On the universality of fixed-energy sandpiles

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Collaboration:

first conjectures with A. Hipke presented in Haifa in 2009

Extended study with

Mahashweta Basu
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Pradeep Mohanty

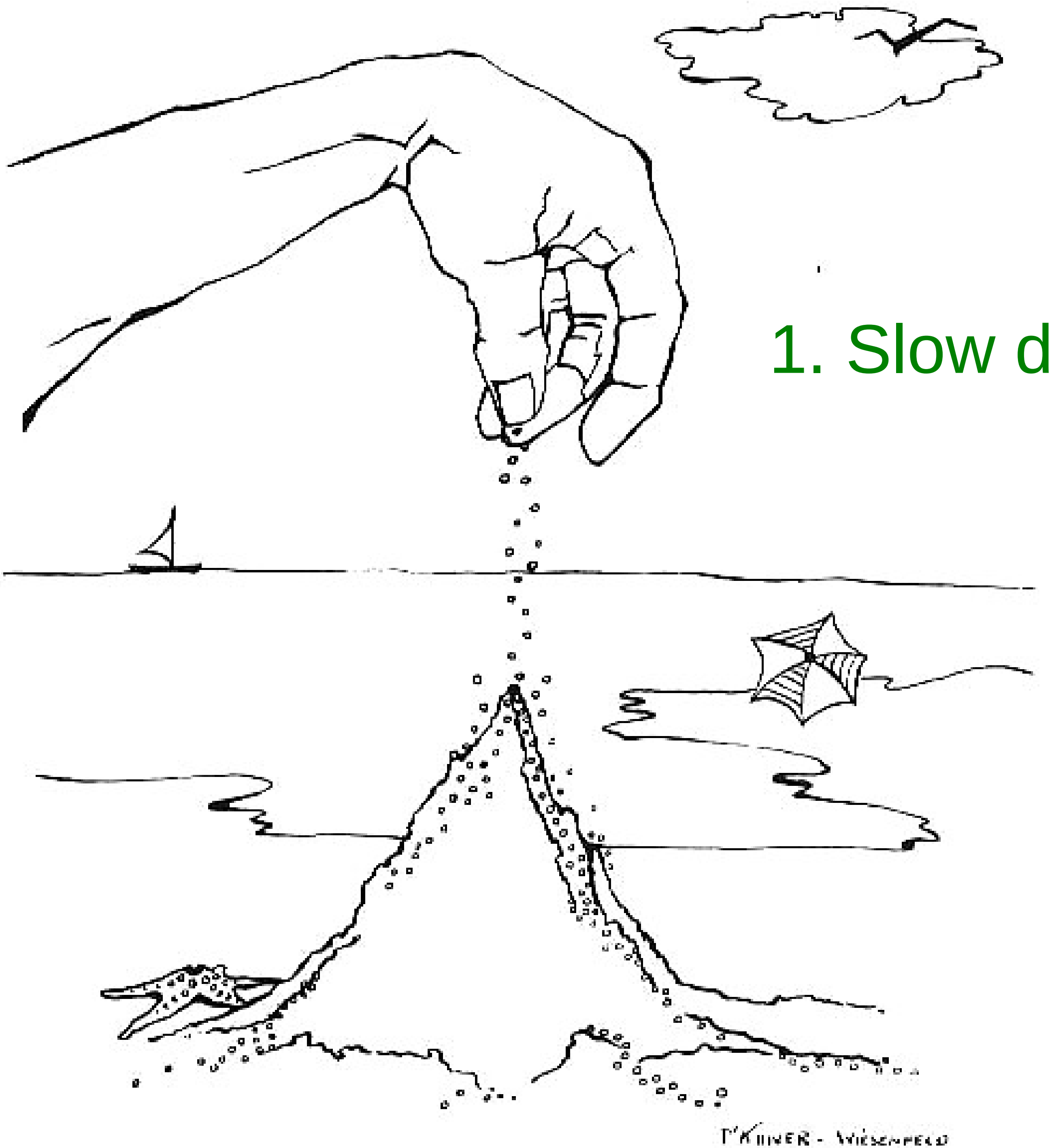
SAHA Institute, Kolkata

ArXiv 1206.3121, to appear in PRL

Outline:

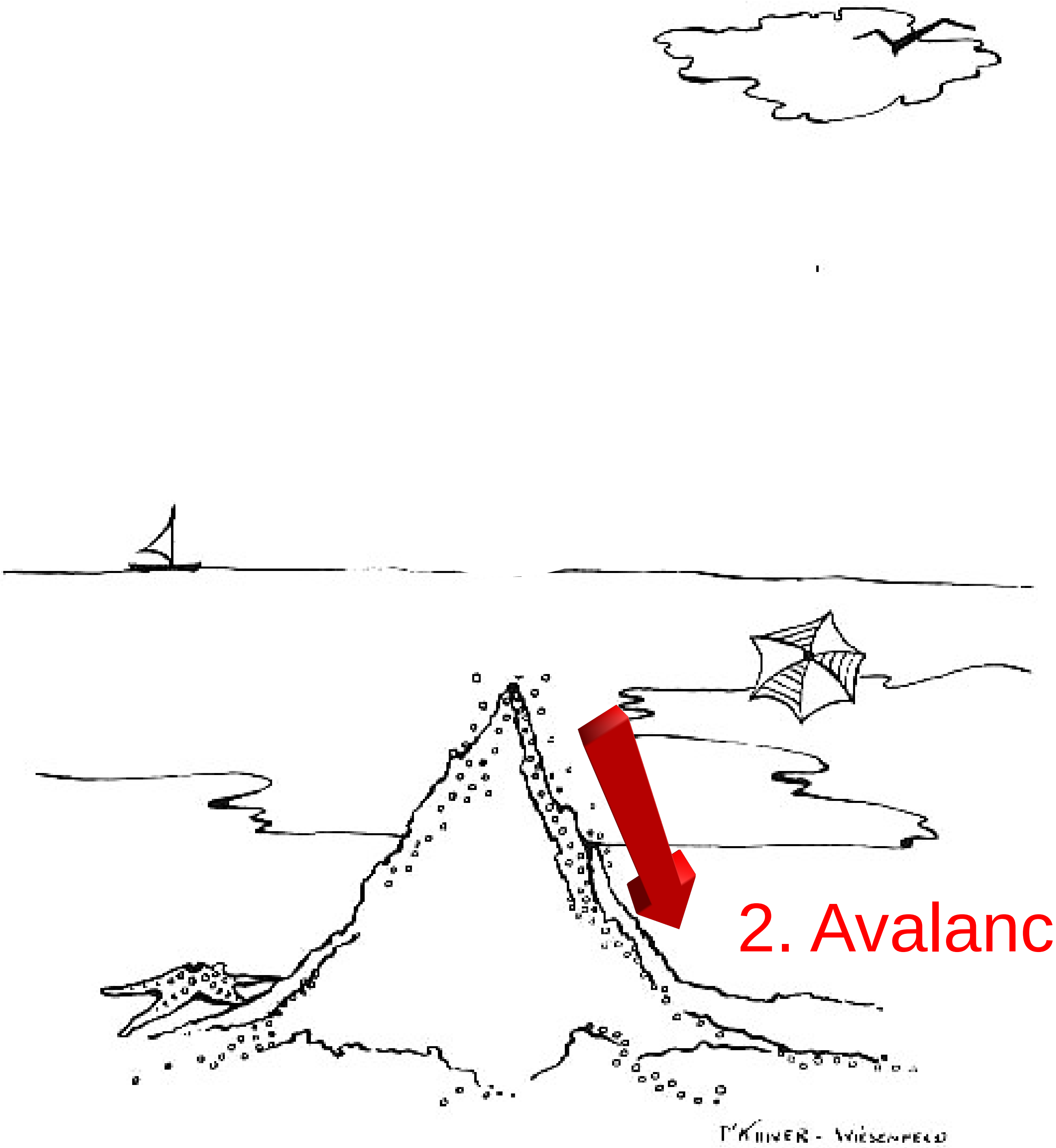
1. SOC and fixed-energy sandpiles.
2. The background field
3. Natural initial states
4. New method to identify DP
5. Conclusions

SOC:



1. Slow driving

SOC:



2. Avalanches

SOC:

3. Dissipation
at the
boundaries

Bak-Tang-Wiesenfeld model

Phys. Rev. Lett. 59, 381 - 384 (1987)

1. Slow driving

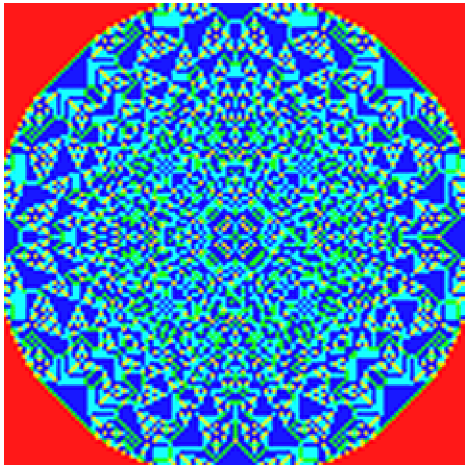
$$h(x,y) \rightarrow h(x,y)+1$$

2. Avalanches

If $h(x,y) \geq 4$ then

$$h(x,y) \rightarrow h(x,y)-4$$
$$h(x+1,y) \rightarrow h(x+1,y)+1$$
$$h(x-1,y) \rightarrow h(x-1,y)+1$$
$$h(x,y+1) \rightarrow h(x,y+1)+1$$
$$h(x,y-1) \rightarrow h(x,y-1)+1$$

3. Dissipation at the boundaries



Cluster sizes

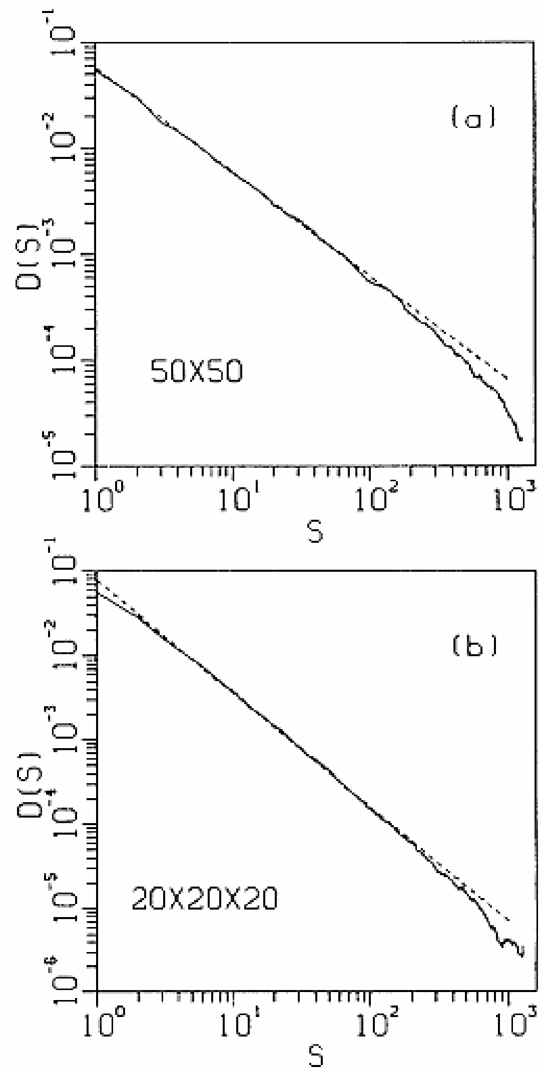


FIG. 2. Distribution of cluster sizes at criticality in two and three dimensions, computed dynamically as described in the text. (a) 50×50 array, averaged over 200 samples; (b) 20×20×20 array, averaged over 200 samples. The data have been coarse grained.

Cluster lifetimes

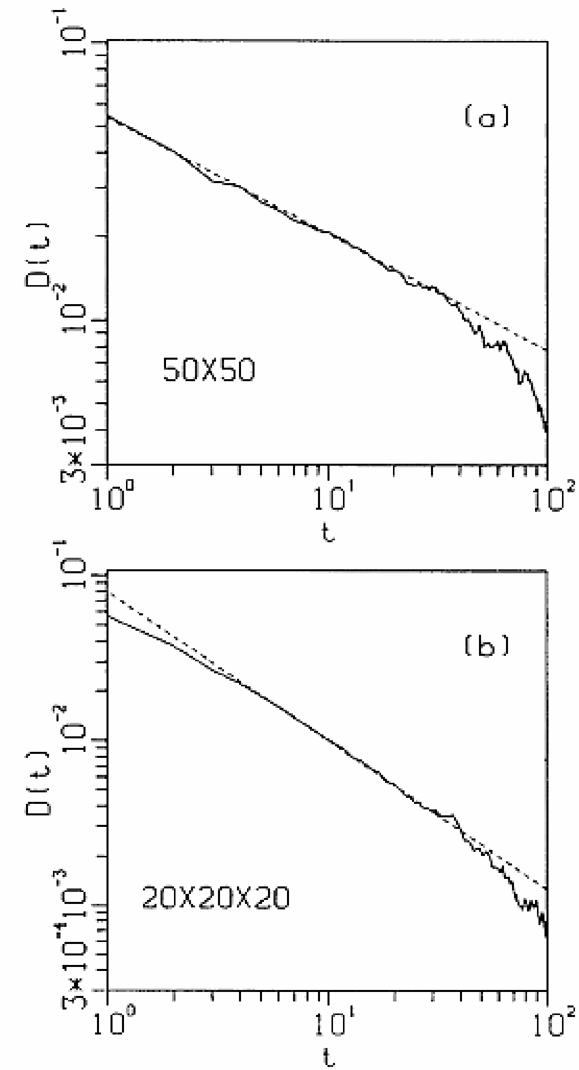


FIG. 3. Distribution of lifetimes corresponding to Fig. 2. (a) For the 50×50 array, the slope $\alpha \approx 0.42$, yielding a "$1/f$" noise spectrum $f^{-1.58}$; (b) 20×20×20 array, $\alpha \approx 0.90$, yielding an $f^{-1.1}$ spectrum

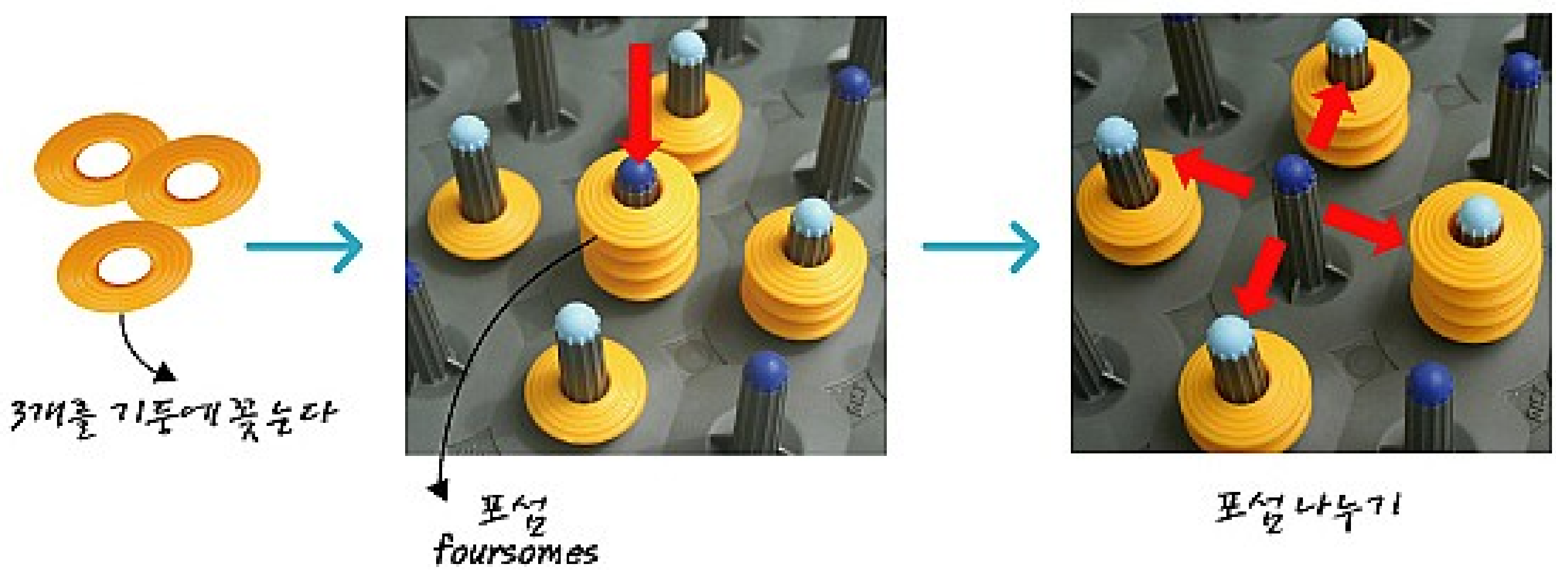
Wolfgang Kinzel Ido Kanter

게임 준비하기

1. 하늘색과 청색 캡을 기둥 위에 교대로 꽂고, 게임판 바닥에는 노란색 캡을 꽂습니다.
2. 청색 캡의 기둥에는 디스크 1개를, 하늘색 캡의 기둥에는 디스크 2개를 꽂습니다.

게임 시작하기

1. 각 게임자에게 **7개**씩 디스크를 나누어 주고 나머지는 주머니에 넣어 뱅크로 둡니다.
2. 자기 차례가 되면 **3개**의 디스크를 1~3개의 기둥에 꽂습니다.
   (만약 디스크가 3개 이하로 남아 있다면 남은 개수 만큼 사용합니다.)
3. 한 기둥에 디스크가 **4개** 꽂히면, 접한 4개의 다른 색 기둥에 디스크를 한 개씩 나누어 꽂습니다.
   (디스크 4개가 한 기둥에 꽂힌 것을 포섬 foursomes 이라고 한다.)



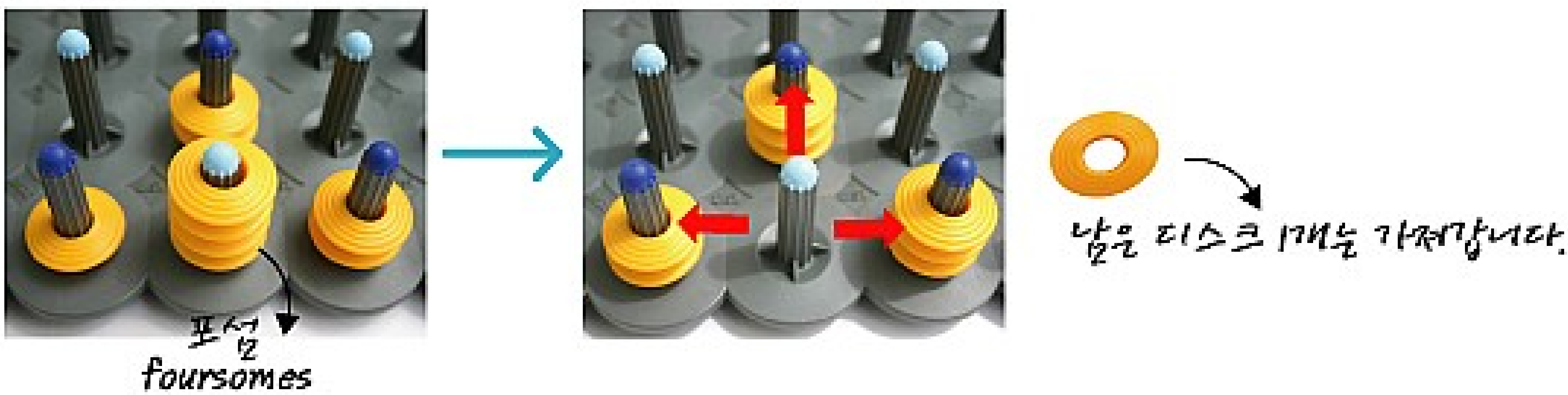
4.게임자가 4개의 디스크를 인접한 4개의 다른 색 기둥에 나누어 꽂을 때마다 뱅크에서 **1개**의 디스크를 가져옵니다.

5.또 다른 색 기둥에 4개의 디스크가 채워지면, 같은 방법으로 4개의 디스크를 나눠 꽂습니다.

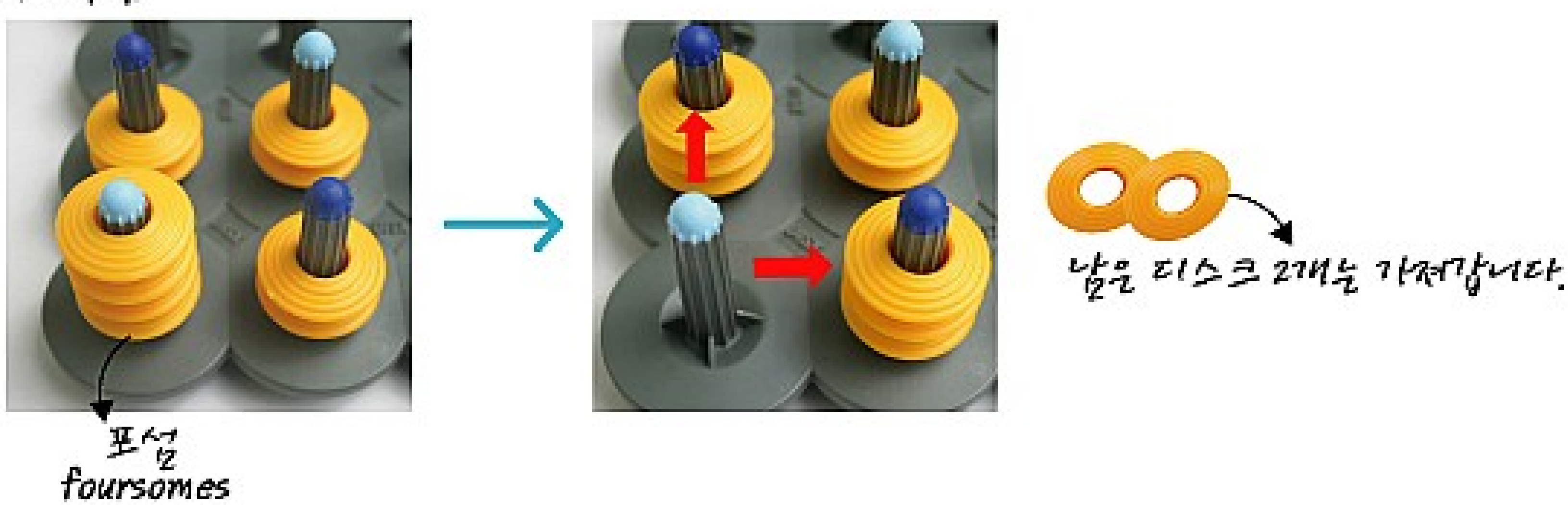
6. 4개의 디스크를 나누어 꽂다가 5개가 꽂힐 경우, 디스크 2개를 가져갑니다.
(쌓여진 5개 중에서 남은 1개와 뱅크에서 받은 것 1개)

7. 게임판의 가장자리나 모서리에 4개의 기둥이 만들어지면

<가장자리>



<모서리>



S. S. Manna

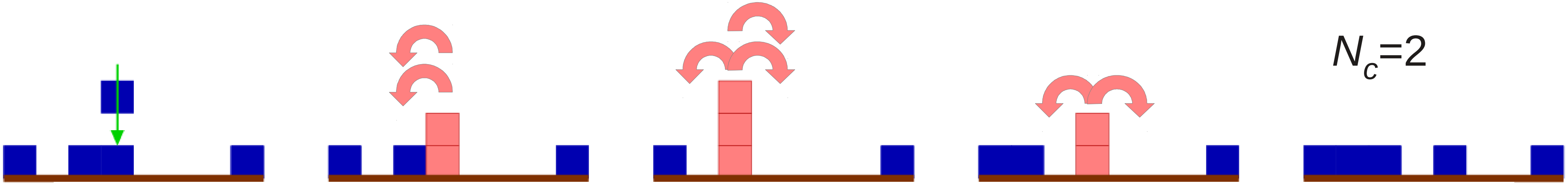
Manna sandpile model

S. S. Manna, J. Phys. A 24, L363 (1991)

...like BTW, but with a probabilistic update

- Sites with $N>N_c$ particles are active.
- Update all active sites simultaneously.
- Move each particle to a randomly selected nearest neighbor

1D:



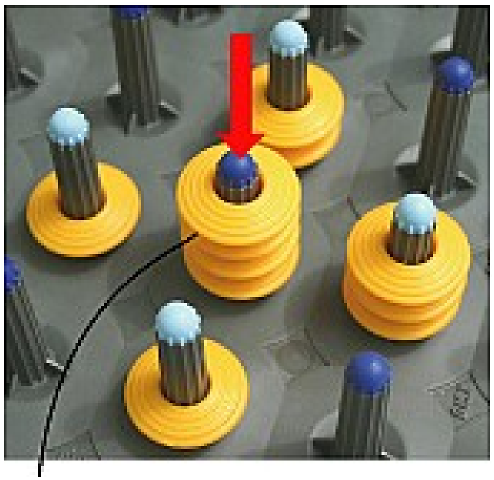
Universality hypothesis

Self-organizing critical systems with

- a single degree of freedom per site,
- local interactions,
- update rules with „enough randomness“,
- no unconventional symmetries

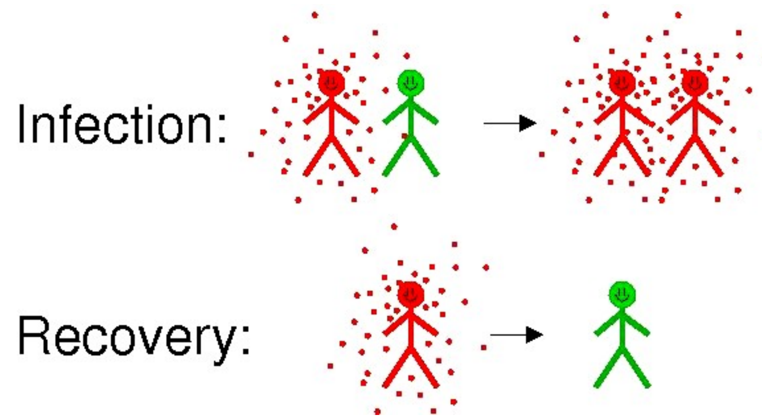
belong to the same universality class,
called **Manna class.**

Manna model



1. Slow driving
2. Sudden avalanches
3. Dissipation at the boundaries

Contact process



Conserved spreading agent

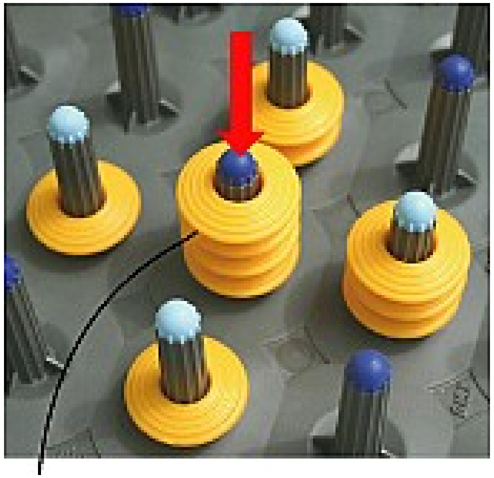
Avalanches controlled by the average density

Conservation law broken by slow driving and dissipation at the boundaries

Non-conserved spreading agent

Spreading controlled by the infection rate

Conserved Manna model



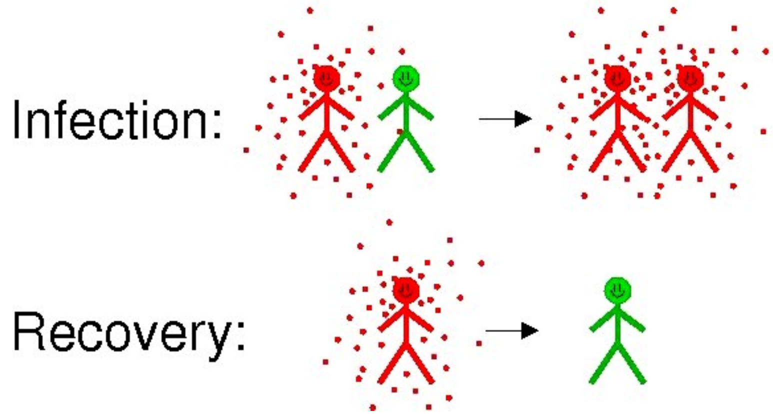
1. Slow driving
2. Sudden avalanches
3. Dissipation at the boundaries

Conserved spreading agent

Transition controlled by the average density

Conservation law broken by slow driving and dissipation at the boundaries

Contact process (DP)



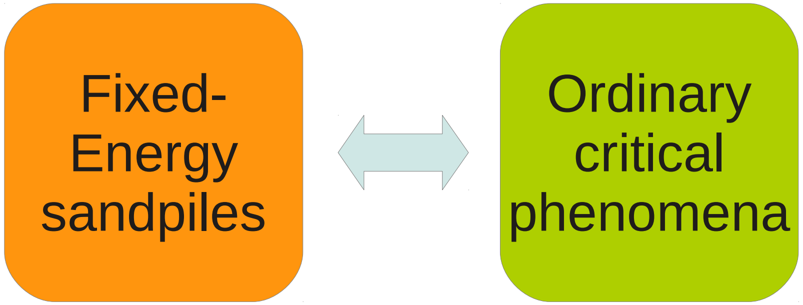
Non-conserved spreading agent

Transition controlled by the infection rate

Concept of „Fixed-Energy Sandpiles“

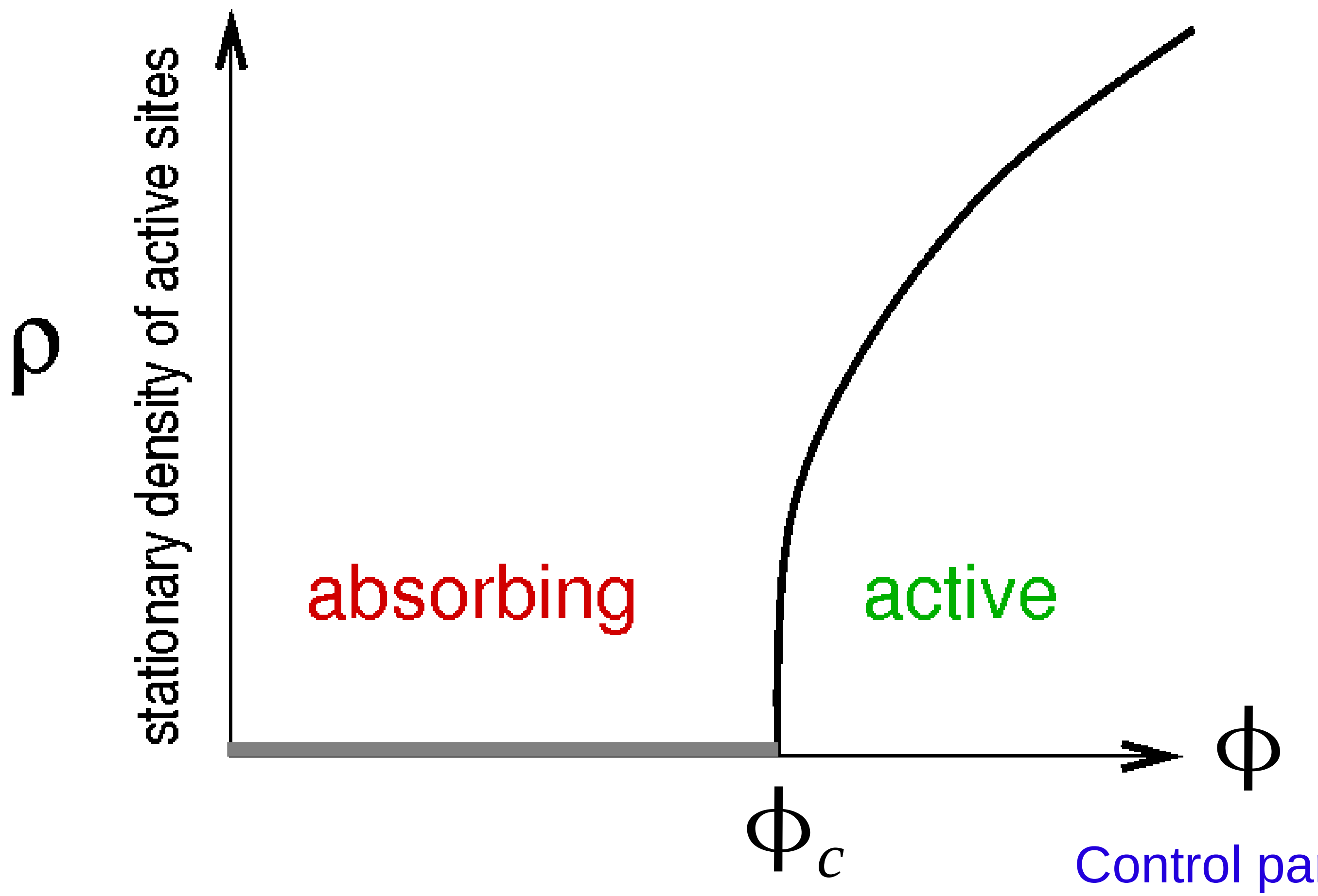
Vespignani et al, PRL 81, 5676 (1998) & Phys. Rev. E 57, 6345

- Use the same update rules for toppling
- No driving
- No dissipation at the boundaries (infinite system or periodic b.c.)
- Use conserved density as control parameter



Concept of „Fixed-Energy Sandpiles“

Order parameter



Control parameter: Density of particles

The most important relations between SOC and ordinary exponents

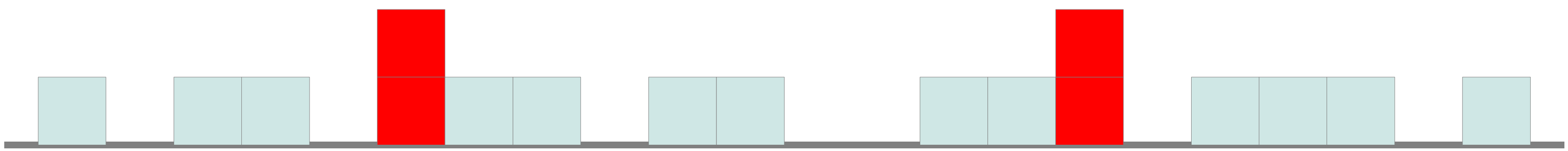
Exponent for avalanche sizes: $$\tau = \frac{1+\theta+2\delta}{1+\theta+\delta} = \frac{d+z}{d+z-\beta/\nu}$$

Exponent for avalanche times: $$\tau_A = 1+\delta = 1+\beta/\nu_{\|}$$

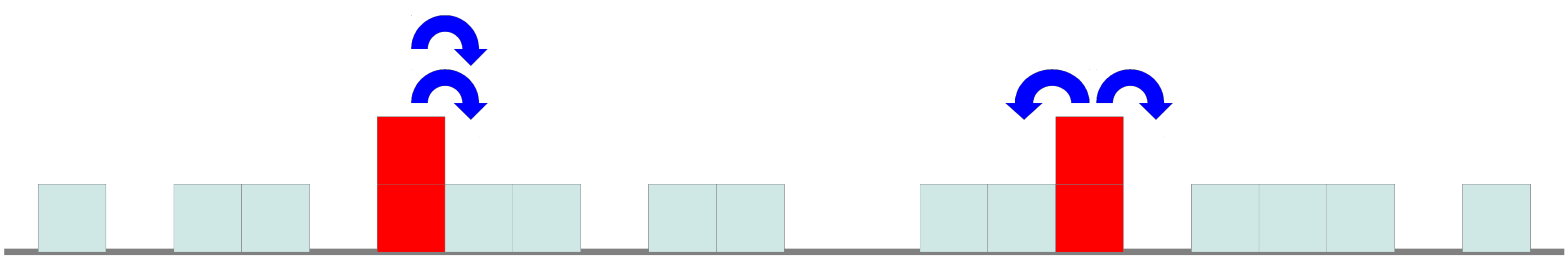
Munoz, Dickman, Vespignani and Zapperi, Phys. Rev. E 59, 6175 (1999)

Conserved Manna Model (CMM)

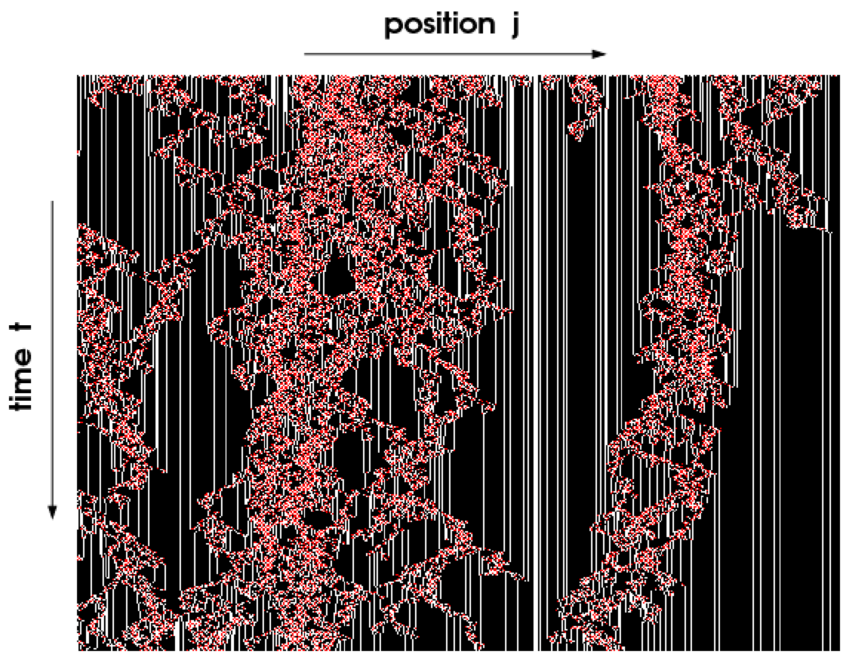
- Prepare an initial state with a certain density Φ



- Apply Manna updates with periodic boundary conditions

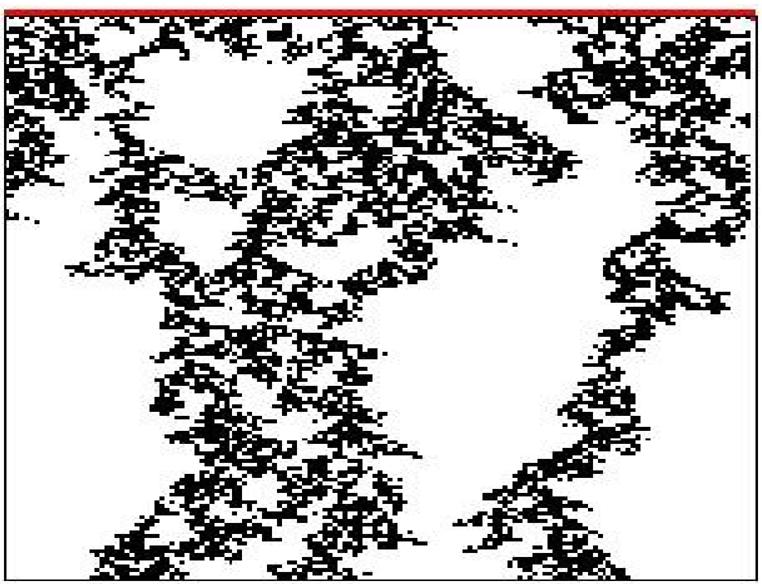


Conserved Manna



Many absorbing states

Contact process



Single absorbing state

Textbooks: Exponents in 1D

Exponent	Conserved Manna (Lübeck)	Directed Percolation
β	0.38(5)	0.276
$\nu_{\parallel}$	2.4(1)	1.733
$\nu_{\perp}$	1.76(9)	1.096
α	0.14(2)	0.159
δ	0.17(2)	0.159
θ	0.35(3)	0.313
z	1.39(4)	1.580

Standard exponents (β, $\nu_{\parallel}$, $\nu_{\perp}$)

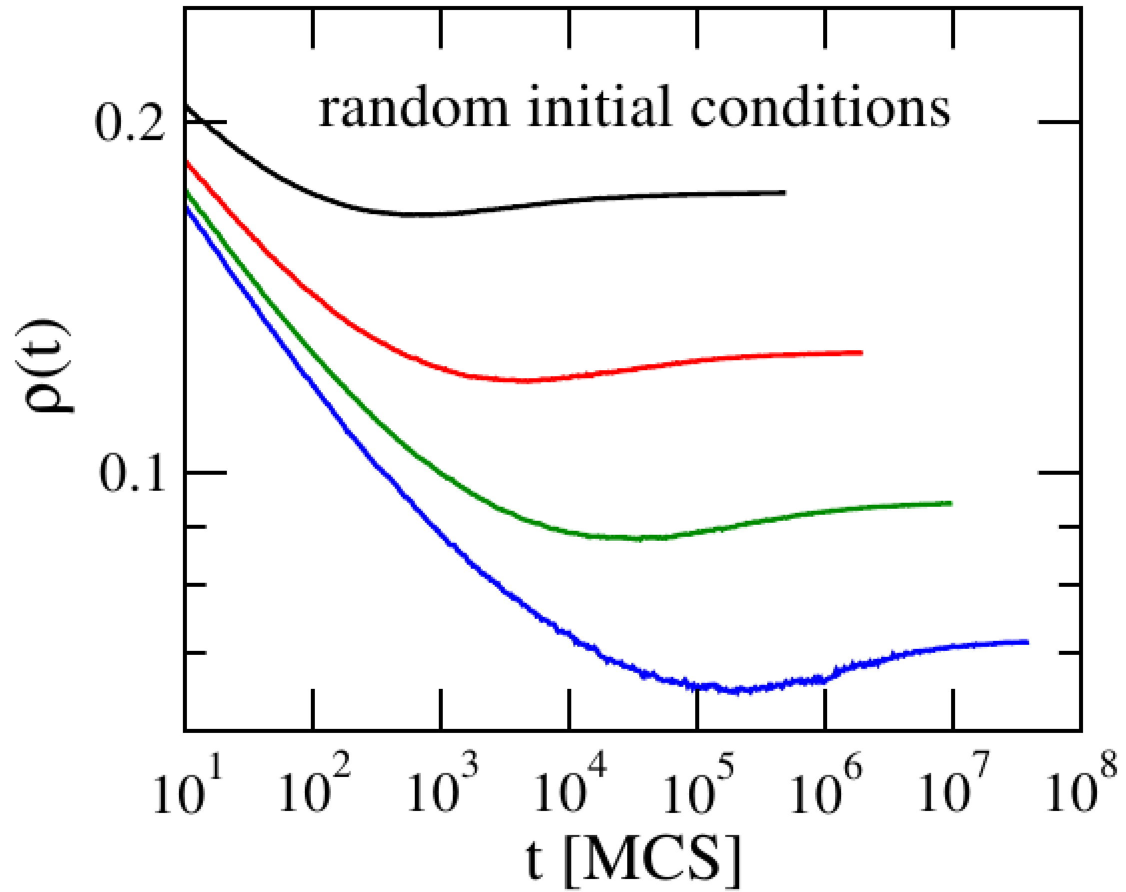
Dynamical exponents (α, δ, θ, z)

Kockelkoren and Chate, ArXiv cond-mat 0306039 (2003)
Ramasco, Munoz, and da Silva Santos, PRE 69, 045105R (2004)
R. Dickman, PRE 73, 036131 (2006)

Exponent	Conserved Manna (Lübeck)		Directed Percolation
β	0.38(5)	0.289(12) 0.29(2) 0.28(2)	0.276
$\nu_{\|\|}$	2.4(1)	1.95(15)	1.733
$\nu_{\perp}$	1.76(9)	1.35(10) 1.31(15)	1.096
α	0.14(2)	0.140(5) 0.14(1)	0.159
δ	0.17(2)		0.159
θ	0.35(3)		0.313
z	1.39(4)	1.50(4) 1.55(3) 1.47(4)	1.580

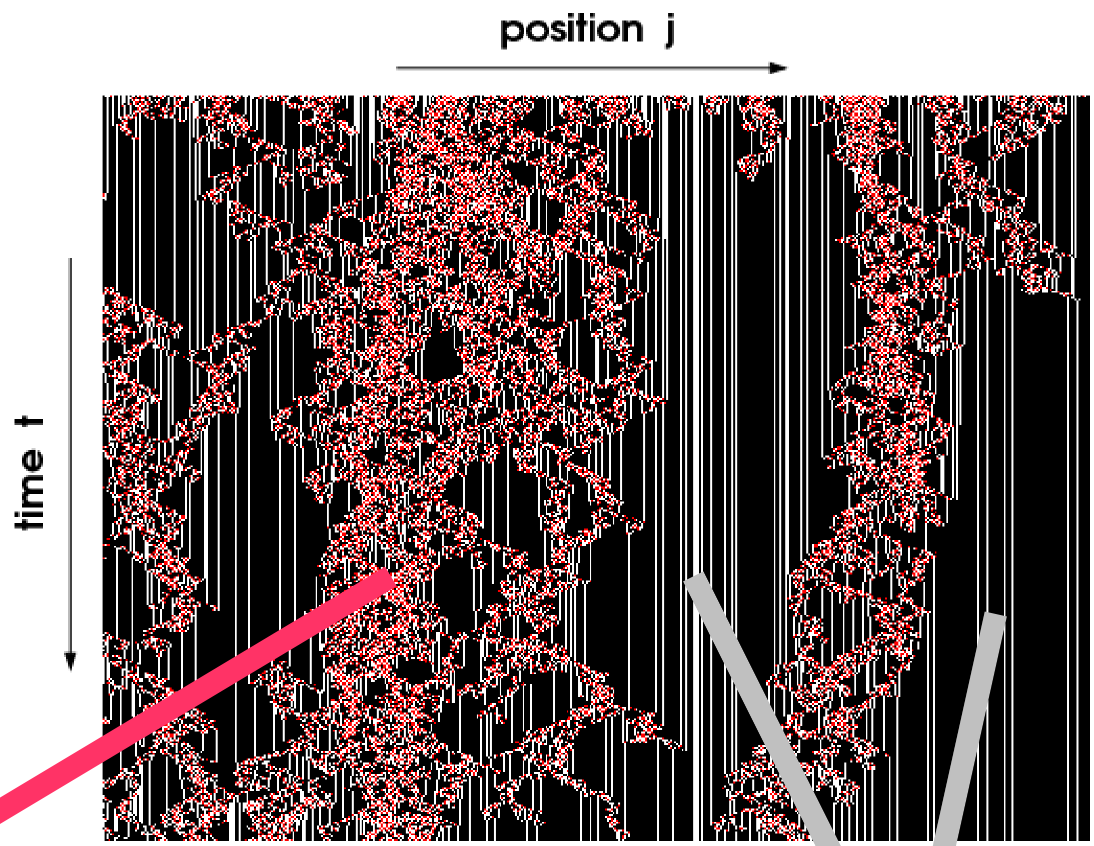
Conserved Manna class – what is disturbing:

- same mean field as for DP
- violation of scaling: $\alpha \neq \delta$ but $\beta = \beta'$
- split of universality in the CTTP
- undershooting



2. The background field

Consider activity and background separately:



Density of active sites ρ(x,t)

Background density Φ(x,t)

DP Langevin equation:

$$\dot{\rho}(\vec{x},t) = a\rho(\vec{x},t) - b\rho^2(\vec{x},t) + D\nabla^2\rho(\vec{x},t) + \eta(\vec{x},t)$$

$a\rho$: A → 0, A → 2A

$b\rho^2$: 2A → A

$D\nabla^2\rho$: Diffusion

η: Noise

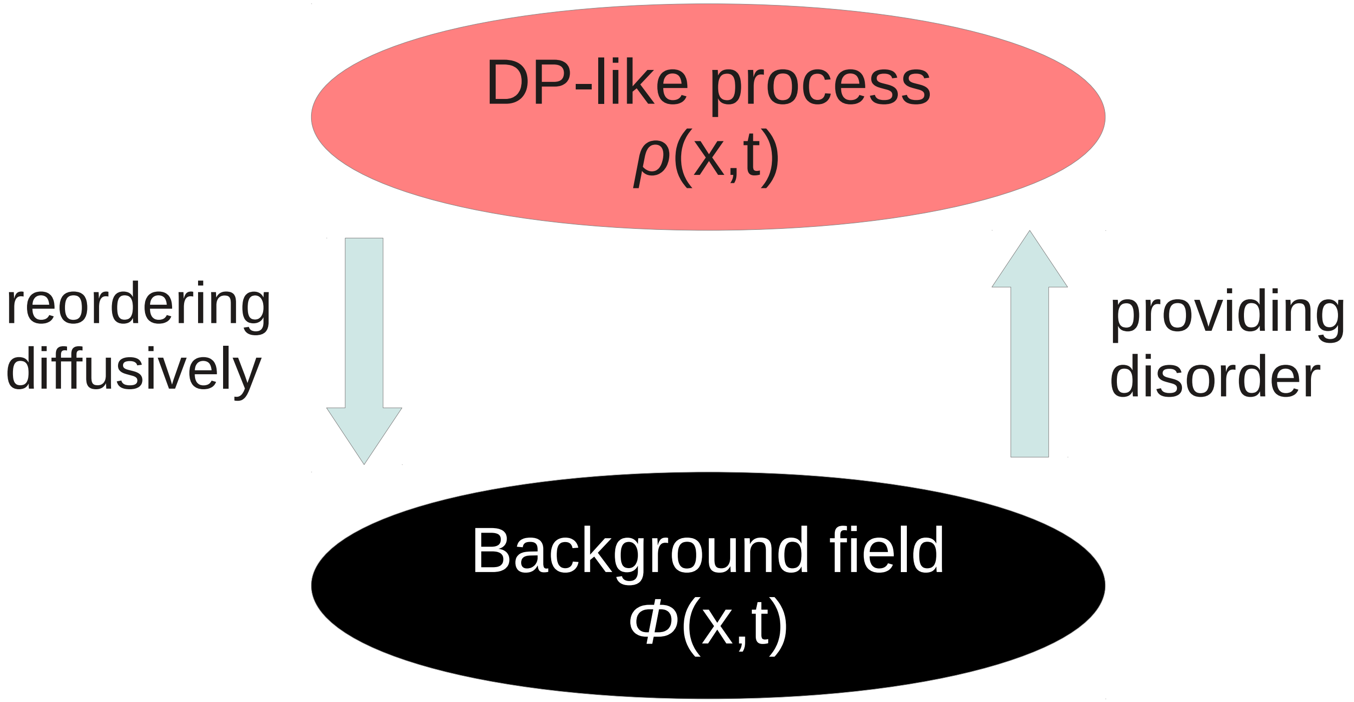
Conserved Manna Langevin equations:

$$\dot{\rho}(\vec{x},t) = \phi(\vec{x},t)\rho(\vec{x},t) - b\rho^2(\vec{x},t) + D\nabla^2\rho(\vec{x},t) + \eta(\vec{x},t)$$

$$\dot{\phi}(\vec{x},t) = D'\nabla^2\rho(\vec{x},t)$$

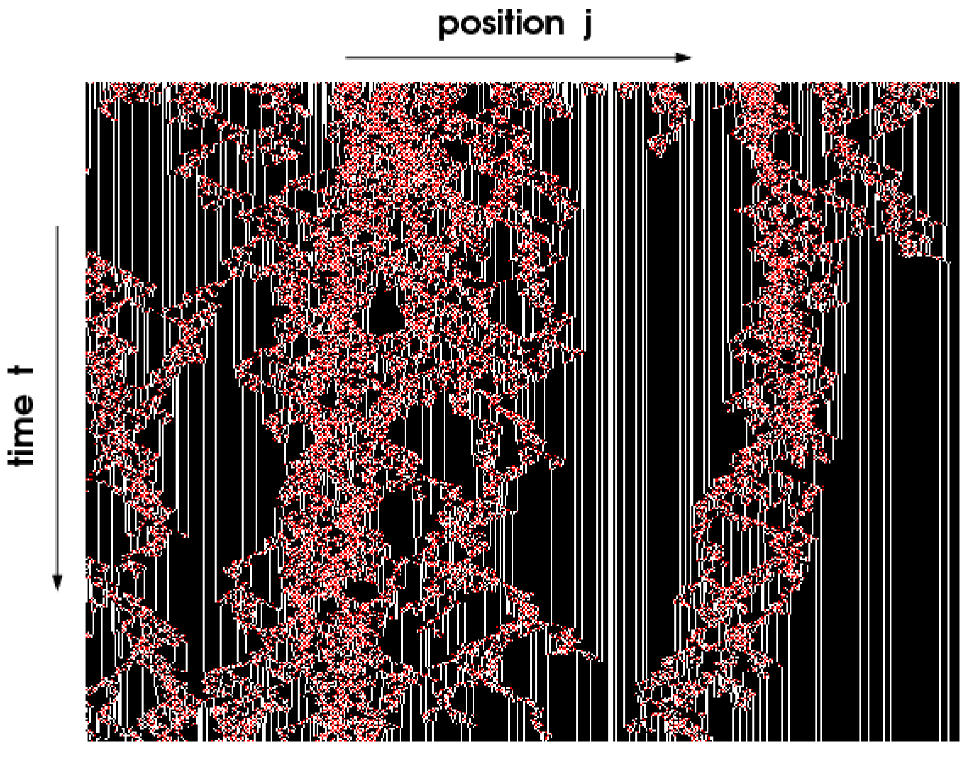
A. Vespignani, et al., Phys. Rev. Lett. 81, 5676 (1998). Phys. Rev. E 62,4564 (2000).

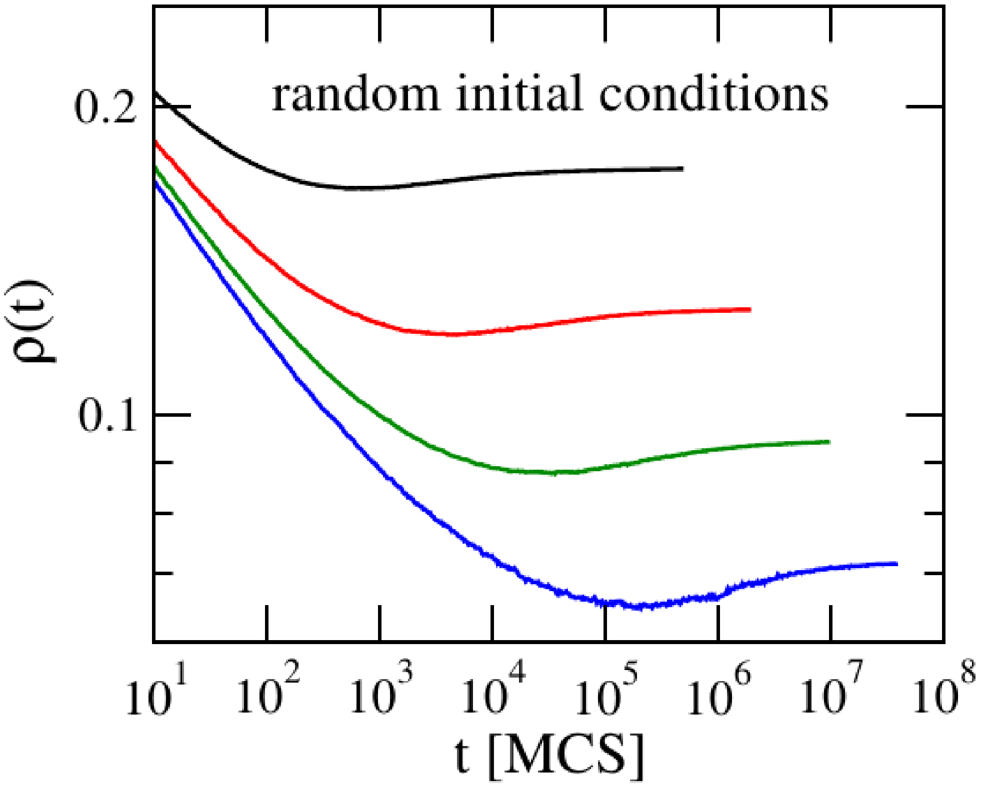
$$\dot{\rho}(\vec{x},t) = \phi(\vec{x},t)\rho(\vec{x},t) - b\rho^2(\vec{x},t) + D\nabla^2\rho(\vec{x},t) + \eta(\vec{x},t)$$
$$\dot{\phi}(\vec{x},t) = D'\nabla^2\rho(\vec{x},t)$$
$$\langle\eta(\vec{x},t)\eta(\vec{x}',t')\rangle = \Gamma\rho(\vec{x},t)\delta^d(\vec{x}-\vec{x}')\delta(t-t')$$



Main idea:

The undershooting is caused by the randomness of the background field.



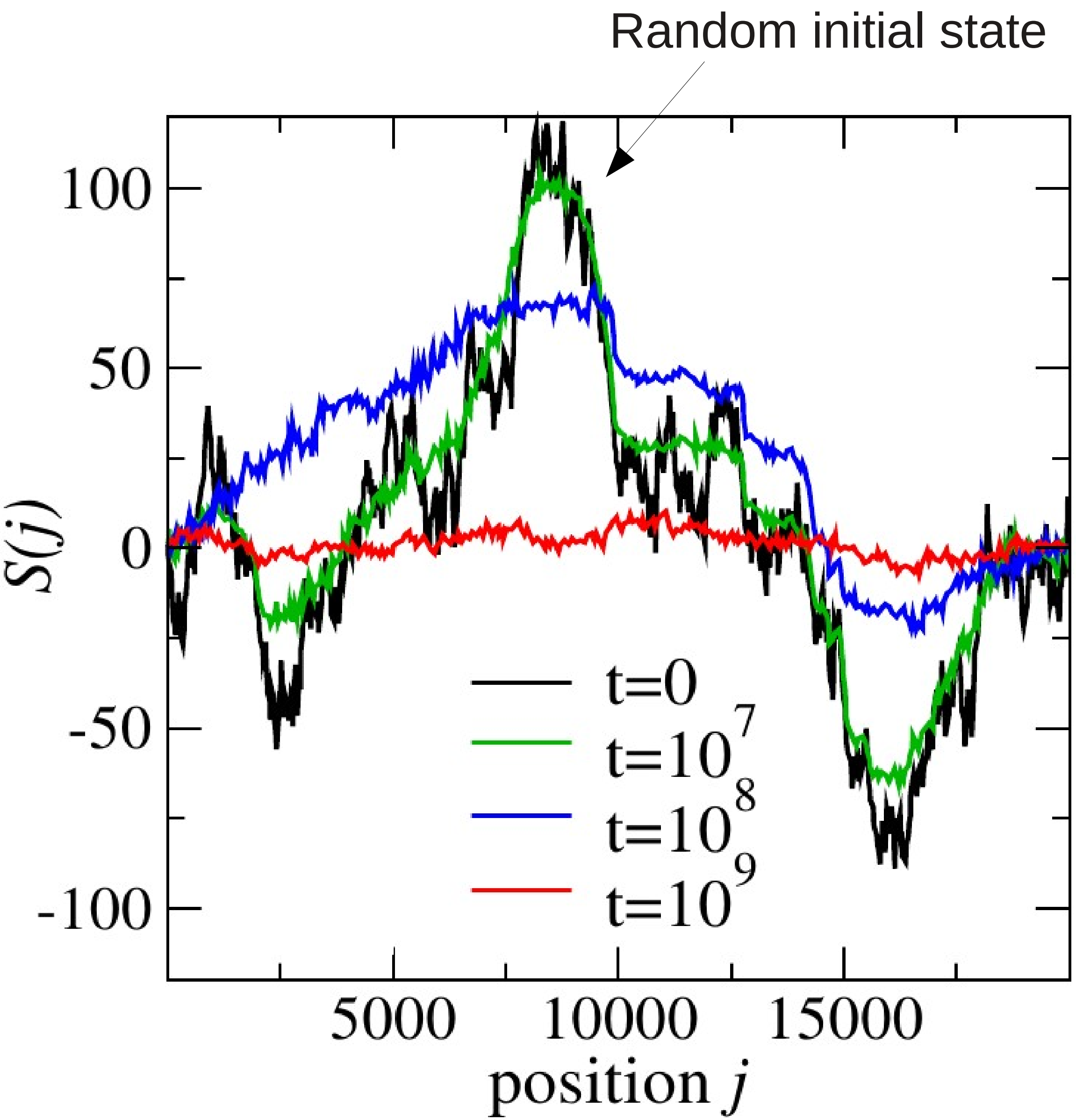


Roughness of the background

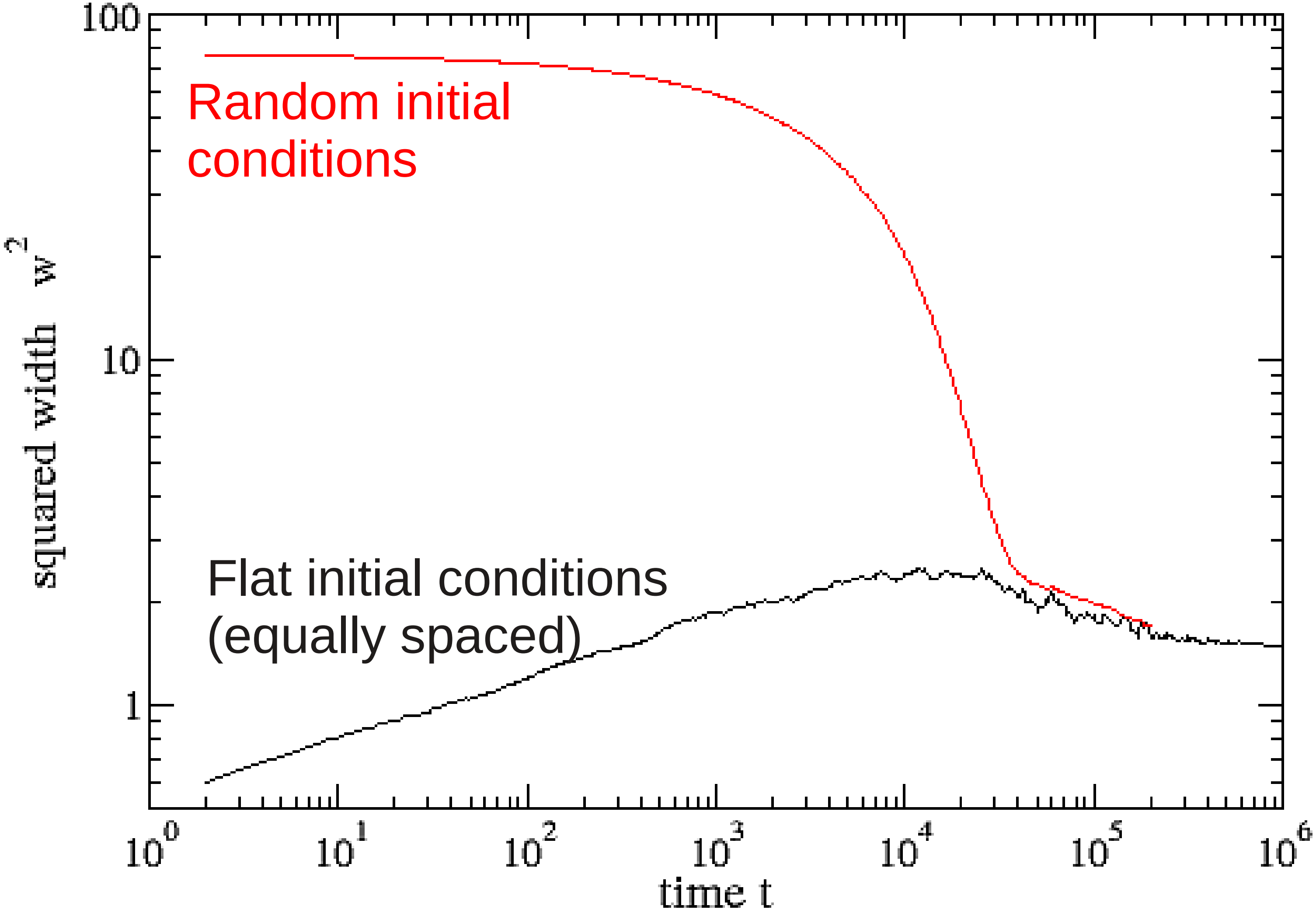
$$\phi=\frac{1}{N}\sum_{i=1}^{N} h_i$$

$$S(j)=\left(\sum_{i=1}^{j} h_i\right)-j\,\phi$$

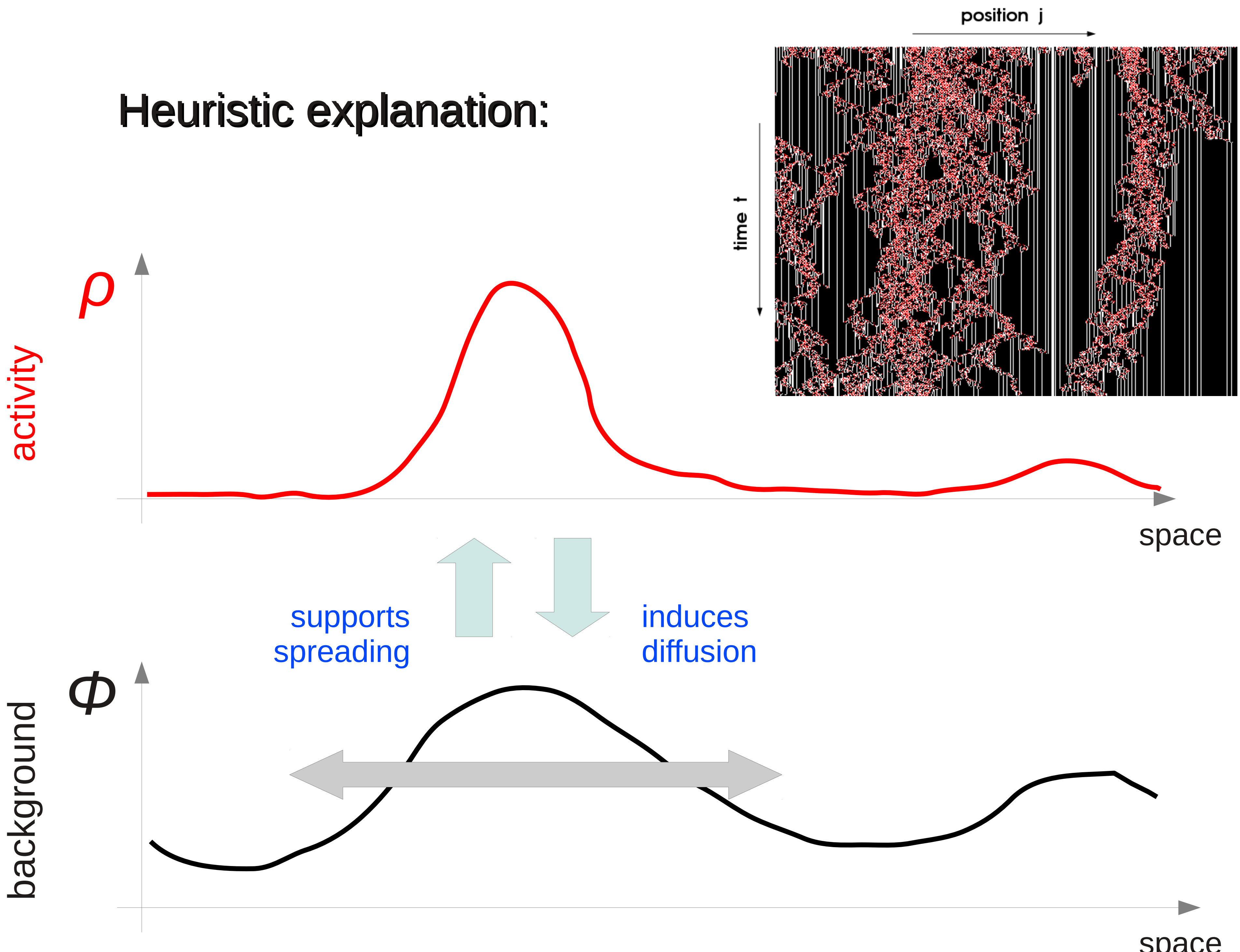
Cumulative sum of particles minus expected average



Width of S(j) at criticality.



Heuristic explanation:



Observation:

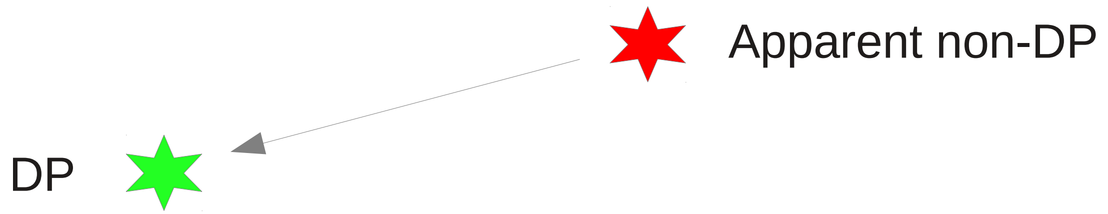
The background field provides a slowly evolving quenched randomness.

The process itself has the tendency to homogenize the background field

This works most efficiently at criticality.

Question:

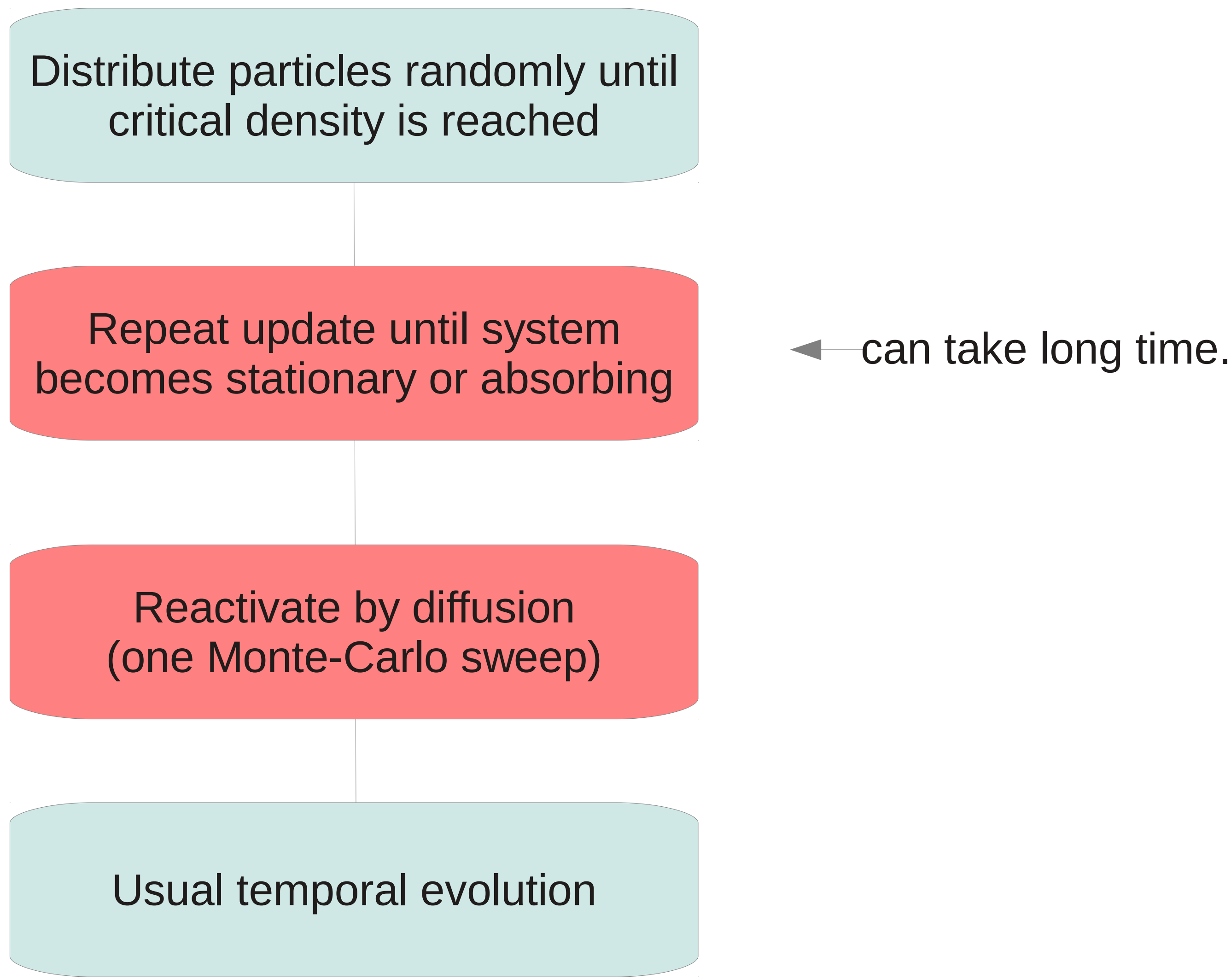
Does it go back to DP?

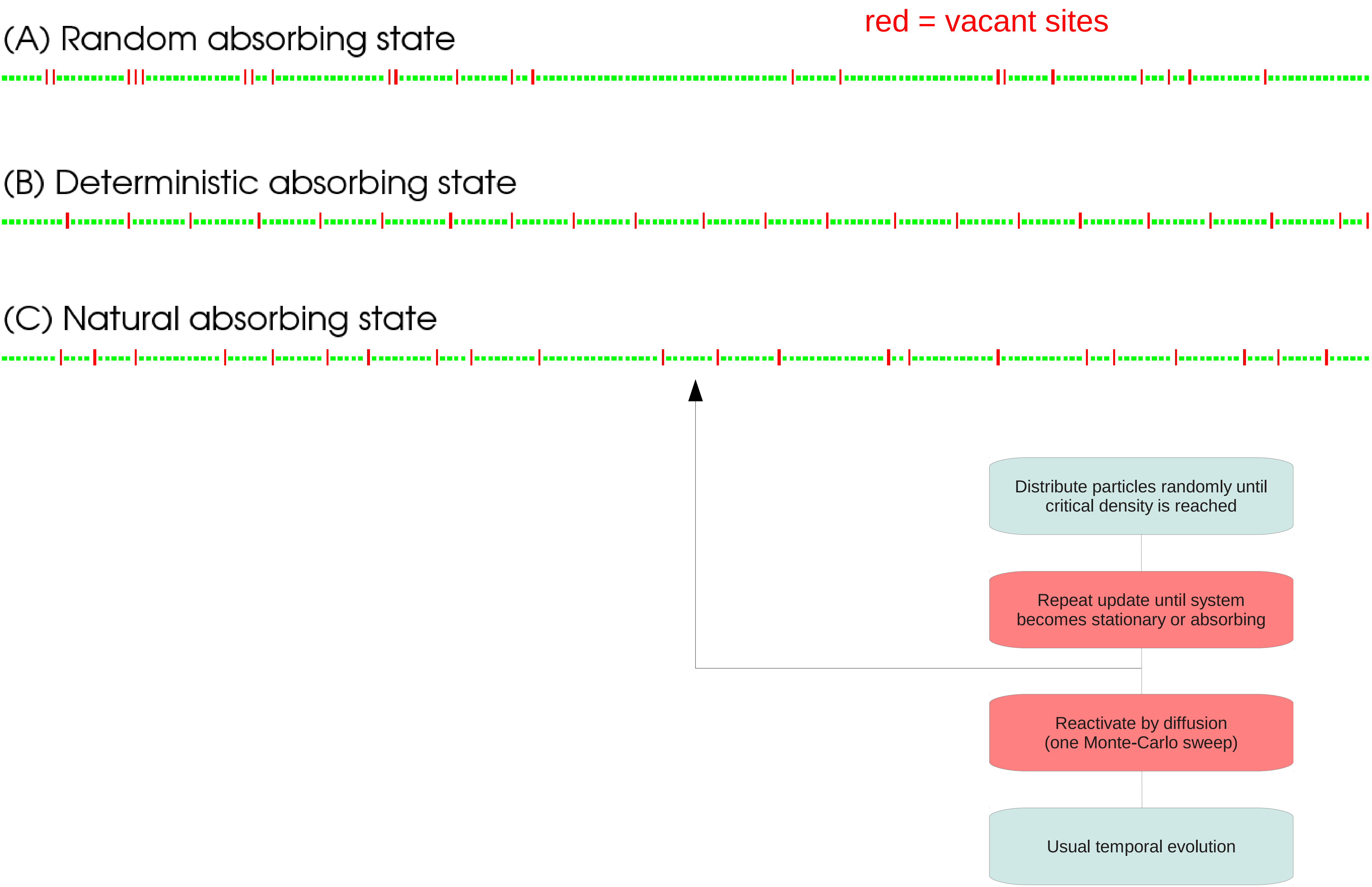


3. Natural initial states

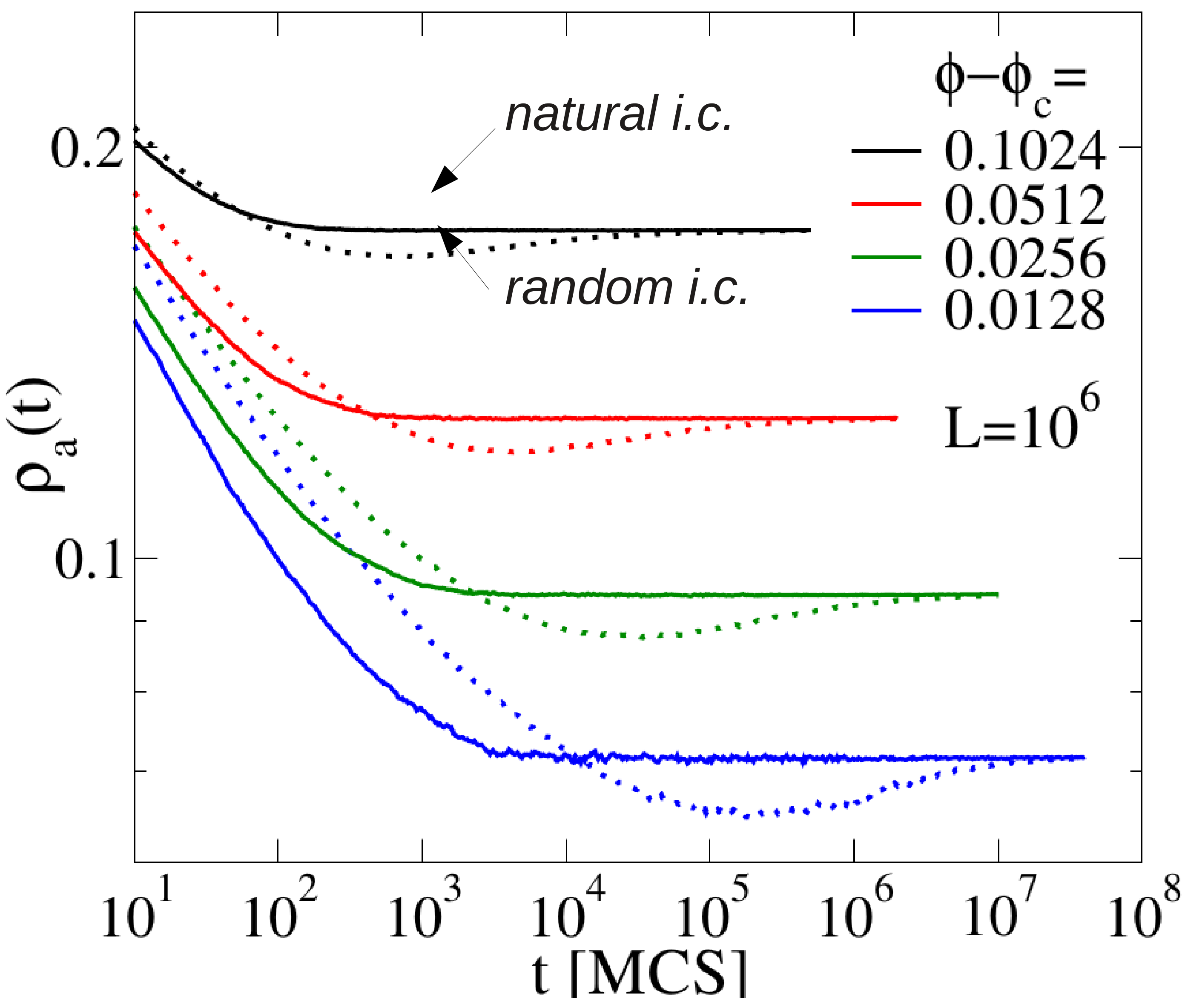
Natural states are configurations where the background is already homogenized by the process itself.

'Natural' homogeneous initial states




red = vacant sites
(A) Random absorbing state
(B) Deterministic absorbing state
(C) Natural absorbing state
Distribute particles randomly until critical density is reached
Repeat update until system becomes stationary or absorbing
Reactivate by diffusion (one Monte-Carlo sweep)
Usual temporal evolution

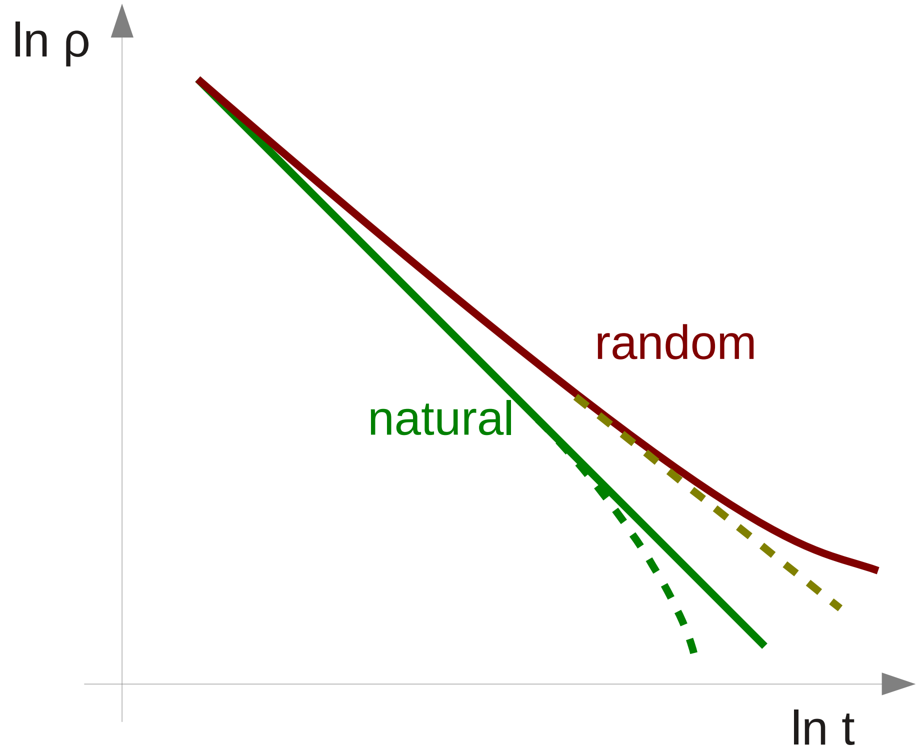
Natural initial states cure undershooting

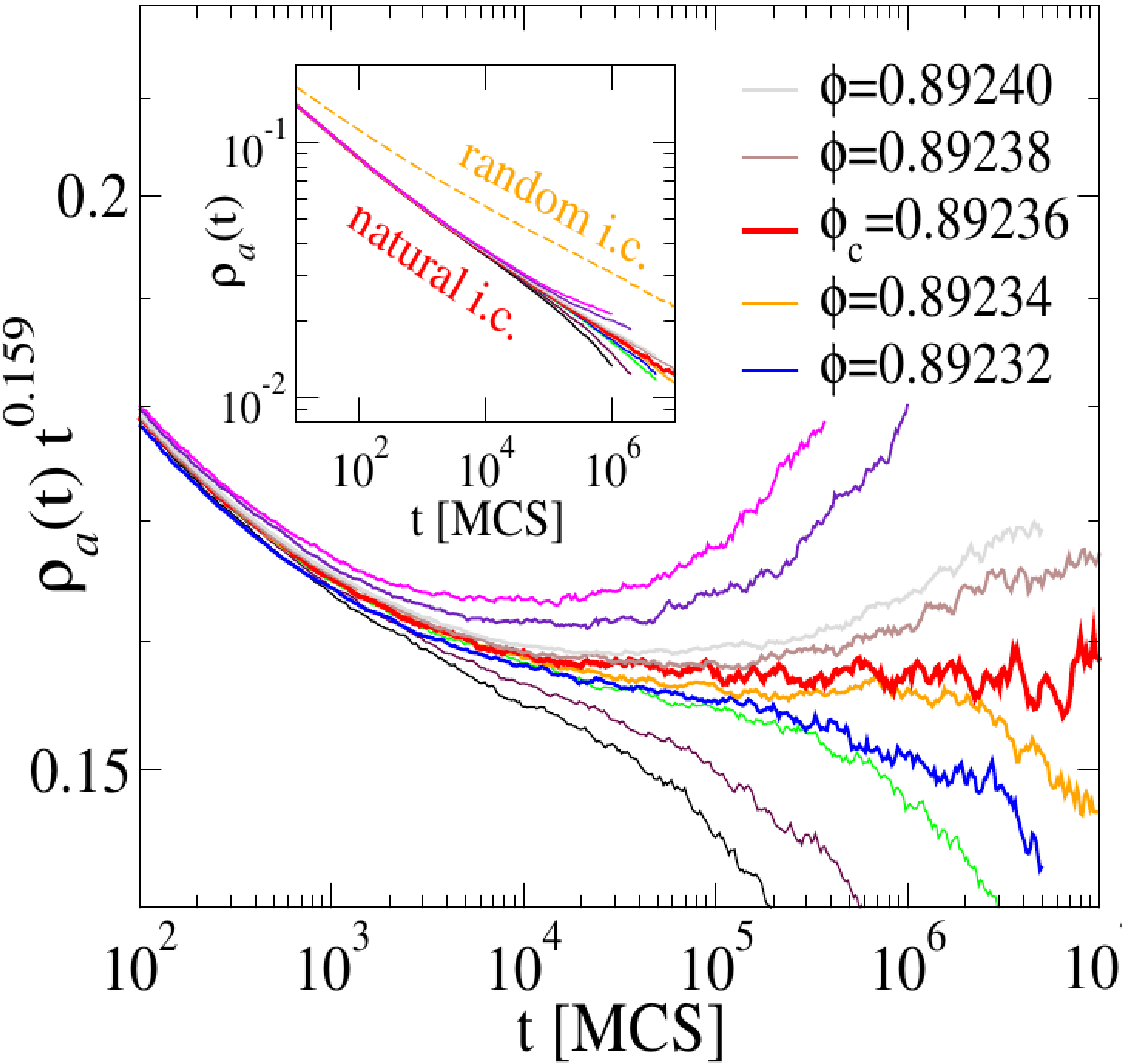


→ slightly different critical density

Review by Lübeck: $\Phi_c = 0.89\mathbf{199}(5)$

Our estimate: $\Phi_c = 0.89\mathbf{236}(4)$

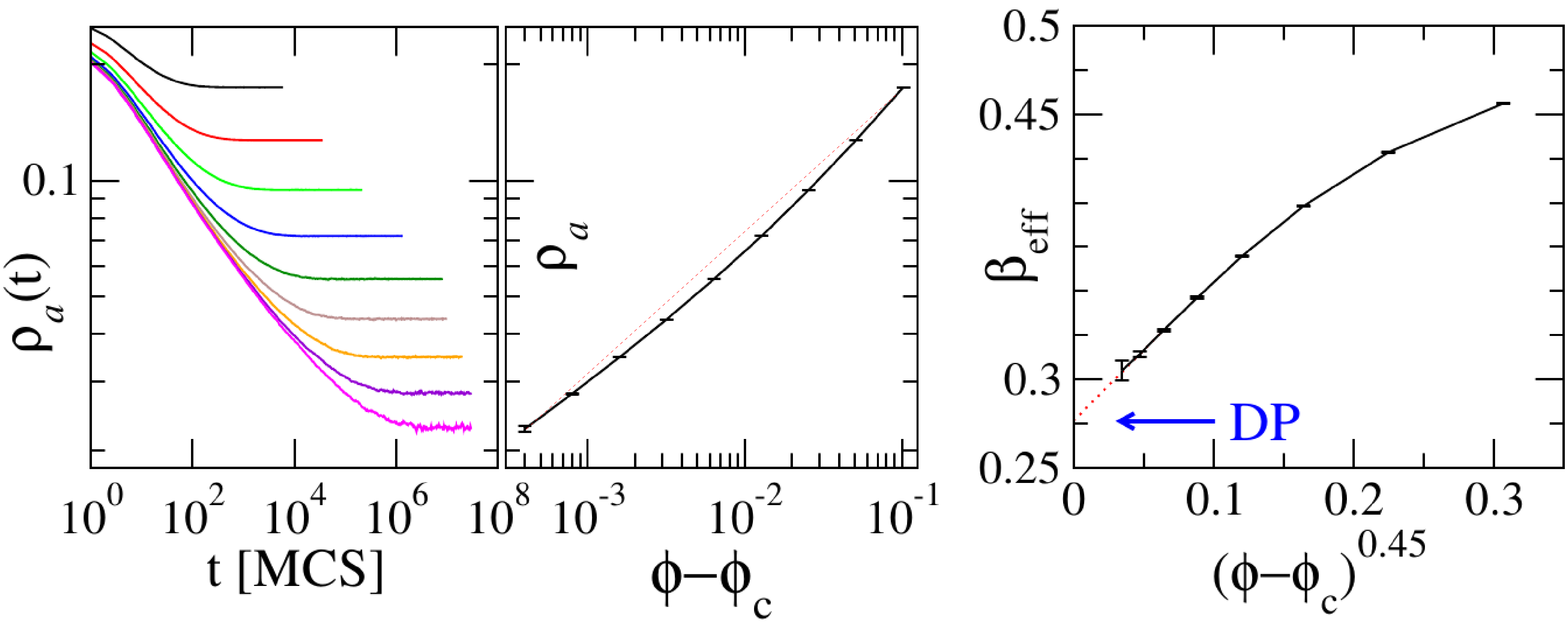




φ=0.89240
φ=0.89238
φ_c=0.89236
φ=0.89234
φ=0.89232
random i.c.
natural i.c.
ρ_a(t)
t [MCS]
ρ_a(t) t^0.159
0.2
0.15
10^2
10^3
10^4
10^5
10^6
10^7
10^-1
10^-2

Static exponent β

$$\rho_{stat} \sim (\phi - \phi_c)^{\beta}$$



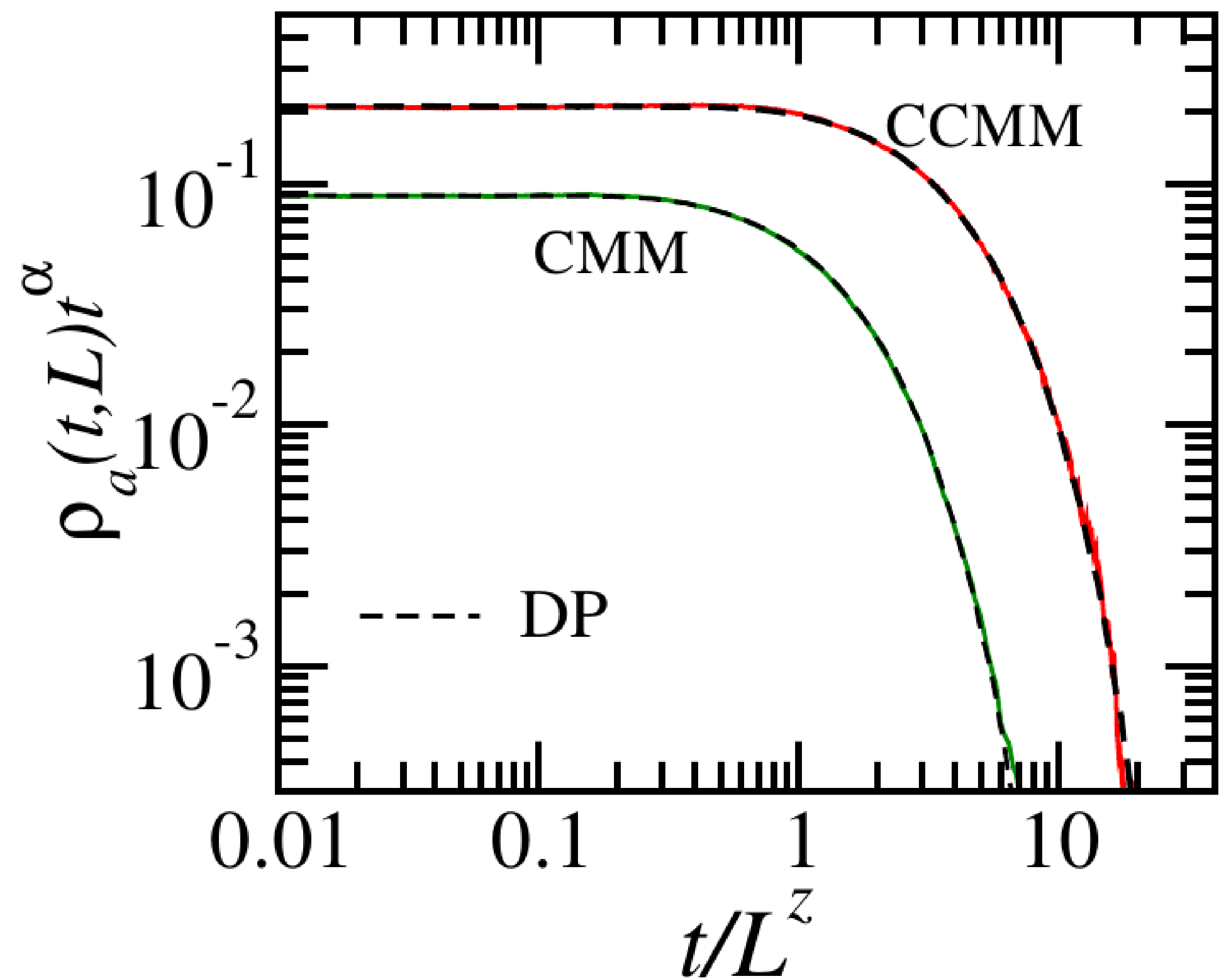
Even with natural initial states, there are unusual corrections to scaling.

Improved estimates for the critical exponents

Exponent	Conserved Manna (old)	Conserved Manna (new)	Directed Percolation
β	0.38(5)	<0.31 → 0.28	0.276486
$\nu_{\parallel}$	2.4(1)	1.75(5)	1.733847
$\nu_{\perp}$	1.76(6)	1.10(1)	1.096854
α	0.14(2)	0.159(3)	0.159464
δ	0.17(2)	0.17(2)	0.159464
θ	0.35(3)	0.34(2)	0.313686
z	1.39(4)	1.51(5)	1.58074

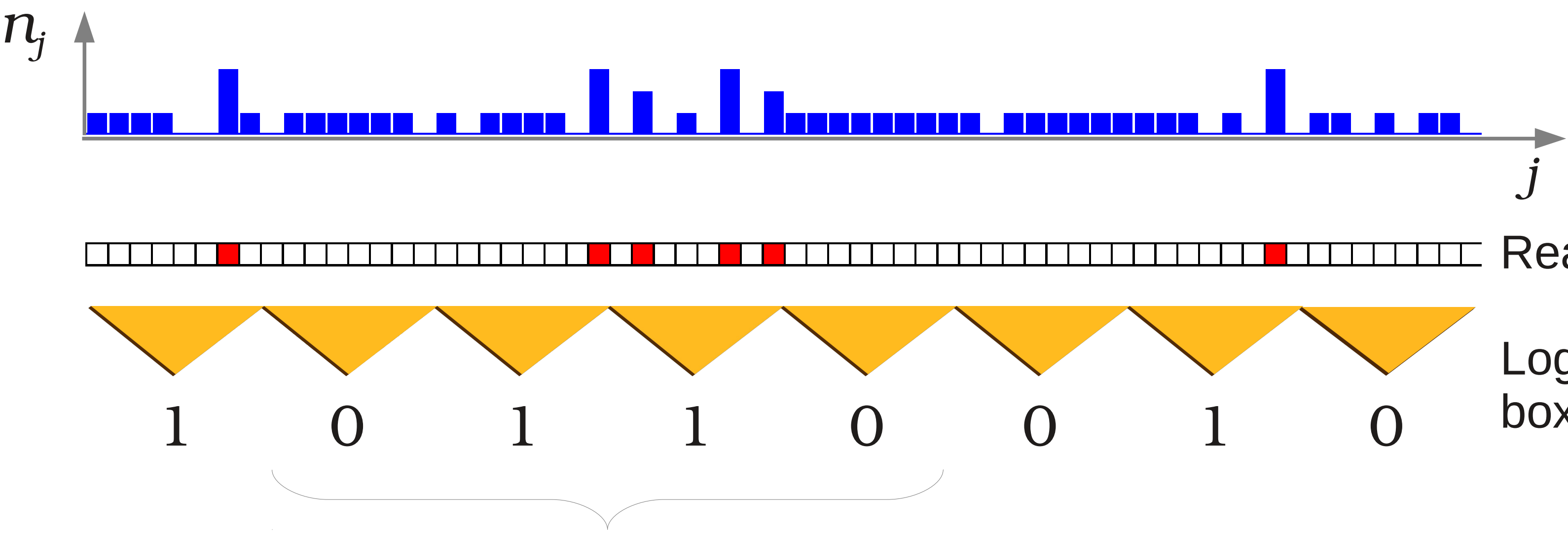
Finite-size scaling function

$$\rho(t,L)=t^{-\alpha}R(t/L^{z})$$



4. New method to identify DP

Real-space block renormalization



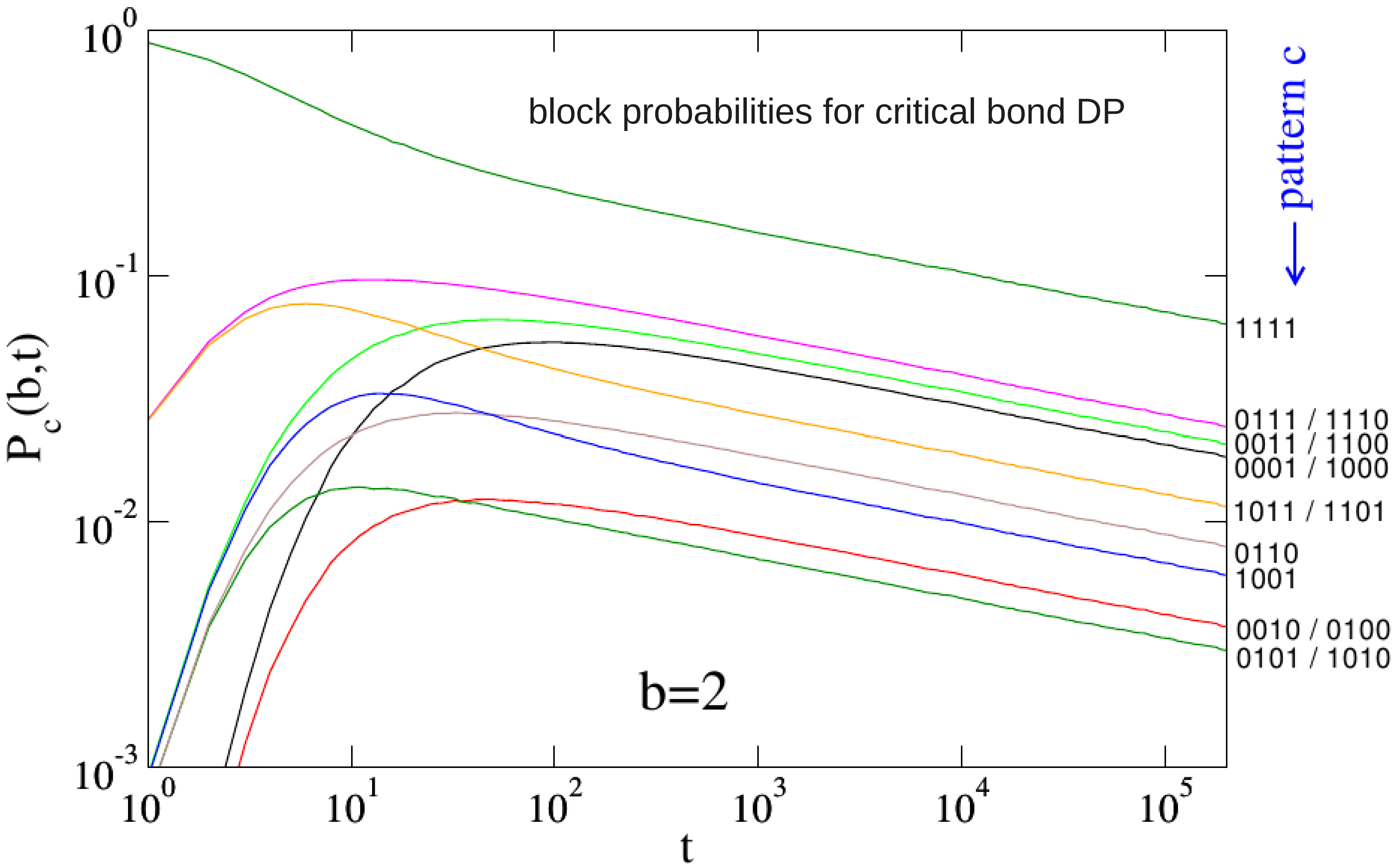
Read active sites

Logical OR in boxes of size *b*.

Read out configuration *c* with *n* bits

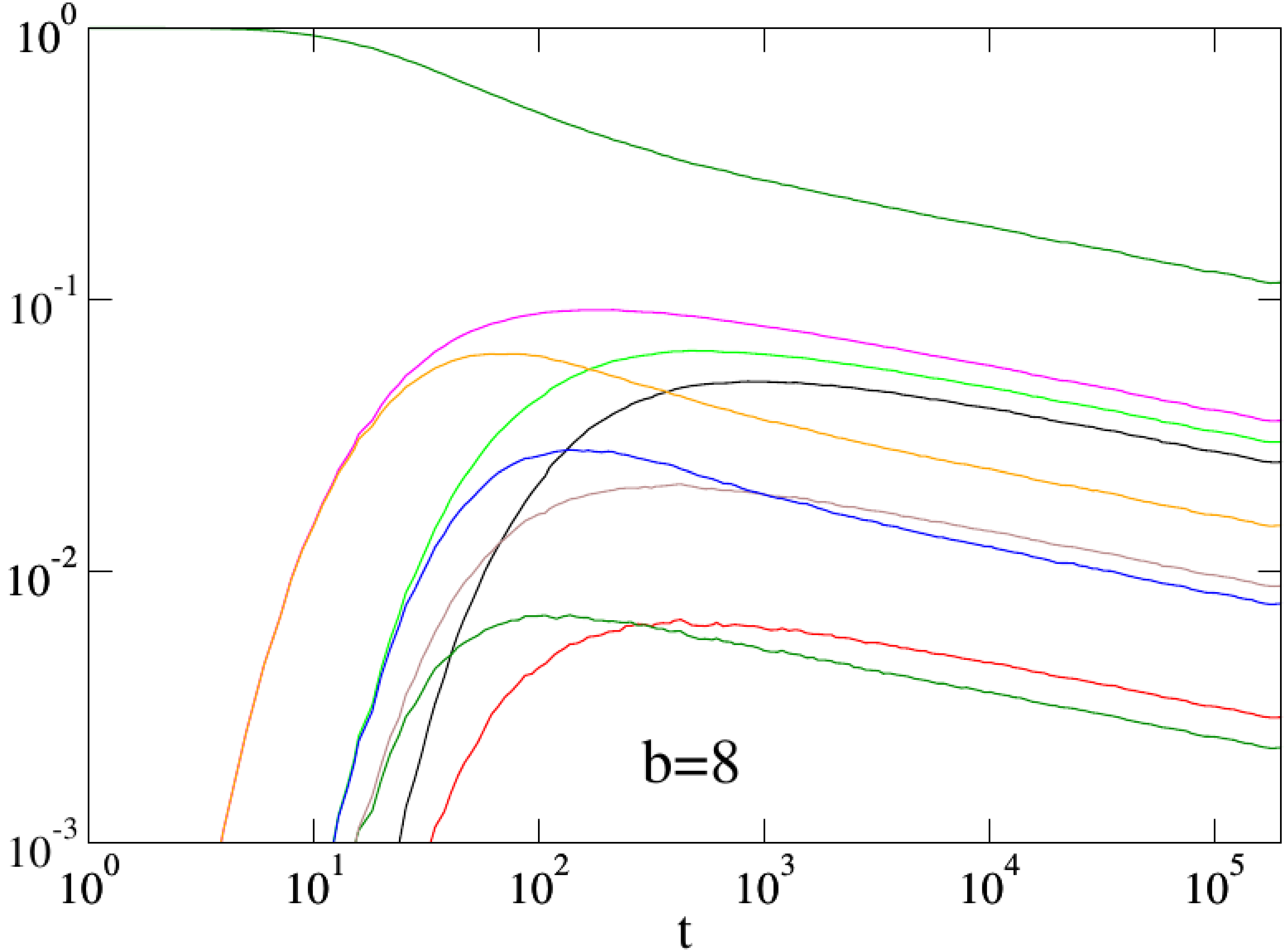
Probability $P_c(b,t)$ to find the coarse-grained bit pattern *c*.

Real-space block renormalization

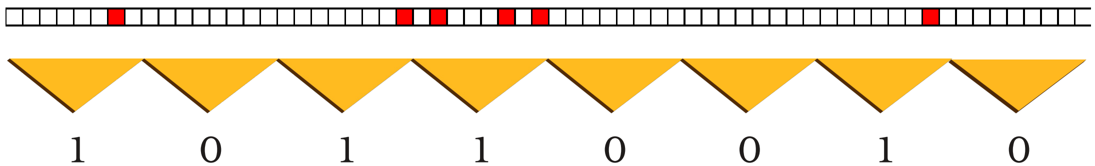


...they all decay like ρ(t)

Real-space block renormalization

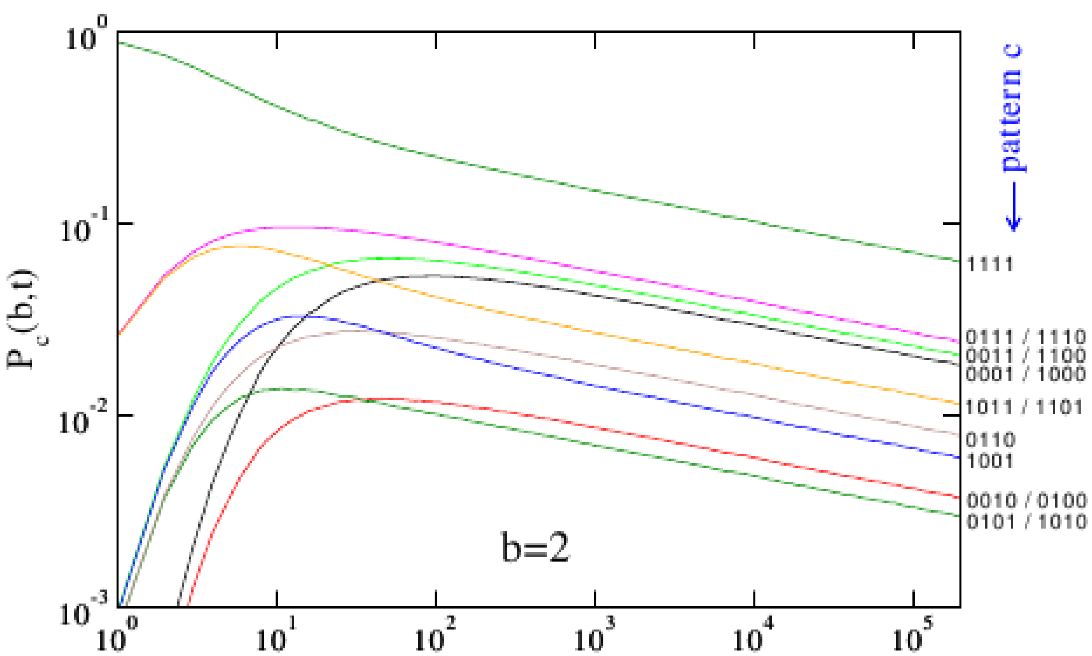


Real-space block renormalization



Logical OR in boxes of size *b*.

Read out configuration *c* with *n* bits

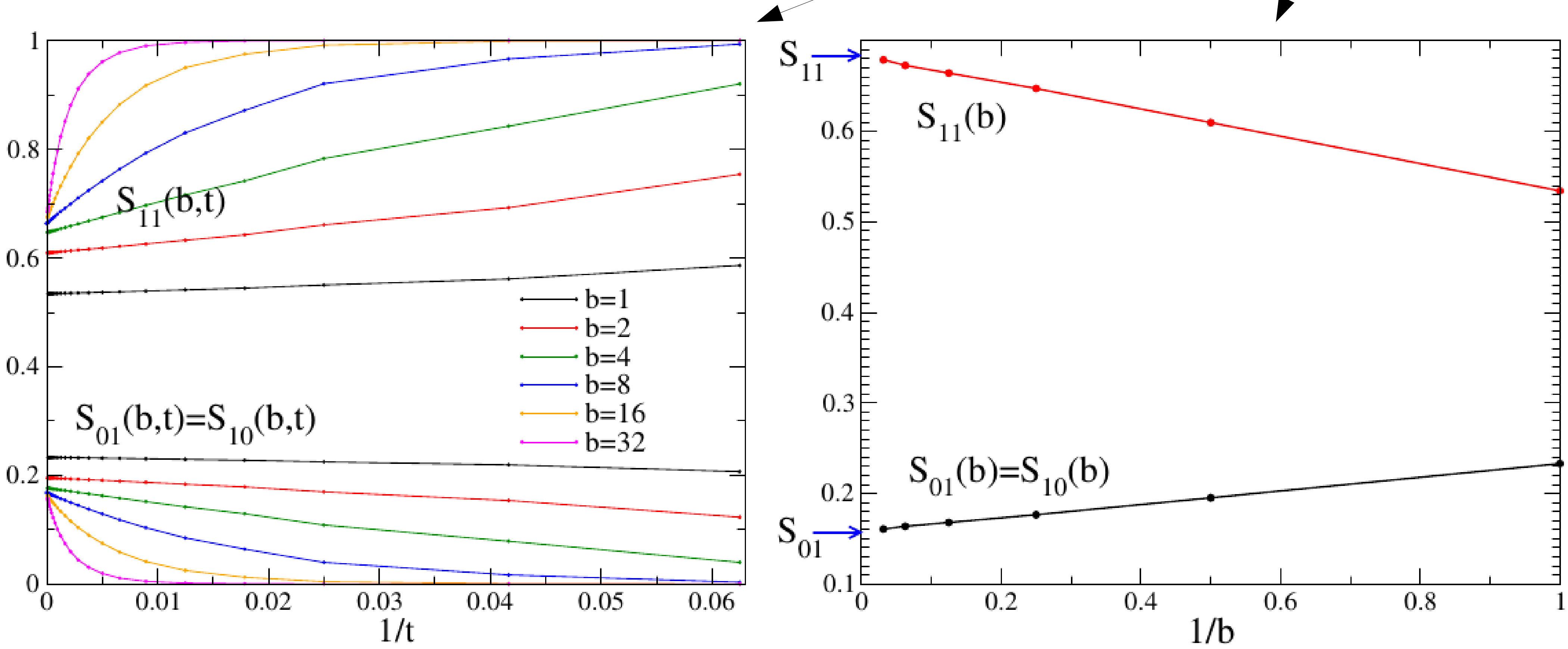


$$S_c(b,t) = \frac{P_c(b,t)}{1 - P_{00\ldots0}(b,t)}$$

$$S_c(b) = \lim_{t\to\infty} S_c(b,t)$$

0010001000101..
active sites

→ RG by logical OR → $P_c(b,t)$ → $S_c(b,t)$ → ($t\to\infty$) $S_c(b)$ → ($b\to\infty$) S_c



The quantities $S_c = \lim_{b\to\infty} S_c(b)$ are universal.

U. Basu and H.H., JSTAT P11023 (2011)

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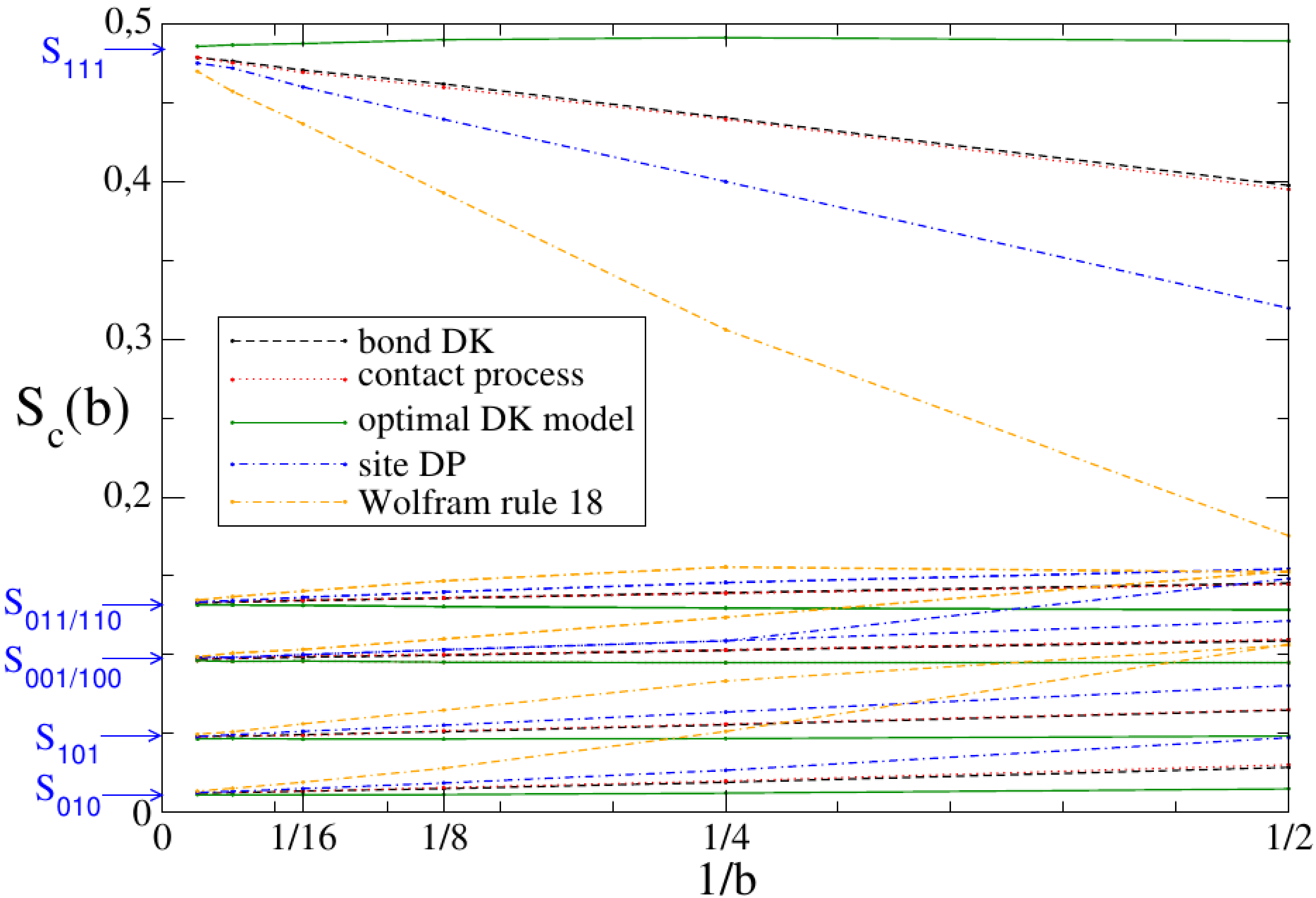
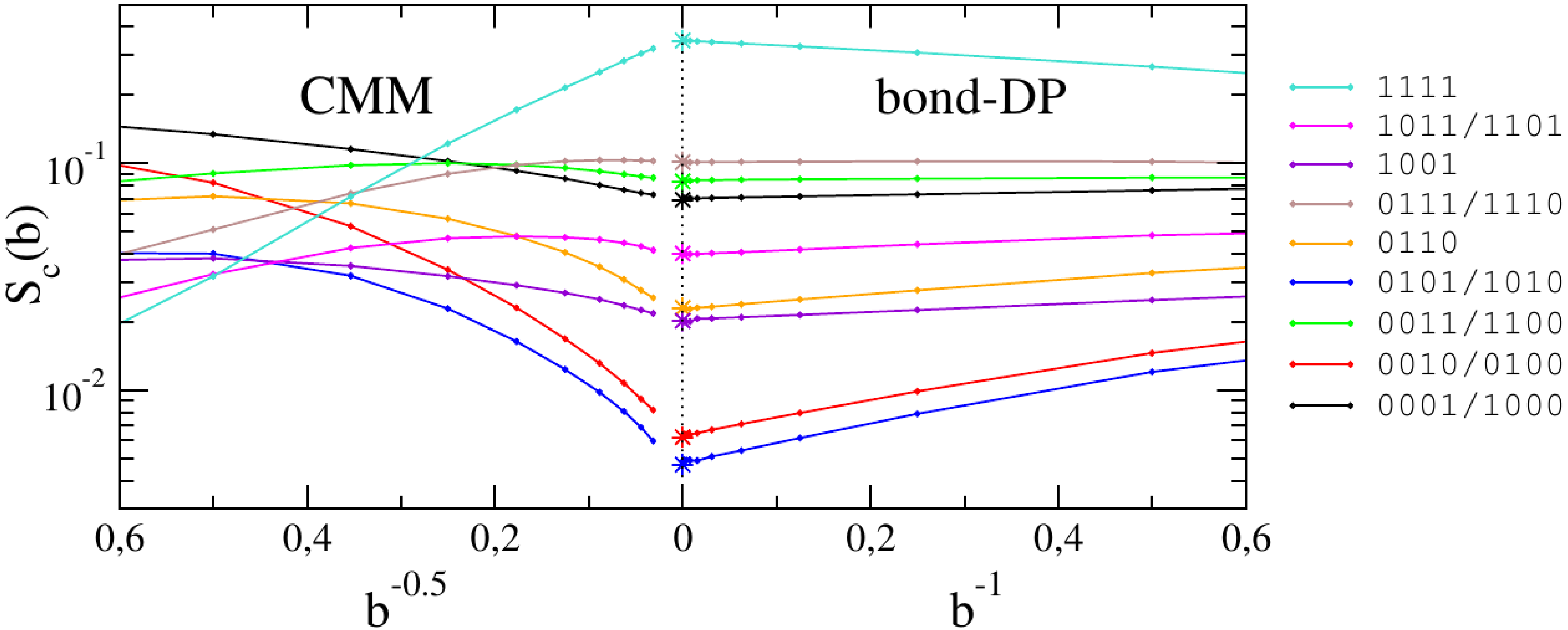
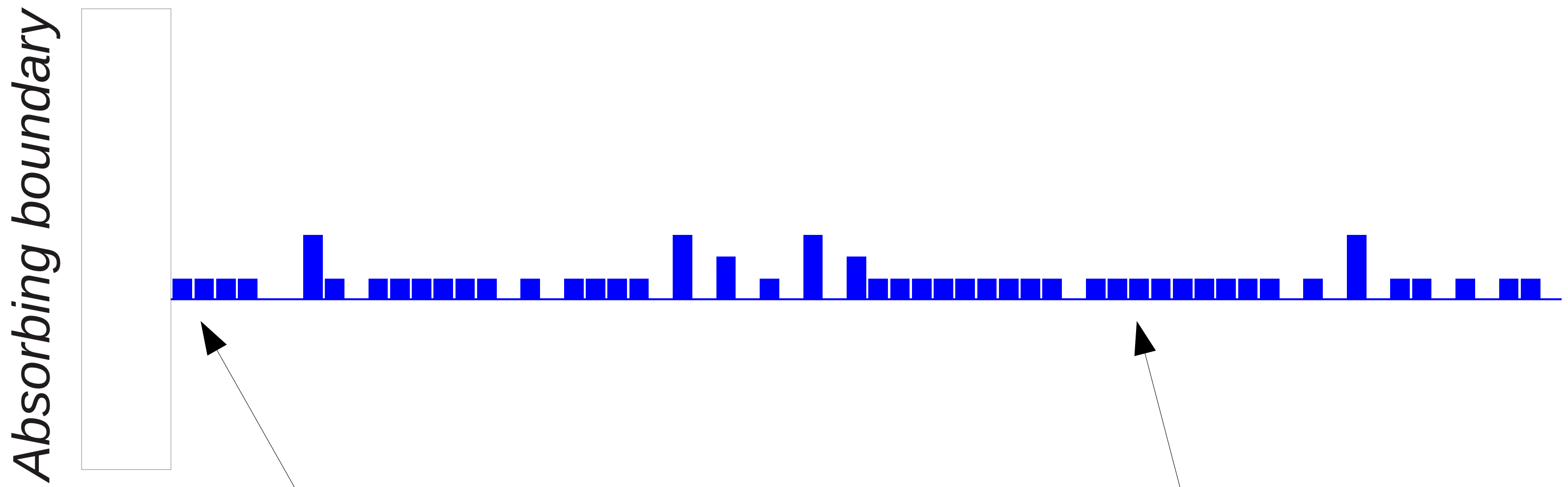


Apply RG method to conserved Manna



Boundary effects

Bonachela and Munoz, Physica A 384, 89 (2005)



At the boundary:

DP: $\rho_s(t) \sim t^{-0.426}$

CMM: $\rho_s(t) \sim t^{-0.85}$

In the bulk:

$\rho(t) \sim t^{-0.159}$

$\alpha_s = \alpha + 1/z$

A. Hipke and HH,
JSTAT P07021 (2009)

Conclusions:

- Conserved Manna sandpiles in 1D.
- Natural initial conditions cure overshooting.
- With natural intial conditions we get a different critical point, questioning previous critical exponents.
- Homogeneous simulations in 1D in perfect agreement with DP.
- RG flow in a block RG scheme confirms DP.
- Seed simulations still unclear.