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Opinion formation in a population with stubborn neutrals and zealots

Hyunsuk Hong

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``Encouraging moderation: Clues from a simple model of ideological conflict'', Seth A. Marvel (UOM), Hyunsuk Hong (CBNU), Anna Papush (Cornell), Steven H. Strogatz (Cornell)

Outline

- Introduction
- A simple model of opinion formation
- Effects by **committed members** and **stubborn neutrals** on the process of opinion consensus
- Mean-field analysis and numerical simulations
- Other generalized models
- Summary (ongoing/future study)

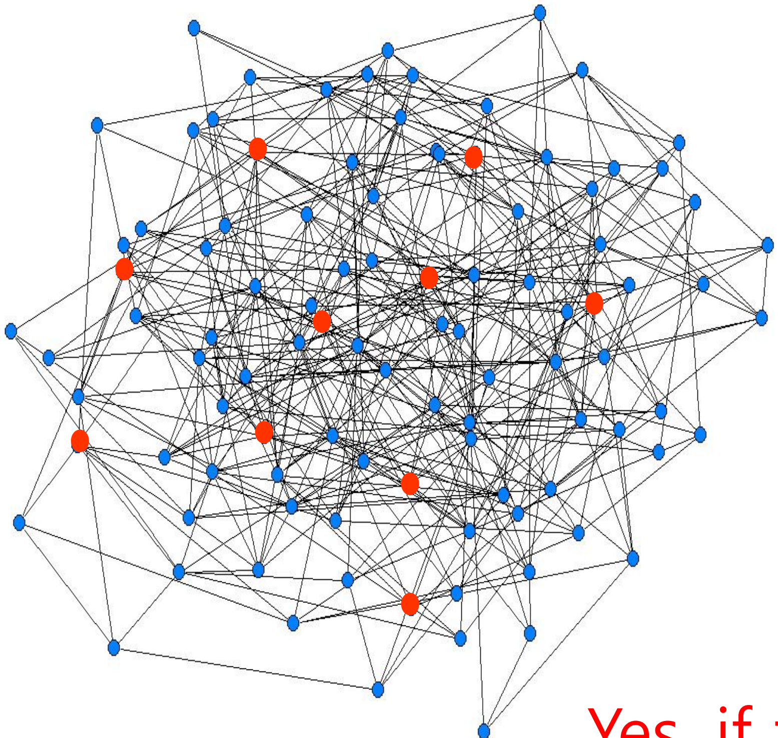
It's majority rule — even if only 10% believe it

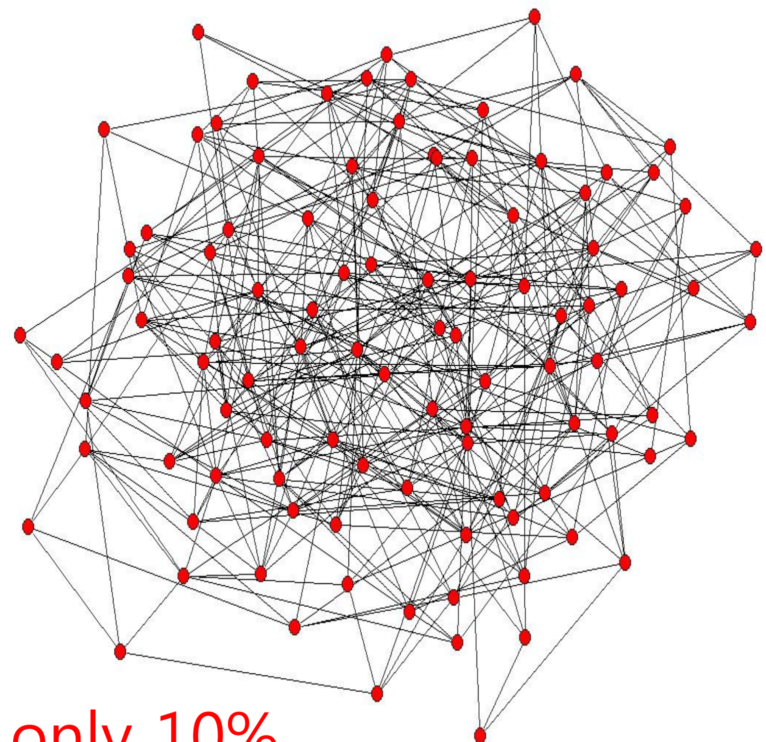
By Emily Sohn, msnbc.com (8/4/2011)

http://www.msnbc.msn.com/id/44024703/ns/technology_and_science-science/t/its-majority-rule-even-if-only-believe-it/

Can a minority group of committed members reverse the majority opinion?

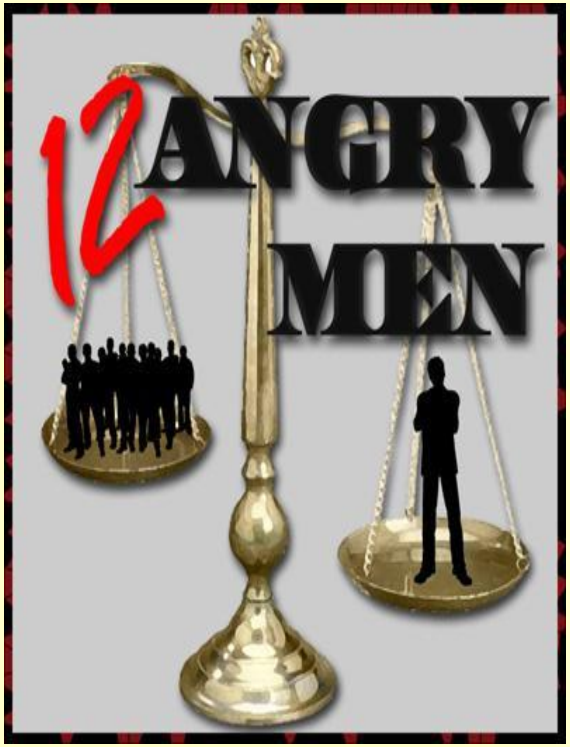
J. Xie, S. Sreenivasan, G. Korniss, W. Zhang, C. Lim, and B.K. Szymanski, Phys. Rev. E **84**, 011130 (2011).





Yes, if there are only 10% true believers! (according to the paper by Xie *et al.*)

• A • B



Eleven jurors vote **guilty**, and only **one** juror votes **not guilty.**

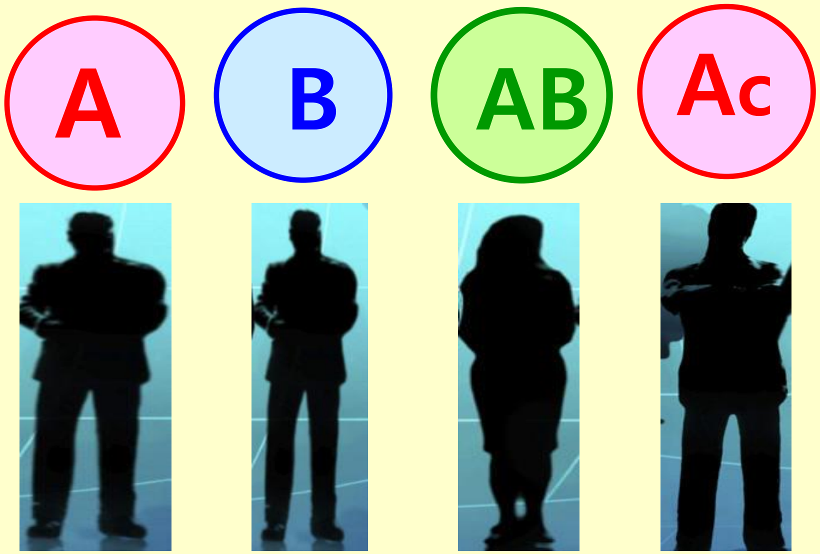
→

<u>**All twelve**</u> **jurors vote** <u>**not guilty**</u>**.**

Questions/Motivations

- Can we make a simple model to show such an interesting behavior?
- Do we have a sort of transition in the model?
- Personal characteristics of the population may affect the tipping point. So, what happens if we consider those things?
- Can we make lower the tipping point?
- Does this transition occur in the real social systems?

Model of Opinion Formation



A : subpopulation that hold extreme opinion A

B : subpopulation that hold the <u>opposing</u> opinion B

AB : those that do not hold either A or B, we call these ``neutrals/moderates".

A_c : those that hold A and are immune to the influence of others, i.e., <u>A zealots</u>.

List of all possible interactions

Speaker	Listener (pre-interaction)	Listener (post-interaction)	Probability
A, Ac	B	AB	1
	AB	AB	s
		A	1-s
B	A	AB	1
	AB	AB	s
		B	1-s

s: ``stubbornness" of the neutrals

Mean-Field Analysis

$$n_A = N_A / N$$
$$n_B = N_B / N$$
$$n_{AB} = N_{AB} / N$$

: fractions of the total population corresponding to the uncommitted **A**, **B**, and **AB**, respectively.

$p = N_{A_c} / N$: fraction of the population corresponding to the committed **A**, i.e., **Ac**

Mean-Field Analysis

uncommitted **A**

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fraction of the total population that hold the opinion A : $n_A + p$

uncommitted **A**

Mean-Field Analysis

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$$n_B = N_B / N$$
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$$\frac{dn_A}{dt} = (1-s)(p+n_A)n_{AB} - n_A n_B,$$

$$\frac{dn_B}{dt} = (1-s)n_B n_{AB} - (p+n_A)n_B, \qquad (1)$$

where $n_{AB} = 1 - p - n_A - n_B$

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Fixed points $\left(\dot{n}_A = 0,\ \ \dot{n}_B = 0\right)$ at:

(i) $\left(n_A, n_B\right) = \left(-p, 0\right)$

(ii) $\left(n_A, n_B\right) = \left(1 - p, 0\right)$

(iii) & (iv) :

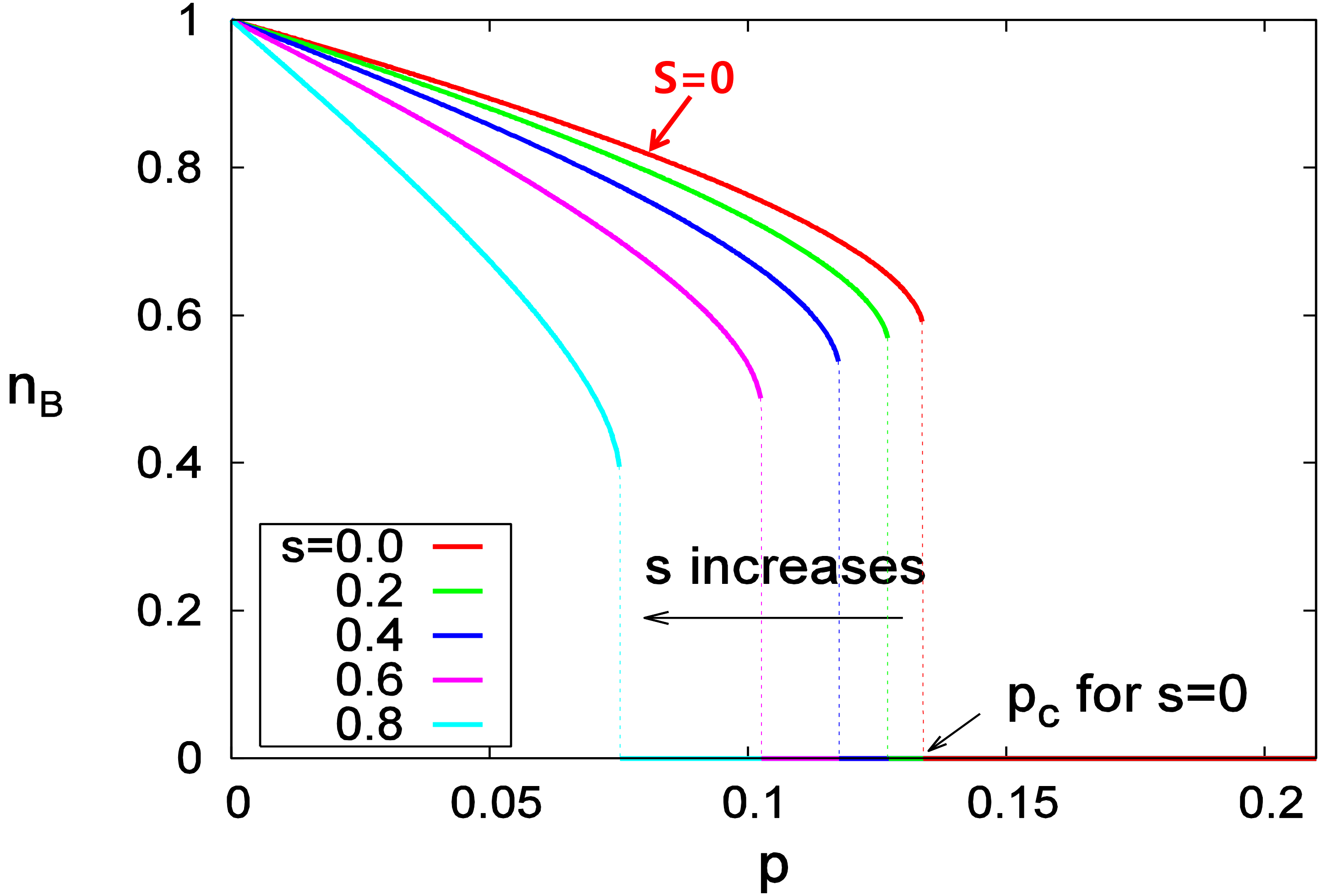
$$n_A = \frac{(1-s) - p(4-3s) \pm \sqrt{f(s,p)}}{2(3-2s)}$$

$$n_B = \frac{(1-s)(1-p) - p - (2-s)n_A}{1-s},$$

where $f(s,p) = (2-s)^2 p^2 - 2(1-s)(4-3s)p + (1-s)^2$

$f\left(s,p\right) = 0$: The fixed points (iii) and (iv) coalesce in a saddle-node bifurcation

Behavior of the density n_B



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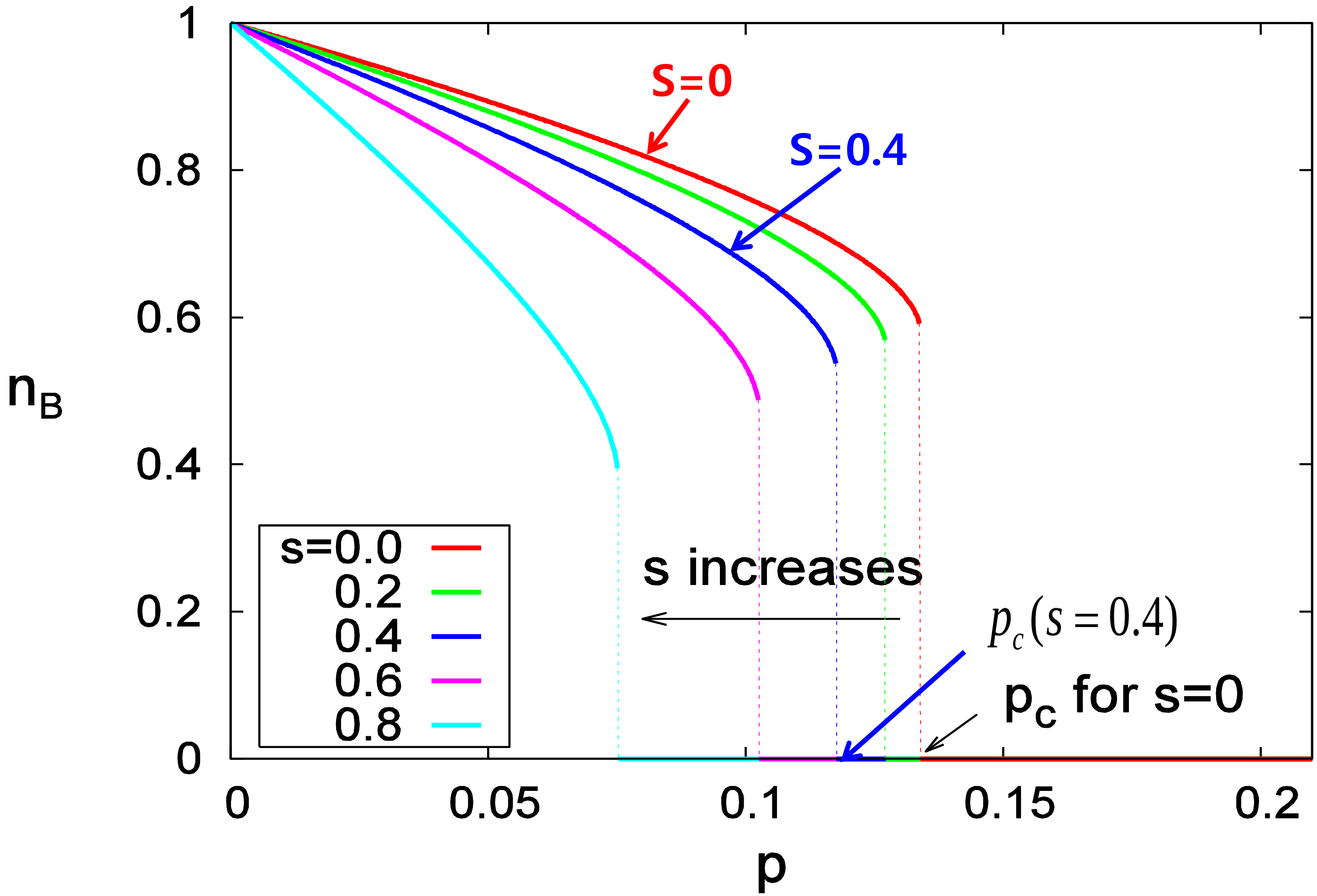

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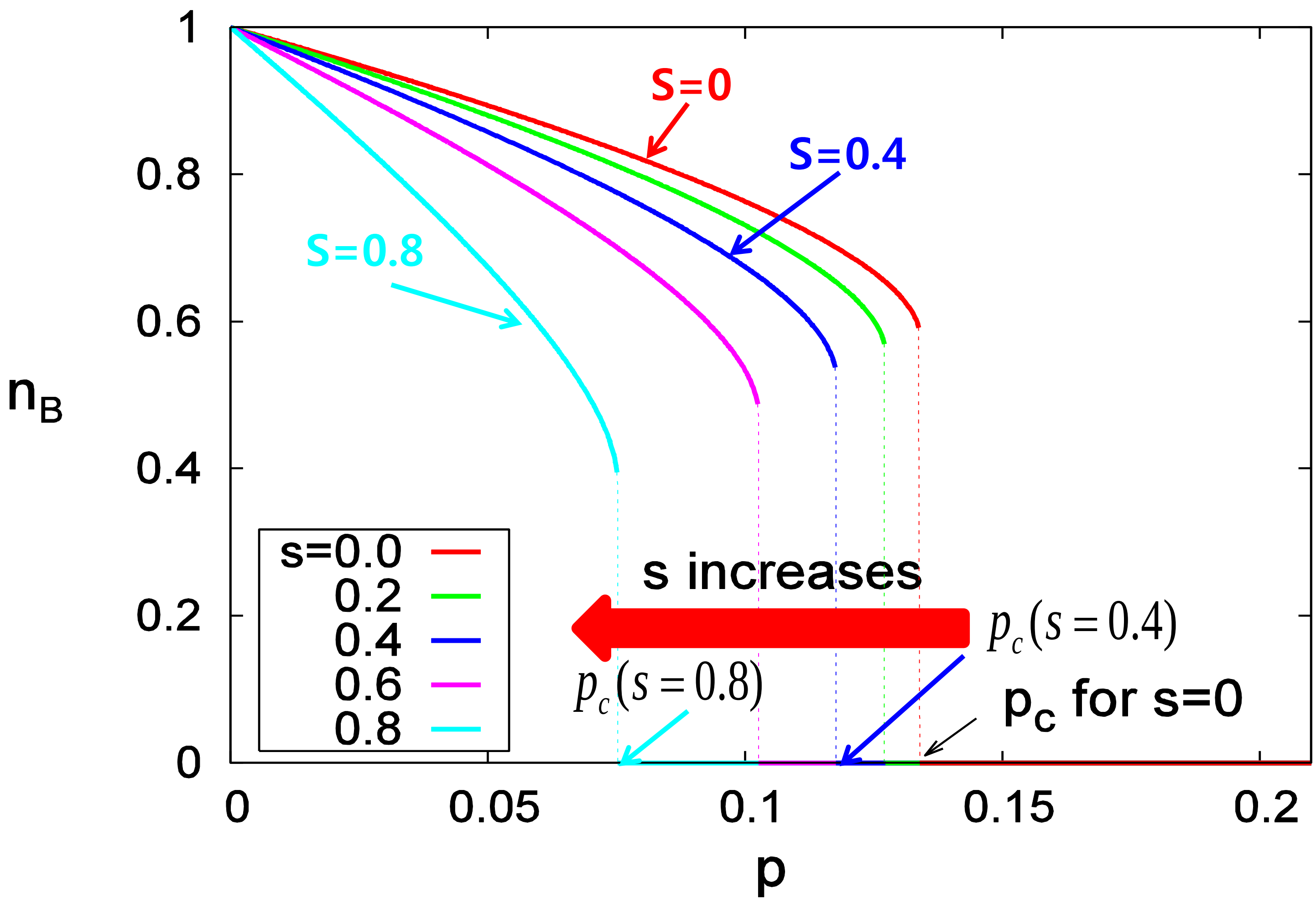
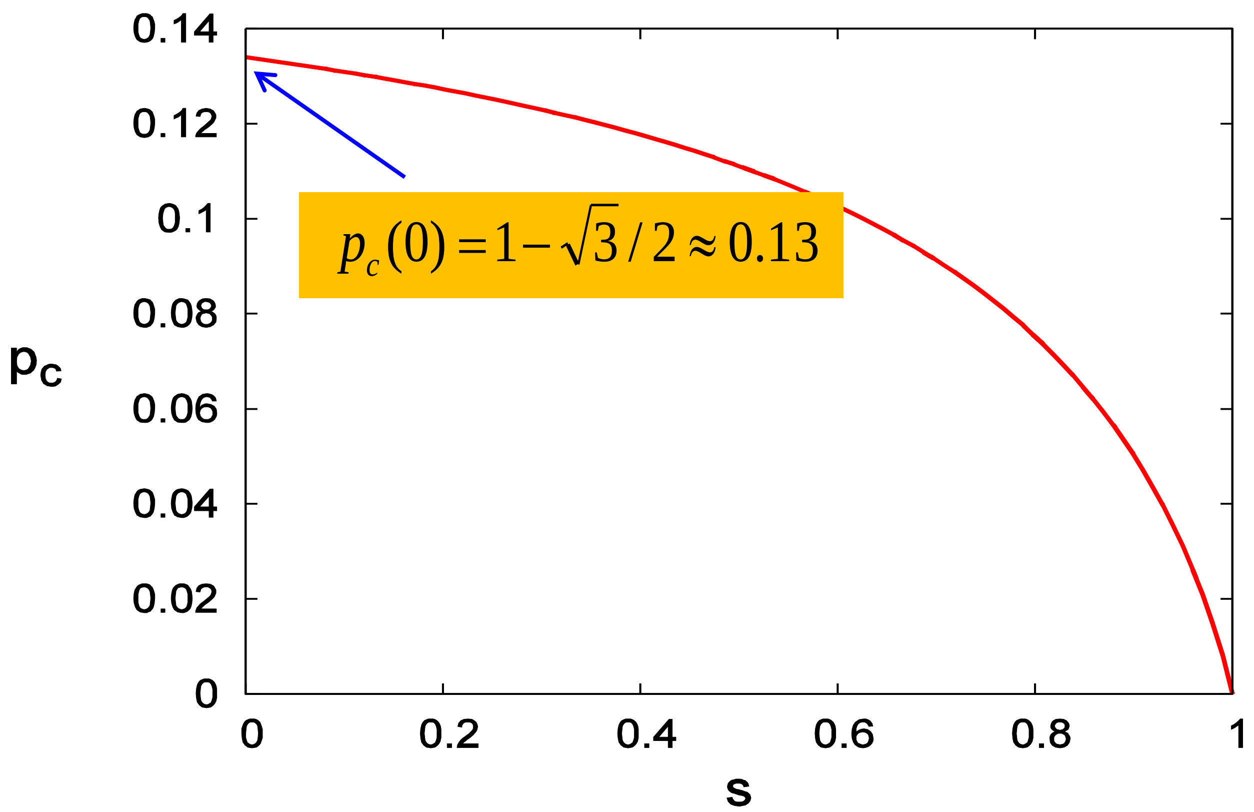


$$f(s,p) = (2-s)^2 p^2 - 2(1-s)(4-3s)p + (1-s)^2$$

Solving $f(s,p)=0$, we obtain

$$p_c(s) = \frac{(3s-4)(s-1) - 2\sqrt{(2s-3)(s-1)^3}}{(s-2)^2}$$



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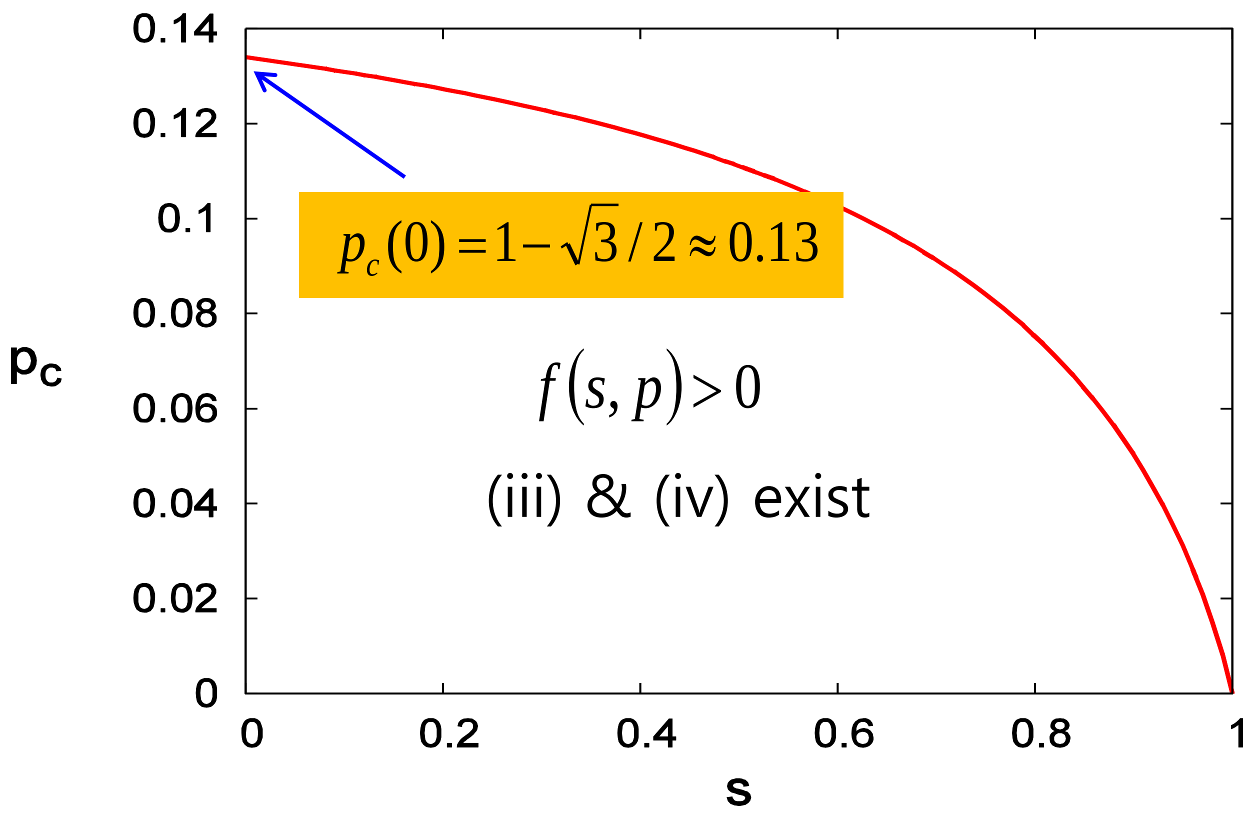

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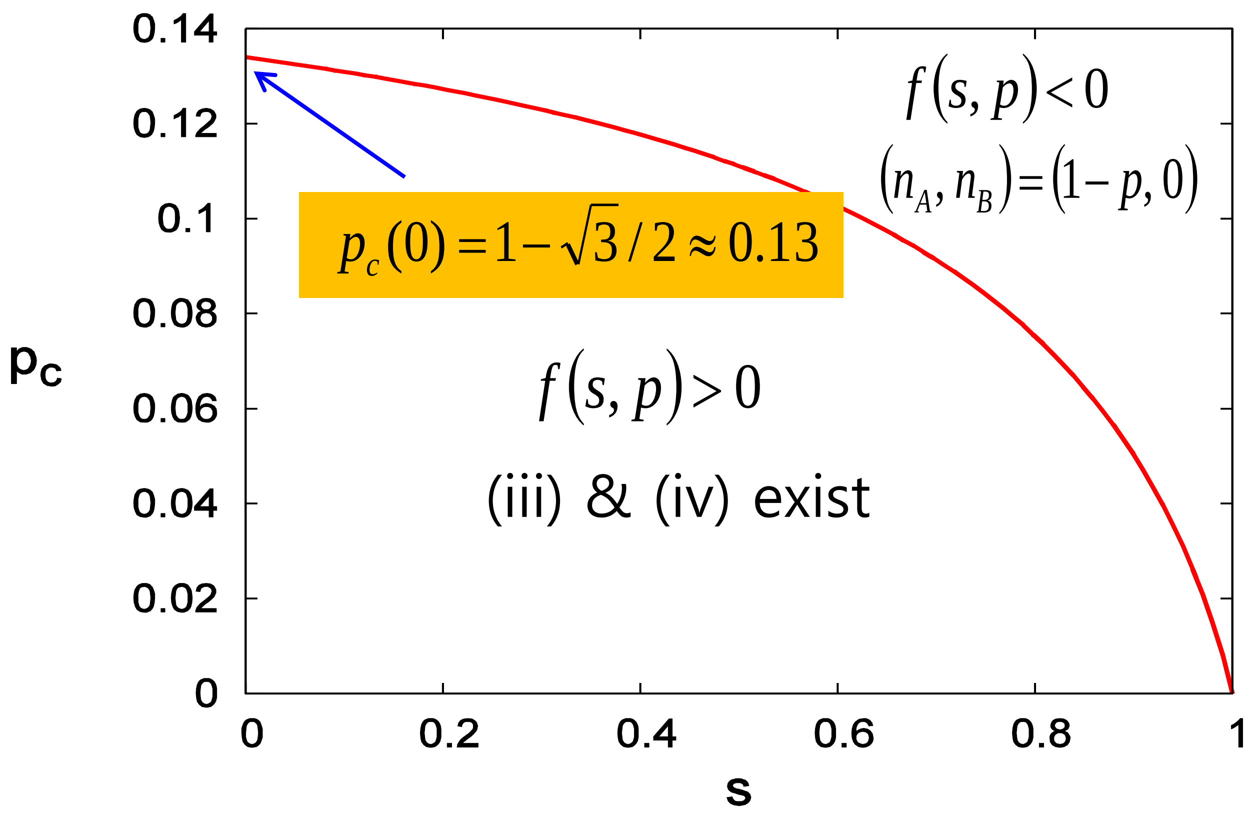

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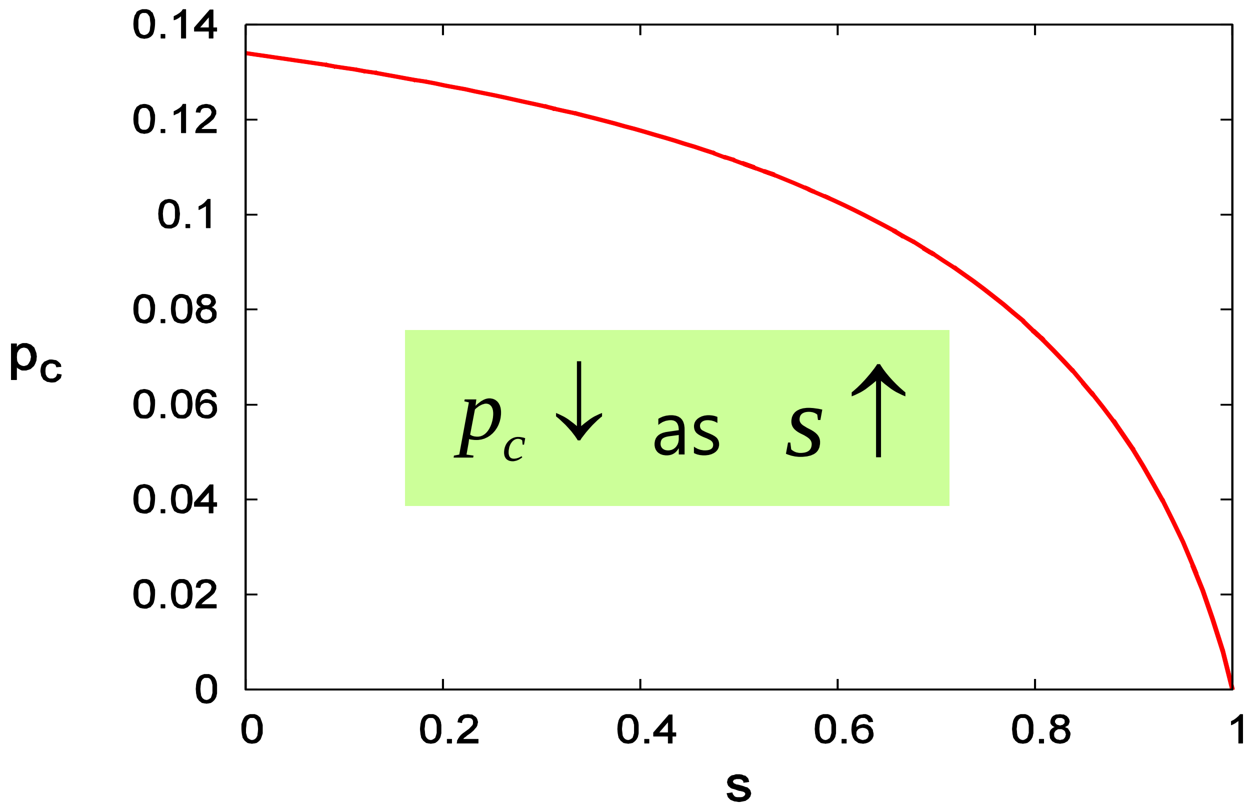
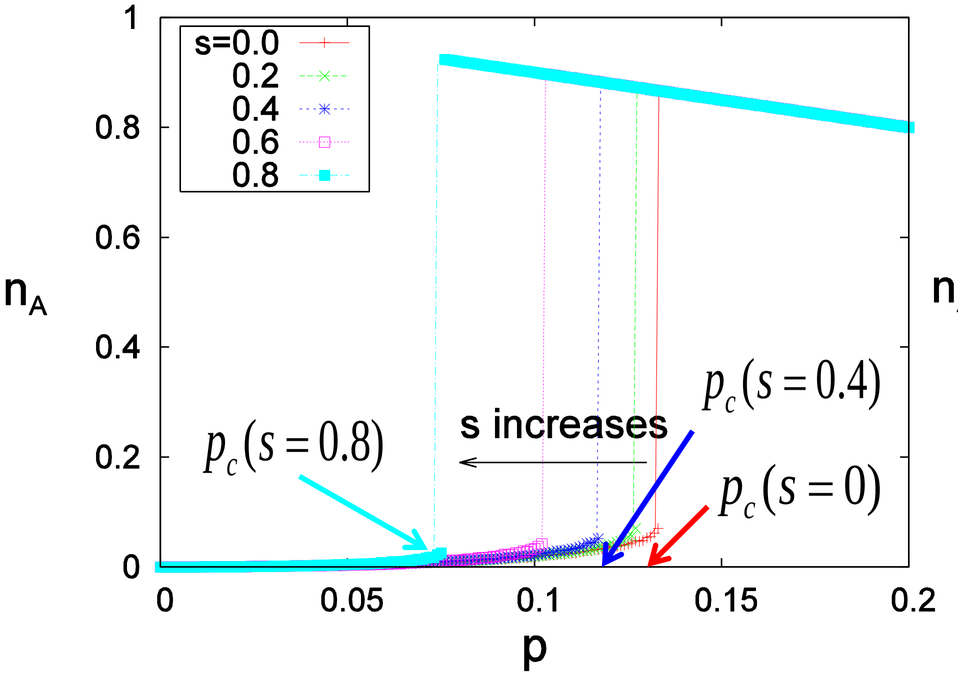
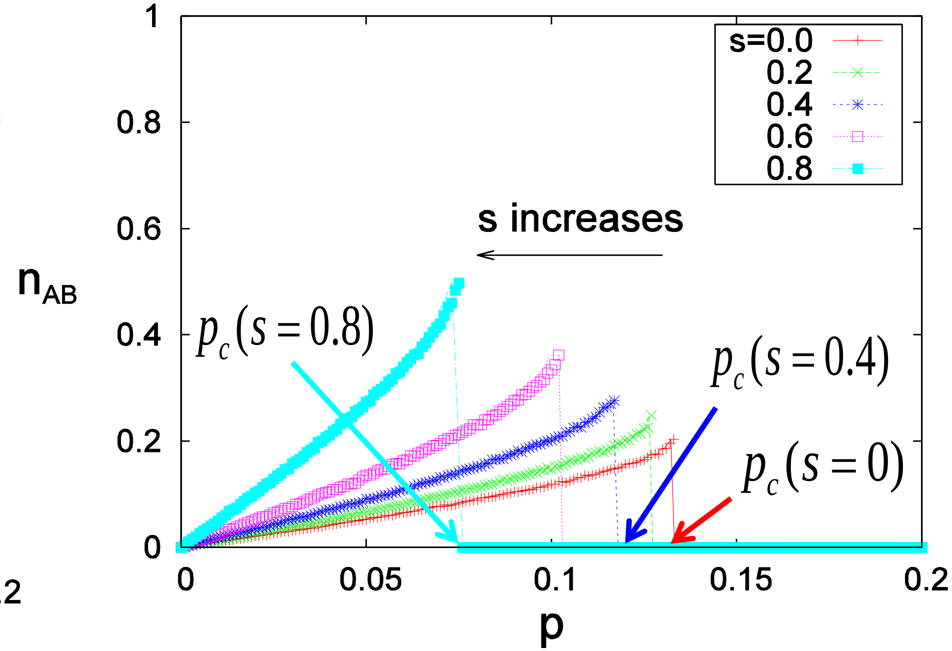
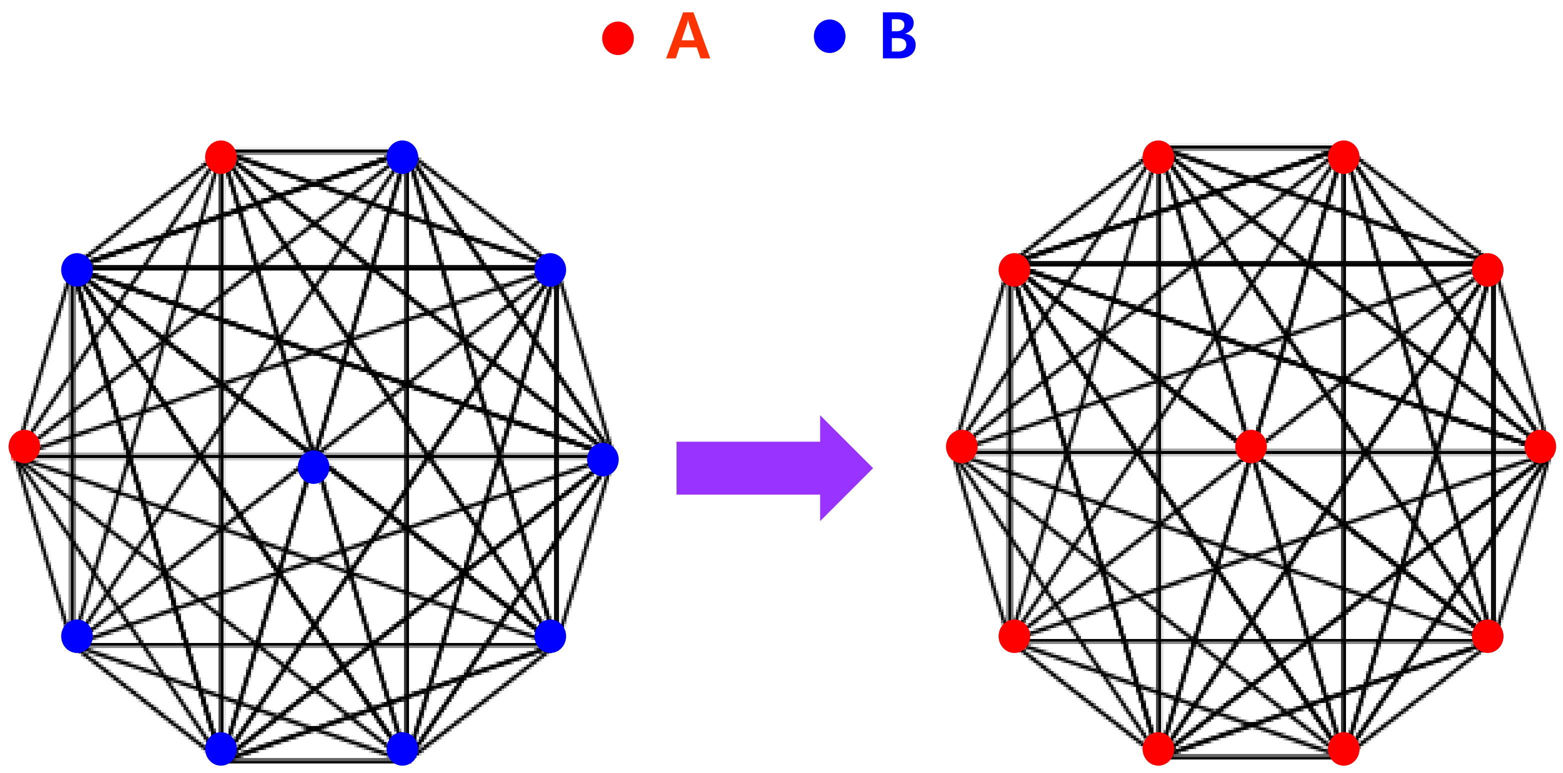


Behavior of the density n_A and n_{AB}





Numerical simulations on the complete graph

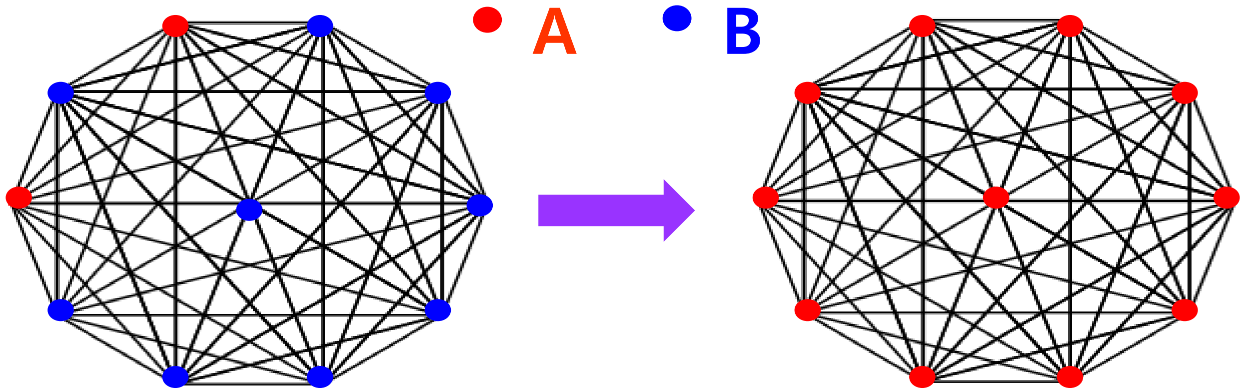


$p < p_c$

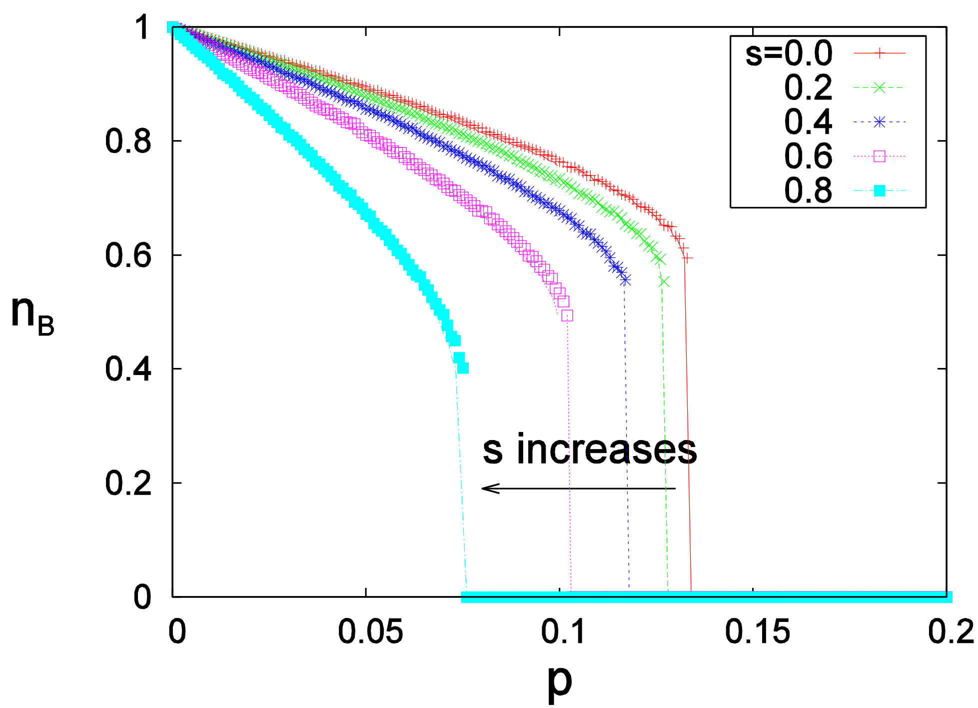
$p > p_c$

Numerical simulations on the complete graph

$p < p_c$



$p > p_c$



Q. Why does such a counterintuitive relationship hold in $p_c - s$?

stubbornness of the neutrals

Increasing **s** → <u>decreases</u> the change from **AB** to **A**: p_c ↑

→ <u>decreases</u> the change from **AB** to **B**: p_c ↓

i.e., depletes both **A** and **B**

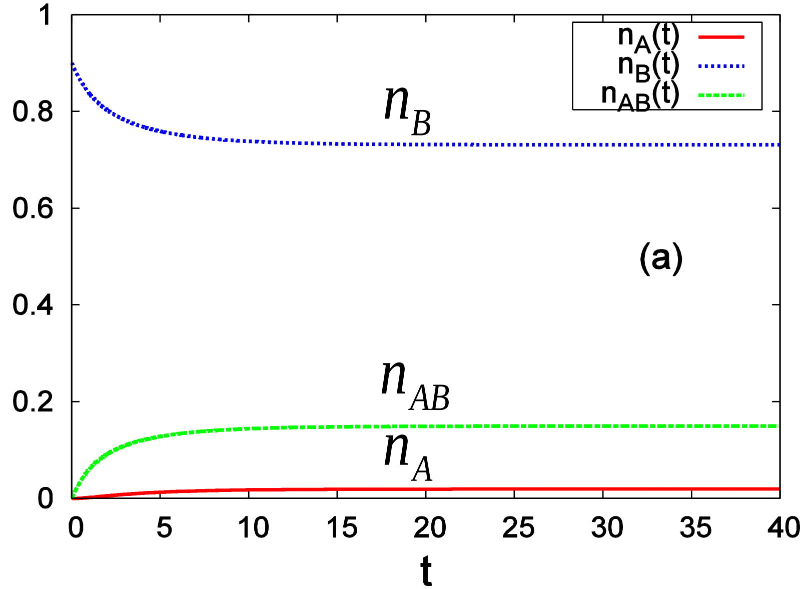
→ Evangelism of the **B** to the **AB** is <u>weakened</u>, comparing to that of the **A** to the **AB**, which makes <u>fewer</u> **A_c** is needed to convert the **AB** to the **A**.

It becomes easier for the zealots to win !!

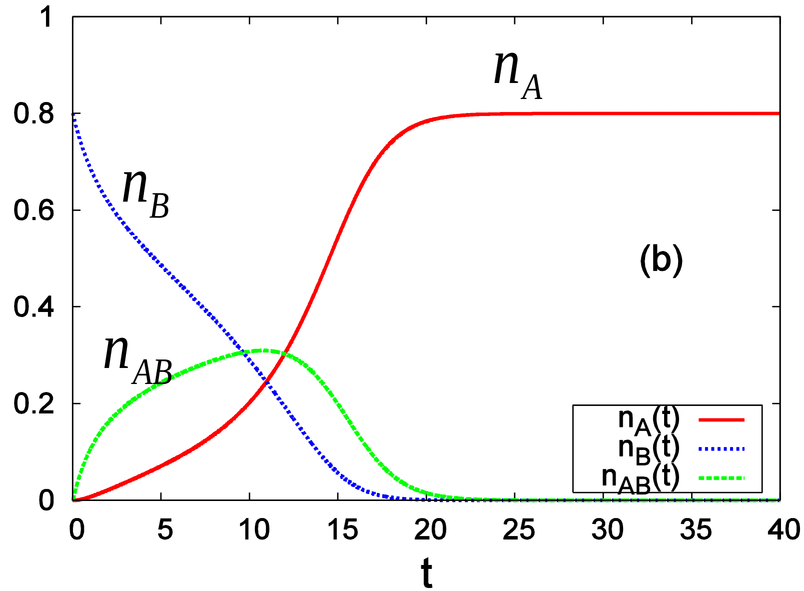
Representative evolution of the system

$s=0.2$

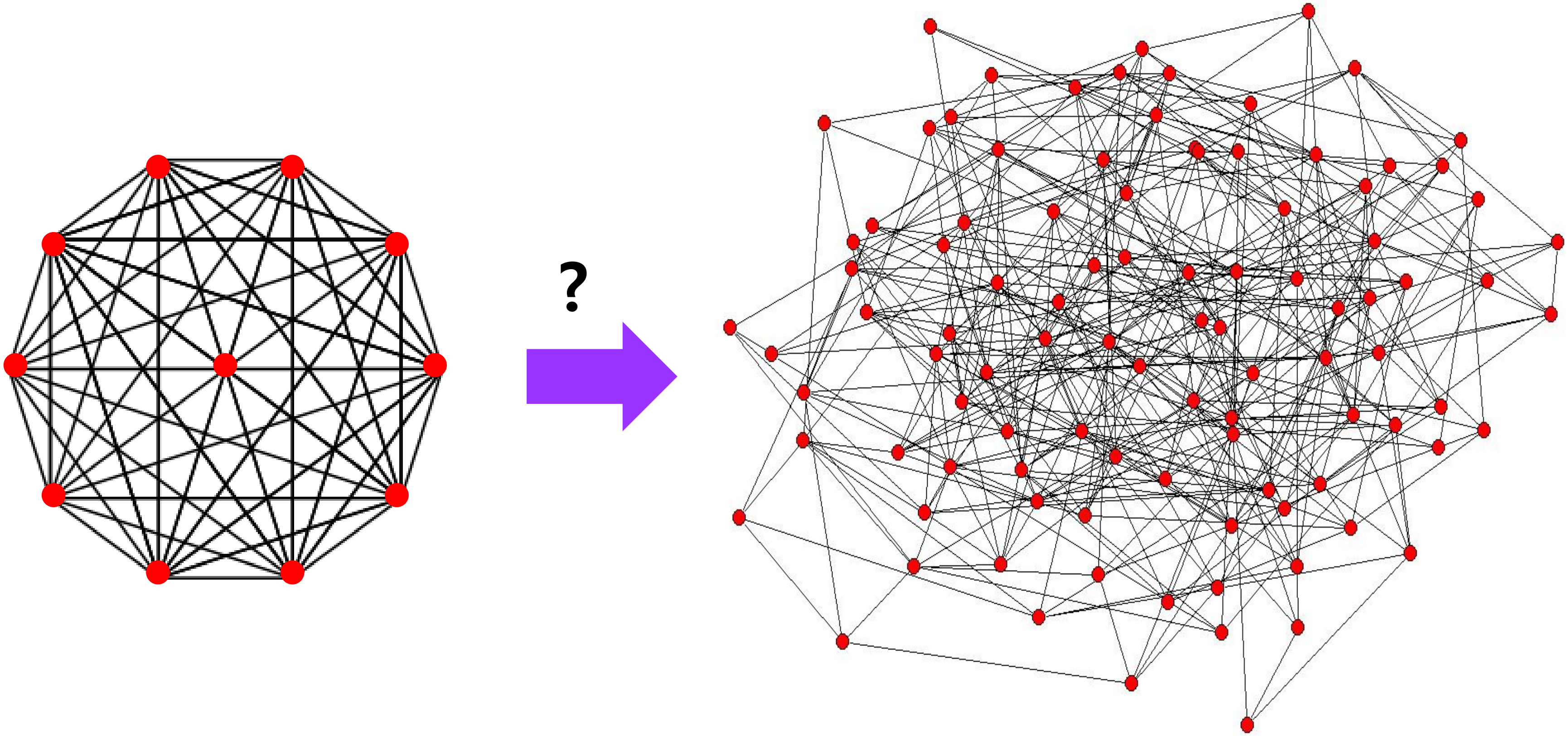
$p=0.1\,(<p_c)$



$p=0.2\,(>p_c)$



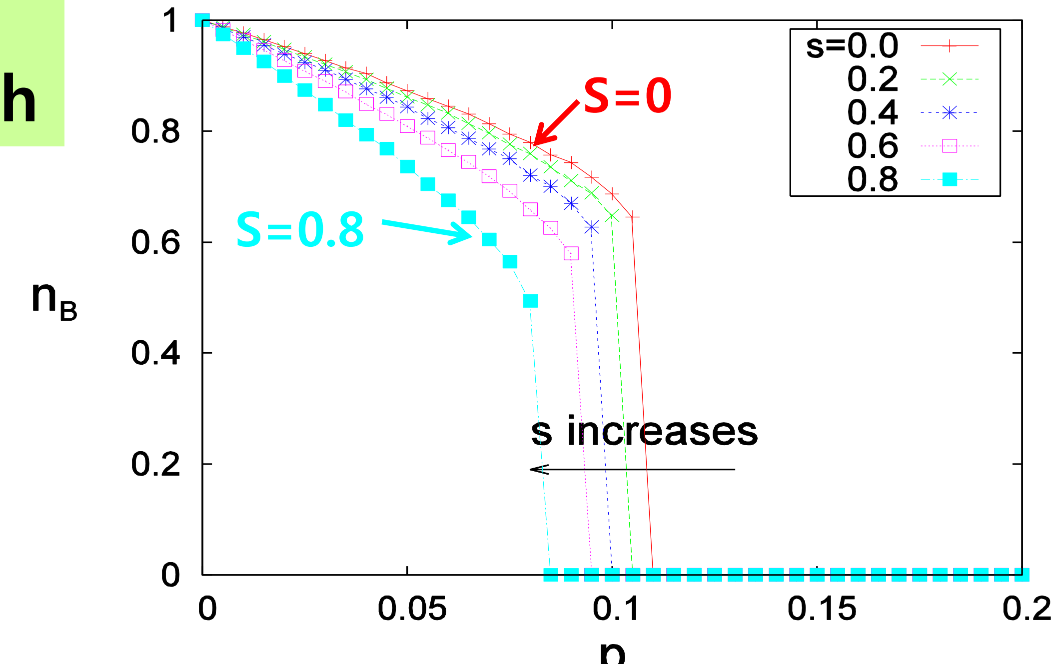
Q. How does the connectivity disorder influence on the $p_c - s$ behavior?

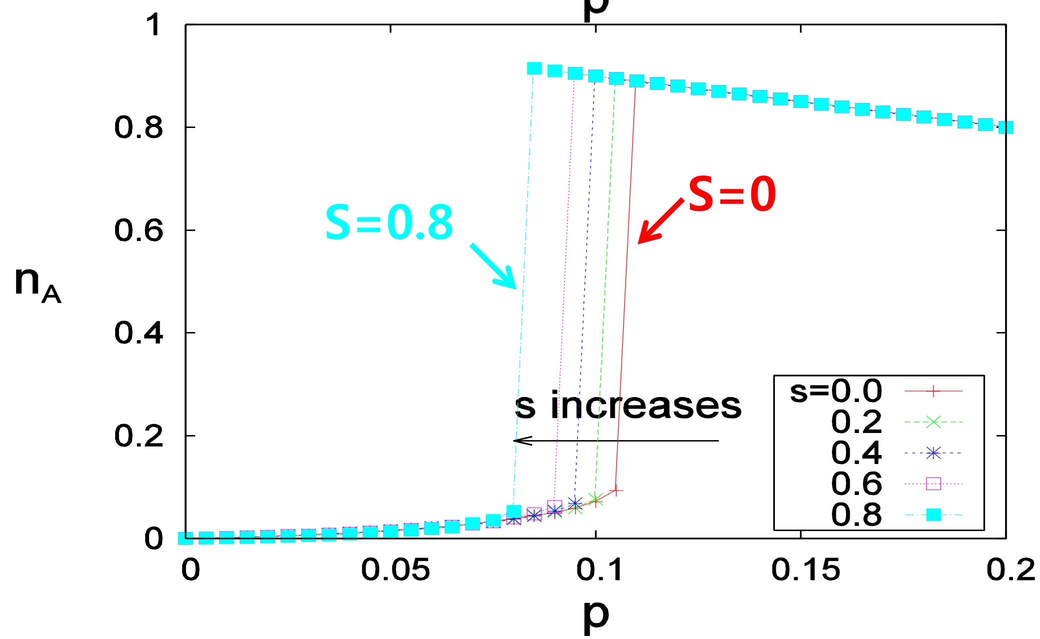


Complete graph

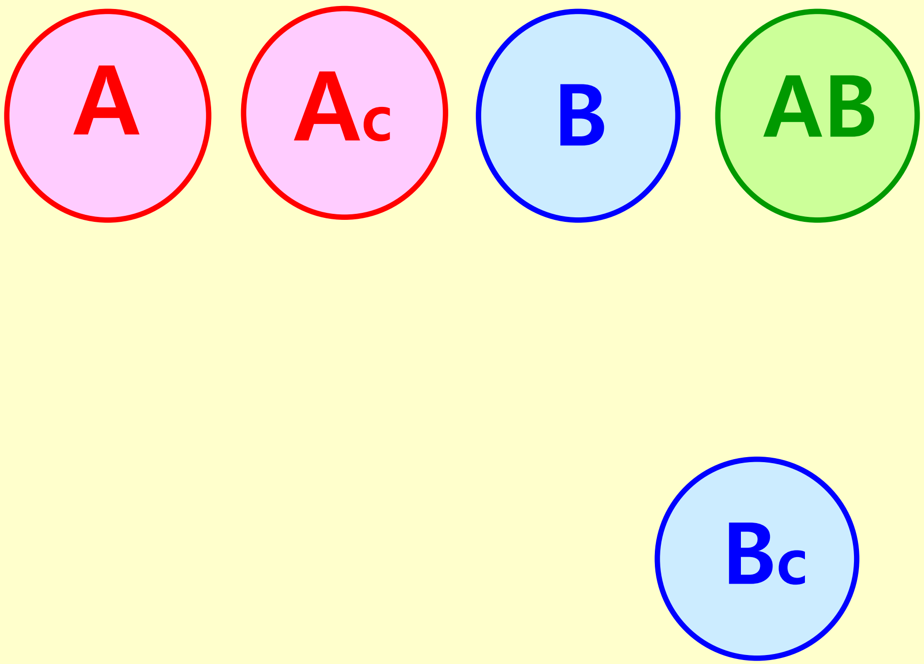
Erdős–Rényi random graph

Erdős–Rényi random graph





Q. What would change in the presence of <u>two zealots</u> A_c and B_c ?



All possible interactions in the presence of two zealots: **A_c** with **p**$(=N_{A_c}/N)$ and **B_c** with **q**$(=N_{B_c}/N)$

Speaker	Listener (pre-interaction)	Listener (post-interaction)	Probability
A, A_c	B	AB	1
	AB	AB	s
		A	1-s
B, B_c	A	AB	1
	AB	AB	s
		B	1-s

Rate equations in the presence of both zealots: A_c and B_c

$$\frac{dn_A}{dt} = (1-s)(p+n_A)n_{AB} - \underline{n_A(q}+n_B)$$

$$\frac{dn_B}{dt} = \underline{(1-s)(q}+n_B)n_{AB} - (p+n_A)n_B$$

where $n_{AB} = 1 - p - q - n_A - n_B$

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where $n_{AB} = 1-p-q-n_A-n_B$

For $q=0$: 1st order transition to the state of **consensus on A (all A)!**

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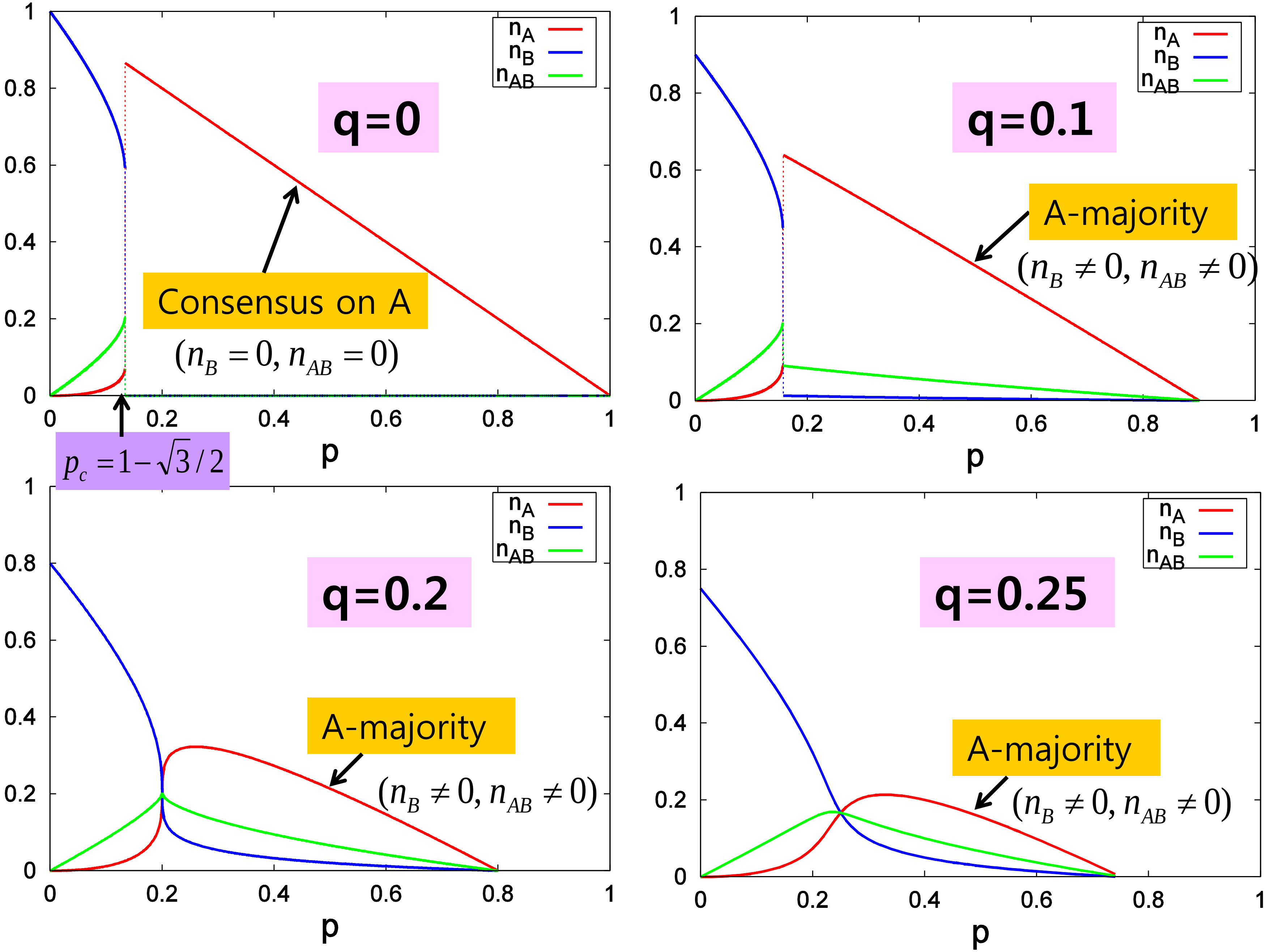
where $n_{AB} = 1-p-q-n_A-n_B$

For $q \neq 0$:

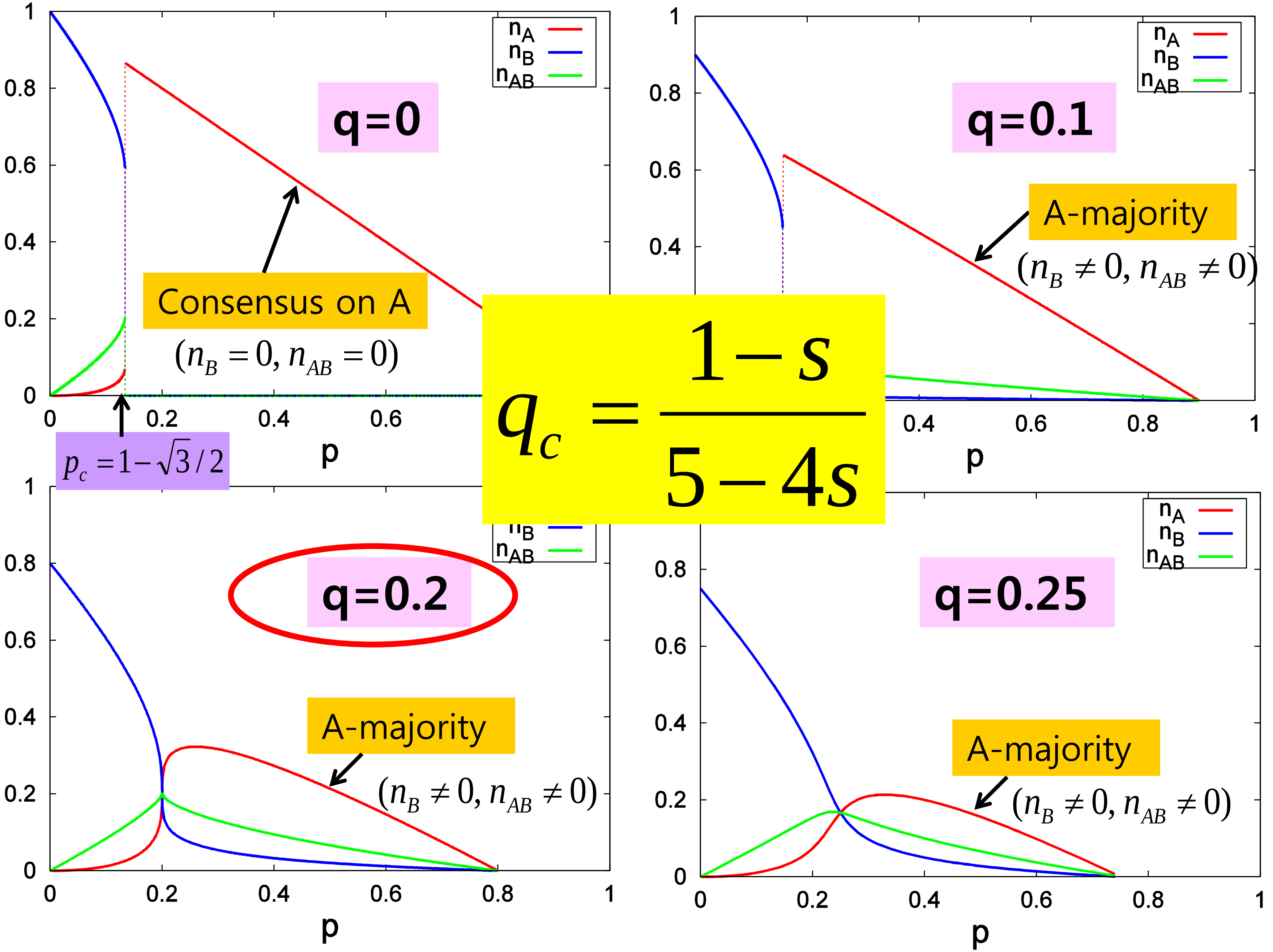
A-majority state for p>q or B-majority state for p<q

For s=0



q=0
Consensus on A
$(n_B = 0, n_{AB} = 0)$
$p_c = 1 - \sqrt{3}/2$
q=0.1
A-majority
$(n_B \neq 0, n_{AB} \neq 0)$
q=0.2
A-majority
$(n_B \neq 0, n_{AB} \neq 0)$
q=0.25
A-majority
$(n_B \neq 0, n_{AB} \neq 0)$
n_A
n_B
n_{AB}
p

For s=0



$$q_c = \frac{1-s}{5-4s}$$

Summary

- A model for the opinion consensus in a population with zealots and stubborn neutrals
- Effects of the stubbornness of the neutrals:
  When the neutrals are more stubbornly moderate, it becomes easier for the zealots to win!
- Other generalization of the model - population with two zealots

Ongoing/future study

- Effects of the network topology on the opinion consensus:
  - our model on various complex networks
- Other generalization – e.g., evangelical neutrals
- Consideration of local/nonlocal interaction
- Applying to real social systems