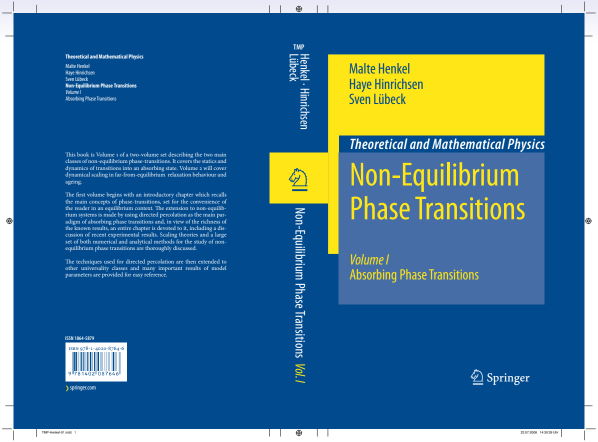
F. Sastre (2012)

logar. LSI fit data, at least down to $y \simeq 1.005$.

7. Conclusions

- physical ageing occurs naturally in many **irreversible** systems relaxing towards **non**-equilibrium stationary states
  considered here : absorbing phase transitions & surface growth
- scaling phenomenology analogous to simple magnets
- **but** finer differences in relationships between non-equilibrium exponents
- a major difference w/ equilibrium : intrinsic **absence** of time-translation-invariance ⇒ **2** scaling dimensions
- shape of scaling functions :
  logarithmic local scale-invariance ?
  performed numerical experiments on auto-response function :
  (i) $1D$ KPZ equation (ii) $1D$ critical directed percolation
  (iii) $2D$ majority voter/Glauber models
- **major** open problem : **Galilei-invariance !**

studies of the ageing properties, via two-time observables, might become a **new tool**, also for the analysis of complex systems !



Vol. 1 : absorbing phase transitions – co-authors H. **Hinrichsen**, S. **Lübeck** 2009

Vol. 2 : ageing & local scaling – co-author M. **Pleimling** 2010

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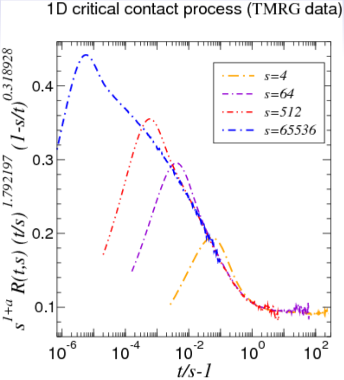
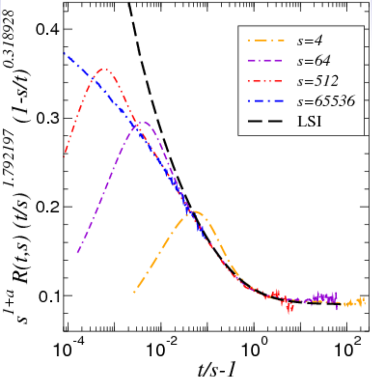


study more closely the limit $t, s \to \infty$, $y = t/s$ fixed ; let $y \to 1$

$$R(t,s) = s^{-1-a} f_R\left(\frac{t}{s}\right) \ , \ h_R(y) := f_R(y) y^{\lambda_R/z} (1 - 1/y)^{1+a}$$

observe good collapse of data, when $y = t/s$ large enough
LSI with $a = a'$ predicts : $h_R(y) = f_0 = \mathrm{cste.}$
$\Rightarrow$ reproduces TMRG data for $y \gtrsim 3 - 4$



$$h_R(y) := f_R(y) y^{\lambda_R/z} (1-1/y)^{1+a} \stackrel{\text{LSI}}{=} f_0 (1-1/y)^{a-a'}$$

with the choice $a'-a=0.26$, LSI works well for $y \gtrsim 1.1$ but **systematic deviations**, still **inside the ageing scaling region**, for smaller values of $y=t/s$ (down to $y \simeq 1.001$) !

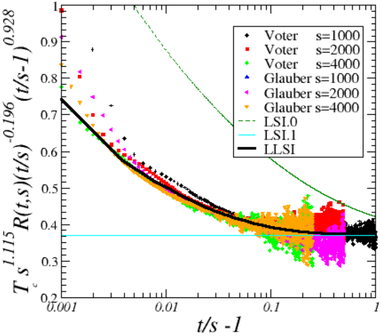
Question : improve the prediction of local scale-invariance (LSI) ?

(C) **assumption :** $R(t,s) = \left\langle \psi(t)\widetilde{\psi}(s) \right\rangle$ **$2D$ majority voter/Glauber model**

(triangular lattice)

good collapse $\Rightarrow$ **no** logarithmic corrections $\Rightarrow$ $\boxed{x' = \widetilde{x}' = 0}$

$$h_R(y) = \left(1 - \frac{1}{y}\right)^{a-a'} \left[h_0 - g_{12,0} \ln(1 - 1/y) - \frac{1}{2} f_0 \ln^2(1 - 1/y) \right]$$



no logarithmic terms for $y \gg 1$

$\Rightarrow$ $\boxed{\xi' = 0}$

can normalise $\widetilde{\xi}' = 1$

F. Sastre (2012)

logar. LSI fit data, at least down to $y \simeq 1.005$.