Diffusion with Stochastic Resetting

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J. Stat. Mech.: Theor. & Exp. P05024 (2012)

Plan

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- Diffusion with Resetting

  $\Rightarrow$ new Nonequilibrium Steady State

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- Various generalizations

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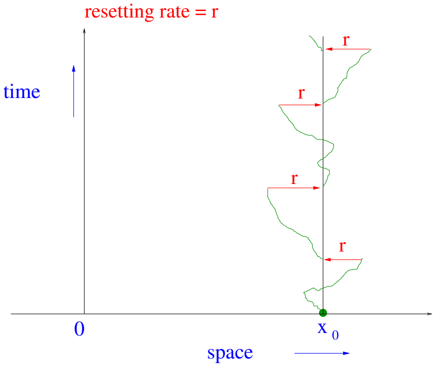
- Various generalizations

- Summary and Conclusion

Diffusion with stochastic resetting: one dimension



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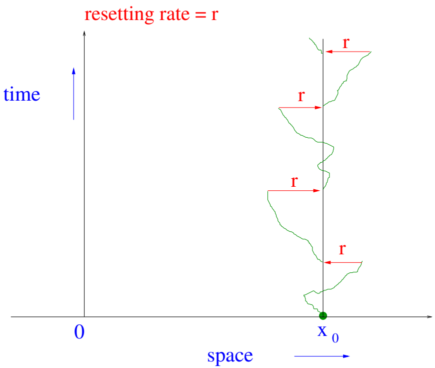

Dynamics: In a small time interval Δt

$$x(t+\Delta t) = x_0 \qquad \text{with prob. } r\Delta t \qquad \text{(resetting)}$$

$$= x(t) + \eta(t)\,\Delta t \qquad \text{with prob. } 1 - r\Delta t \qquad \text{(diffusion)}$$

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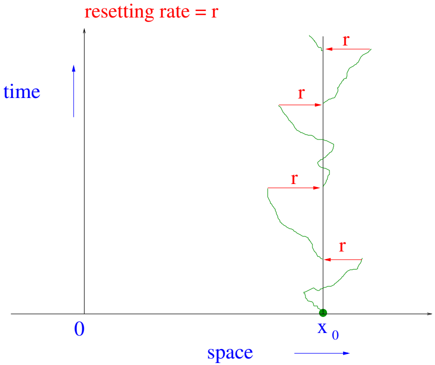

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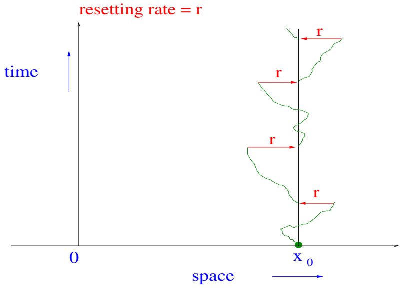
$\eta(t) \to$ Gaussian white noise: $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = 2\,D\,\delta(t-t')$

Prob. density $p(x,t)$ with restting rate $r > 0$



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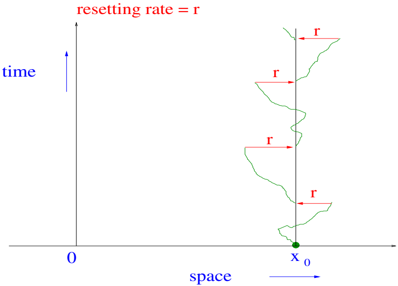

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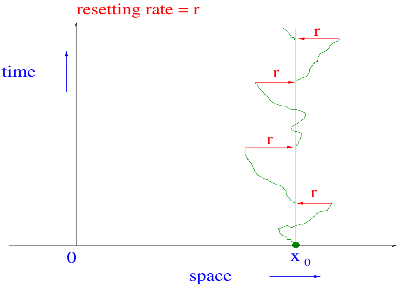

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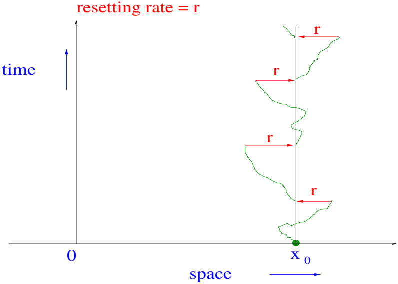

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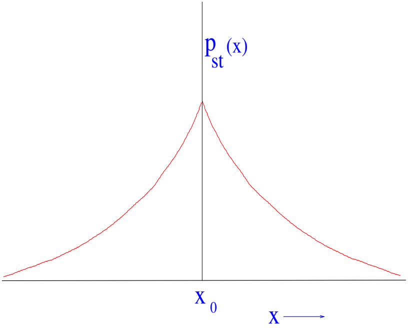


$\rightarrow$ nonequilibrium steady state

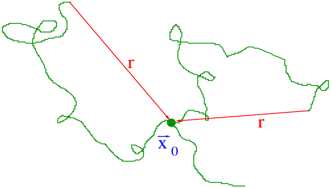
$\Rightarrow$ current carrying with detailed balance $\rightarrow$ violated

$p_{\rm st}(x) = \alpha_0\,\exp[-V_{\rm eff}(x)]$

effective potential:

$V_{\rm eff}(x) = \alpha_0 |x - x_0|$

Generalization to higher dimensions



particle starting at $\vec{x}_0$ diffuses in d dim. and resets to $\vec{x}_0$ with rate r

$p(\vec{x}, t) \rightarrow$ prob. density. at time t

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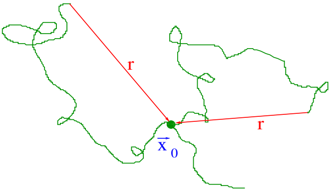

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$$p_{\rm st}(\vec{x}) = \frac{(\alpha_0)^d}{(2\pi)^d}\,\left[\alpha_0\,|\vec{x}-\vec{x}_0|\right]^{\nu}\,K_\nu\left(\alpha_0\,|\vec{x}-\vec{x}_0|\right)$$

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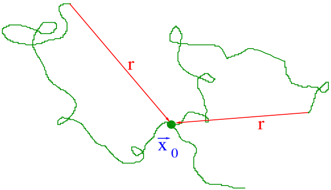

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where $\alpha_0 = \sqrt{r/D}$, $\nu = 1 - d/2$ and $K_\nu(z) \to$ modified Bessel function

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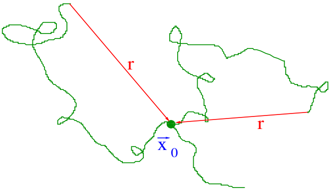

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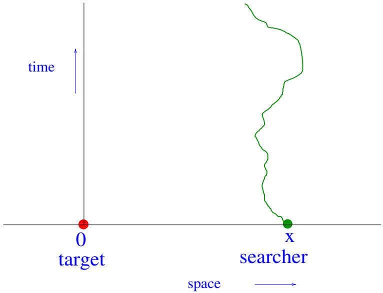
- For $d = 1$, it reduces to $\rightarrow p_{\rm st}(x) = \frac{\alpha_0}{2}\,\exp[-\alpha_0\,|x - x_0|]$

Search of a fixed target by a purely diffusive searcher in $d = 1$ without resetting



$Q(x,t) \rightarrow$ survival prob. of the target

backward Fokker-Planck equation

$\partial_t Q(x,t) = D\partial_x^2 Q(x,t)$ for $x \geq 0$

boundary cond. :
$Q(x=0,t) = 0$ and $Q(x \rightarrow \infty, t) = 1$

intial cond. :
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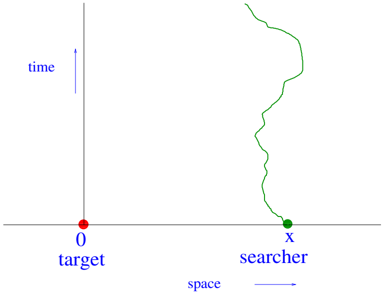

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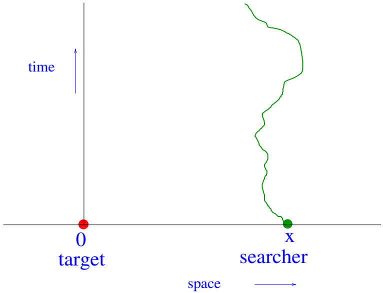

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$$F(x,t) = -\partial_t Q(x,t) = \frac{x}{\sqrt{4\pi D t^3}} \exp[-x^2/4Dt] \xrightarrow[t\to\infty]{} t^{-3/2}$$

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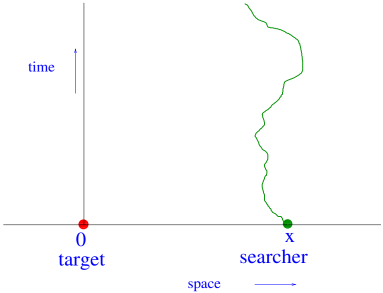

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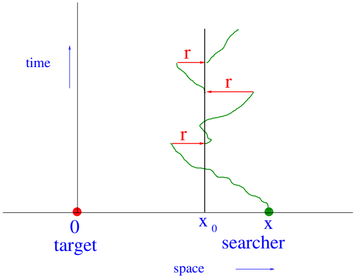
- mean capture time $\to \bar{T} = \int_0^\infty t\, F(x,t)\, dt = \infty$

Target search via diffusion with resetting



- starting position $x \rightarrow$ 'variable', resetting to x_0 with rate r

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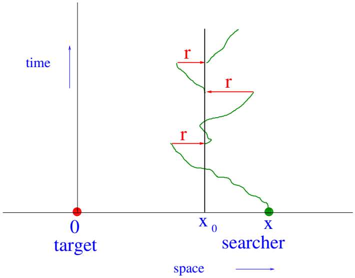

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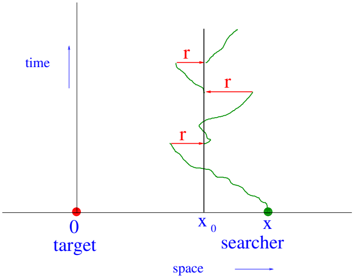

- starting position x → 'variable', resetting to x_0 with rate r
- $Q(x,t)$ → survival prob. of the target
- backward Fokker-Planck Eq.:

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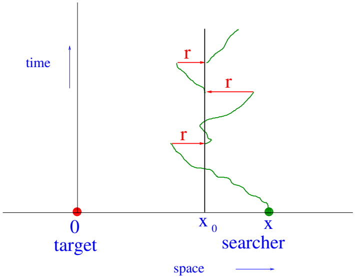

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$\Rightarrow$ As $t \to \infty$, $Q(x_0,t) \approx \exp\left[-r\,t\,e^{-\sqrt{r/D}\,x_0}\right]$

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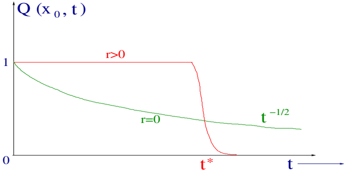
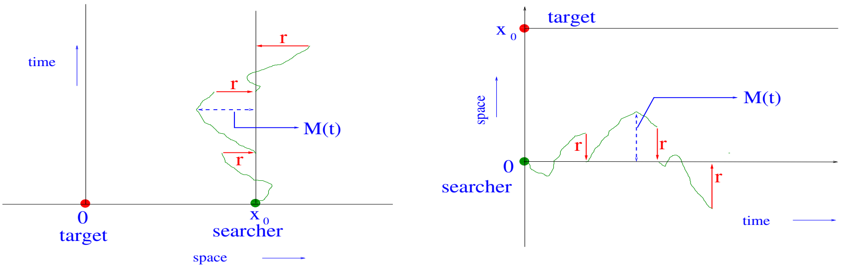


For $r > 0$, $Q(x_0,t) \approx \exp[-t/t^*]$

where $t^* \approx (1/r)\, e^{\sqrt{r/D}\,x_0}$

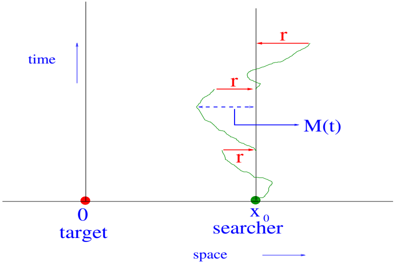
ROTATE & SHIFT

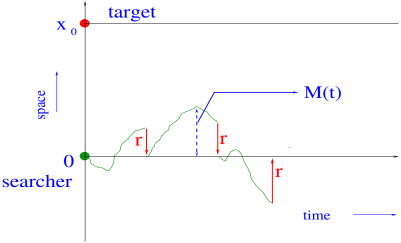


- $M(t) \to$ maximum of the process up to time t

Survival Probability $\Longleftrightarrow$ Extreme Value Statistics

ROTATE & SHIFT





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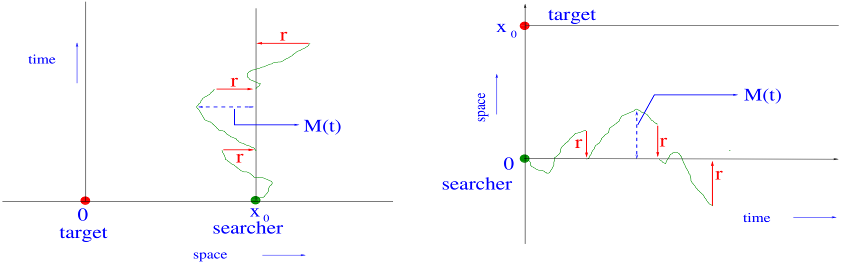

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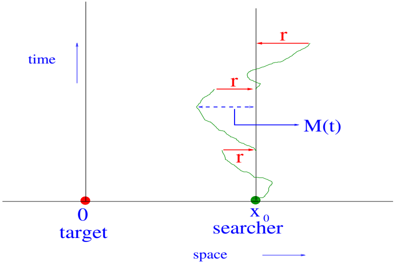

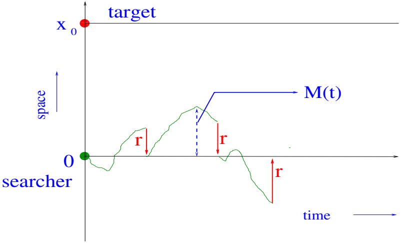

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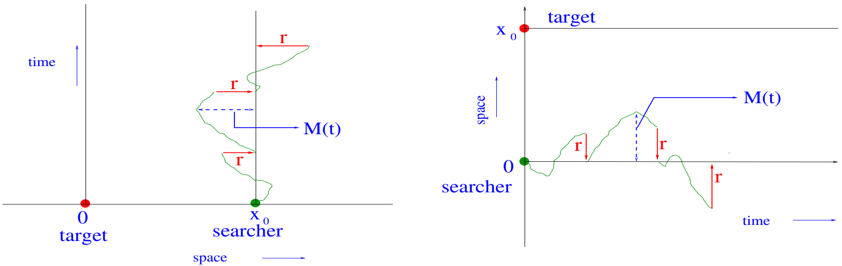
- Correlation time $\tau = 1/r \to N_{\rm eff} = t/\tau = rt$ effectively independent blocks in the time interval $[0, t]$

ROTATE & SHIFT



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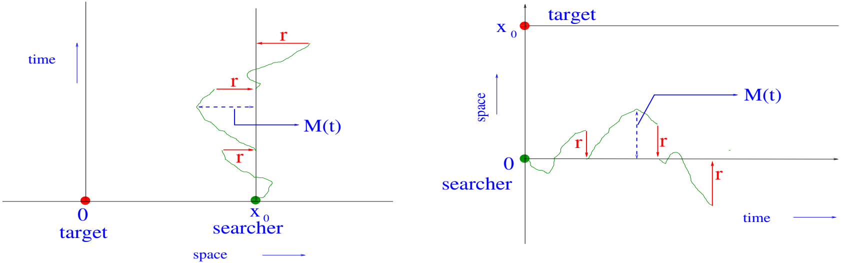

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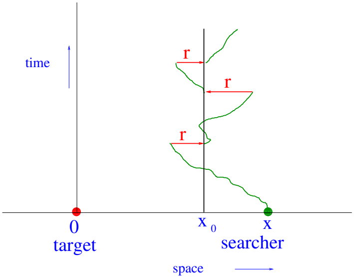
$\Rightarrow$ classical Gumbel distribution for the maximum of a set of N_{eff} exponentially distributed independent random variables

Mean capture/search time



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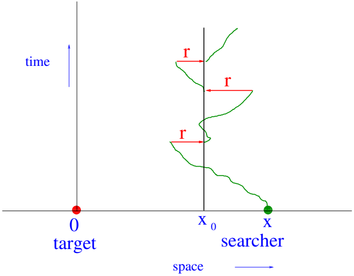

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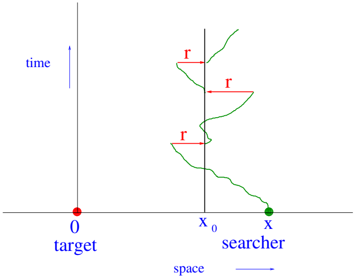

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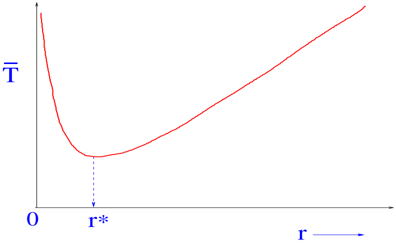
$\Rightarrow$ mean capture time is ∞ for $r = 0$, but finite when $r > 0$

Optimal resetting rate



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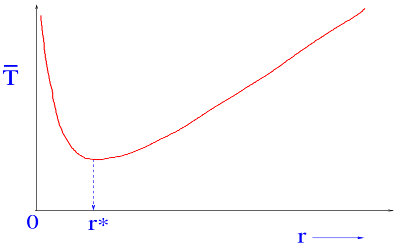

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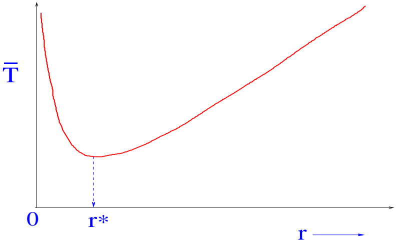

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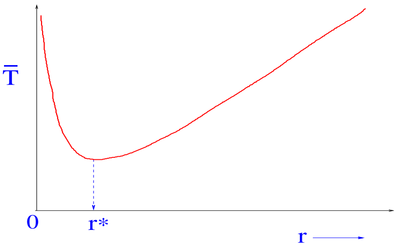

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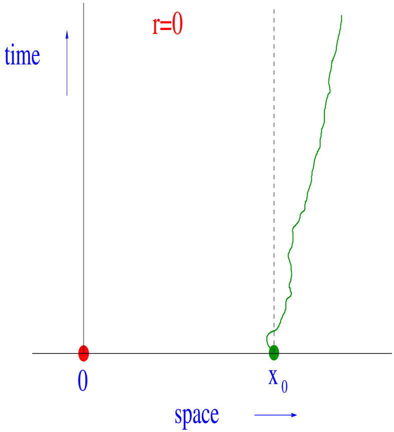
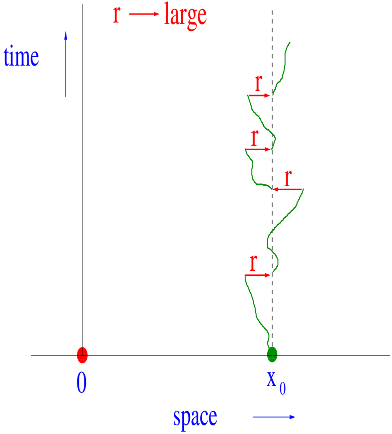
optimal resetting rate r^* is given by:

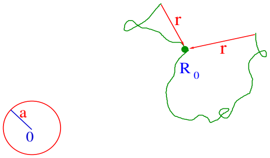
$$\boxed{r^* = \gamma^2 \frac{D}{x_0^2}} \quad \text{where} \quad \boxed{\gamma - 2\left(1 - e^{-\gamma}\right) = 0} \Rightarrow \gamma = 1.59362\ldots$$

(M.R. Evans and S.M., Phys. Rev. Lett. 106, 160601 (2011))

Typical trajectories for $r \to 0$ and $r \to \infty$



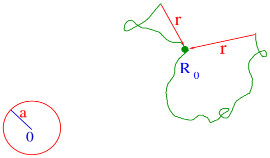




stationary target of radius a at 0 in $d > 2$

searcher starts at $R_0 > a$, diffuses, and resets with rate r

Target search via diffusion with resetting in $d > 1$



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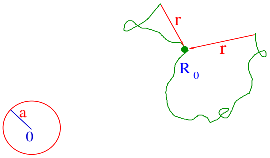

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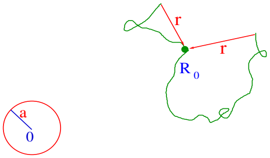

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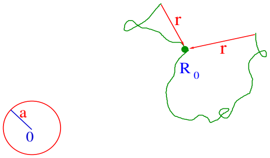

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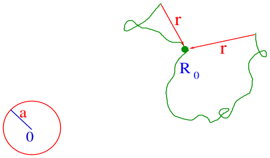

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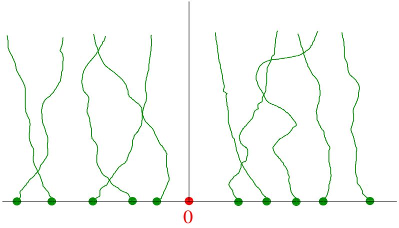
- Once again, there is an optimal r^* that minimizes $\bar{T}(r, R_0)$ in all d

Target search by multiple searchers $d = 1$



stationary target at 0 surrounded by a sea of independent searchers (traps), initially distributed with uniform density ρ

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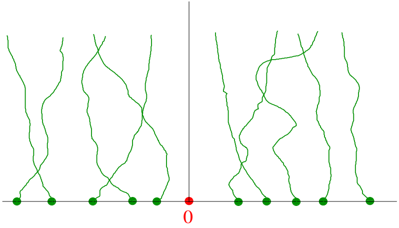

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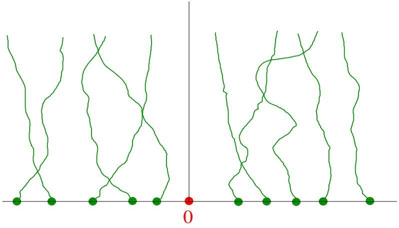

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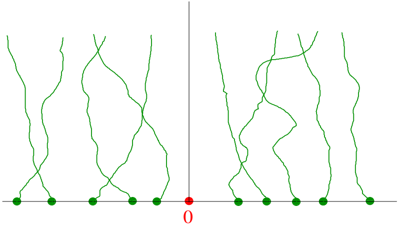

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- Eventually $N \rightarrow \infty$ and $L \rightarrow \infty$ with their ratio $N/L = \rho$ fixed

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- For diffusive searchers without resetting: $Q(x,t) = \mathrm{erf}(|x|/\sqrt{4Dt})$

$$\boxed{\langle P_s(t)\rangle = \exp\left[-4\,\rho\,\sqrt{Dt/\pi}\right]}$$ $\rightarrow$ stretched exponential decay

(Zumofen, Klafter, Blumen '83, Tachiya '83, Burlatsky & Ovchinnikov '87)

Link to Extreme Value Statistics

- $\langle P_s(t)\rangle = \exp\left[-2\,\rho\,\int_0^\infty \left(1 - Q(x,t)\right)\, dx\right] = \exp\left[-2\,\rho\, E[M(t)]\right]$

Link to Extreme Value Statistics

- $\langle P_s(t)\rangle = \exp\left[-2\,\rho\,\int_0^\infty (1 - Q(x,t))\; dx\right] = \exp\left[-2\,\rho\, E[M(t)]\right]$
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$\Rightarrow$ general result valid for any trap process

(J. Franke and S.M., JSTAT, P05024 (2012))

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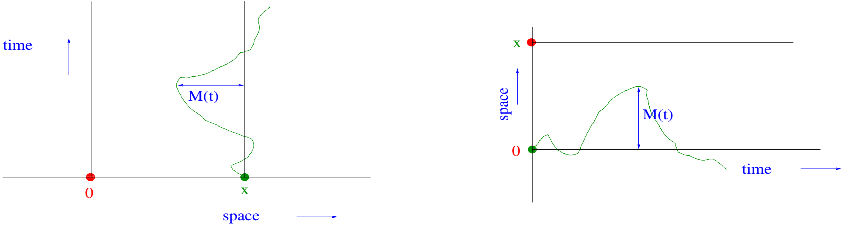

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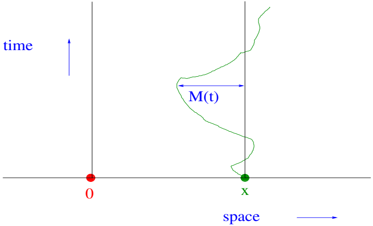

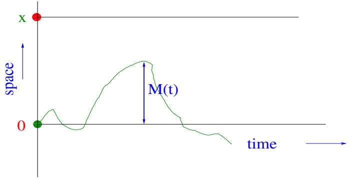

- Several exact results for $E[M(t)]$ for subdiffusive and superdiffusive (Lévy flights) processes (J. Franke and S.M., 2012)

Average vs. Typical

- average of a random variable may be different from typical

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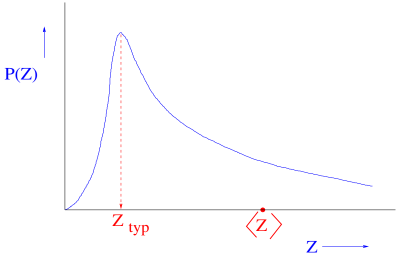

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but has a long tail such that

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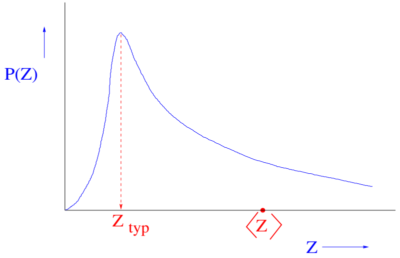

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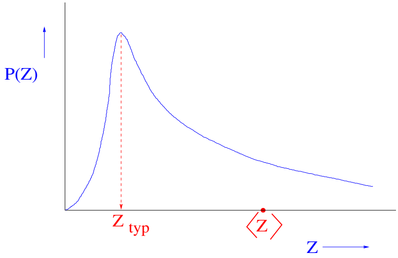

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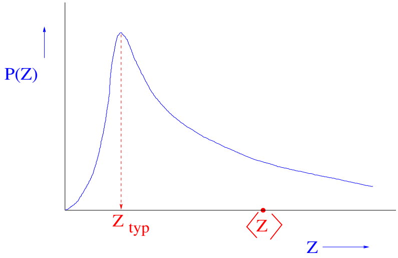

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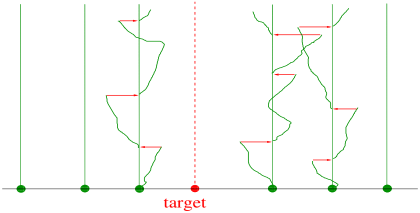
$\Rightarrow \langle P_s(t)\rangle \sim P_s^{\rm typ}(t)$ and both decay stretched-exponentially

Target search by multiple searchers with resetting



stationary target at 0 surrounded by a sea of independent diffusive searchers or traps (each with reset rate r), initially distributed with uniform density ρ

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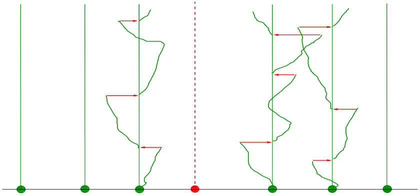

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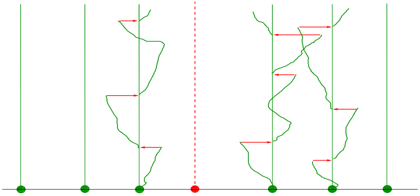

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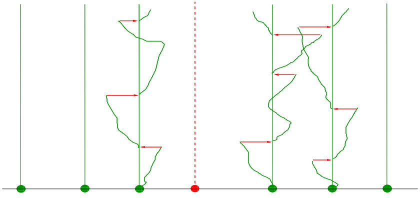

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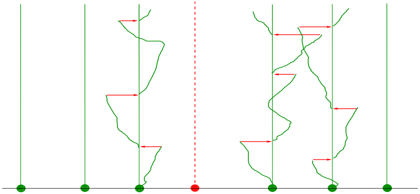

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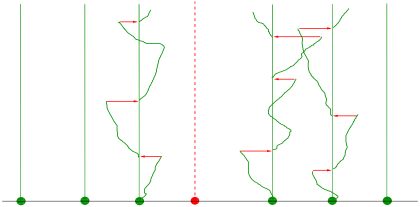

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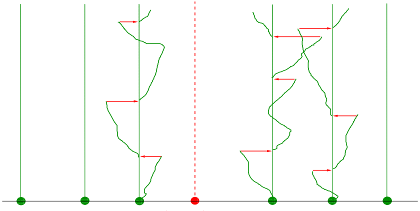

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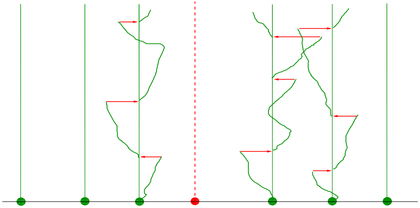

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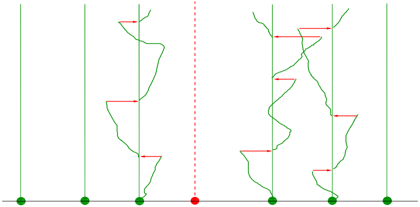

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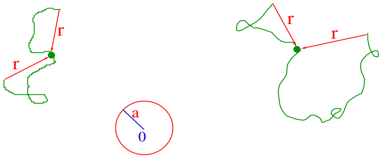
Rare trajectories dominate the average

Multiple searchers with reset in $d > 1$



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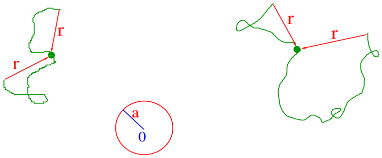

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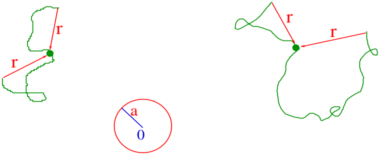

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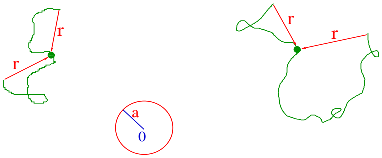

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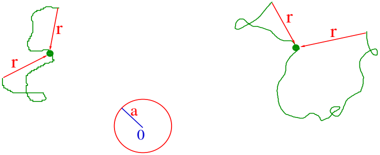

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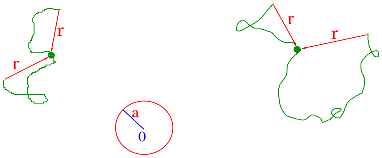

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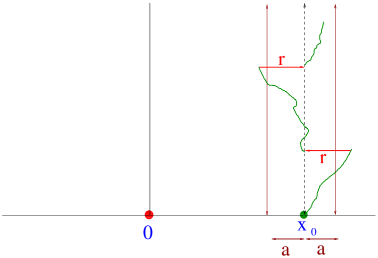

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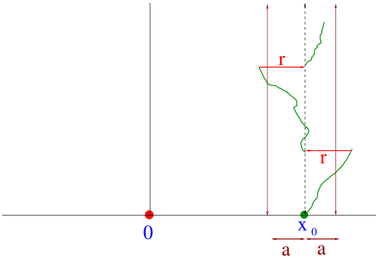

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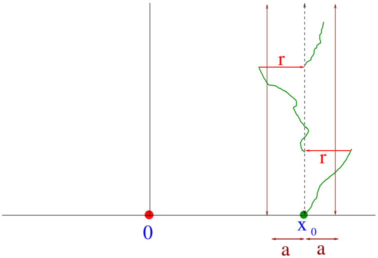

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(M.R. Evans & S.M., J. Phys. A: Math. Theo. 44, 435001 (2011))

Summary and Conclusion

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