Absorbing phase transitions in diluted conserved threshold transfer process

Sang Bub Lee

Department of Physics Education,
Kyungpook National University, Daegu 702-701

Influence of quenched disorder on APT

First introduced by Noest on Stochastic Cellular Automata by applying the Harris criterion, established for the equilibrium spin systems, on the absorbing phase transitions.

Harris criterion : presence of quenched impurities yields that pure fixed point is unstable if the specific heat exponent α is positive.

Harris, J Phys C 7, 1671 (1974)

For the contact process : $\alpha = d - 2\nu\,;\ \nu_\perp = \begin{array}{ll} 1.097\,(1\mathrm{D}), & \alpha \simeq 0.9 \\ 0.734\,(2\mathrm{D}), & \alpha \simeq 0.53 \\ 0.581\,(3\mathrm{D}), & \alpha \simeq 0.26 \\ 0.5\,(4\mathrm{D}), & \alpha = 0 \end{array}$

Disorder would thus be relevant if Harris criterion is applicable to APT.

Possible concerns :

- Is the replacement of v by $v_\perp$ in the hyperscaling relation valid?
- Is it valid to apply the Harris criterion established in equilibrium systems to nonequilibrium systems as well.

● APT in contact process on disordered lattice (diluted CP)

Moreira and Dickman, MC simulation PRE 54, R3090, PRE 57, 1263

In the Griffith region $(\lambda(0) < \lambda < \lambda_c(x))$, the survival probability and the density of active particles exhibited non-universal power-law divergences.

Hooyberghs et al. : renormalization calculations from a mapping of disordered contact process onto a random quantum spin chain using the Hamiltonian formalism (PRL 90, 100601, PRE 69, 066140, 2005)

- the transition is controlled by an infinite randomness.
- At infinite randomness fixed point, activated scaling $\tau \sim \xi^z \Rightarrow \ln \tau \sim \xi^\psi$ yields the particles and survival probability diverge logarithmically.
- Conventional scaling relation is replaced with the ultra-slow dynamic scaling:

$$\rho(\delta,\ 1/t,\ 1/L) = b^{-x}\rho(\delta b^{1/\nu_\perp},\ b^z/t,\ b/L)$$
$$\Rightarrow \rho(\delta,\ 1/\ln t,\ 1/L) = b^{-x}\rho(\delta b^{1/\nu_\perp},\ b^z/\ln t,\ b/L)$$

$$P_s(t) \sim (\ln t)^{-\theta'},\ N(t) \sim (\ln t)^{\eta'},\ R^2(t) \sim (\ln t)^{\sigma'}$$

- For weak disorder, the static critical exponents vary with the strength of disorder.

Vojta and Dickison : in 1D, PRE 72, 036126 (2005)

Quenched spatial disorder is introduced by making the birth rate λ as a random function of the lattice site r.

$$P(\lambda(\vec{r})) = (1-p)\delta(\lambda(\vec{r})-\lambda) + p\delta(\lambda(\vec{r})-c\lambda)$$

p : spatial density of impurity sites, *c* : relative strength

Overview

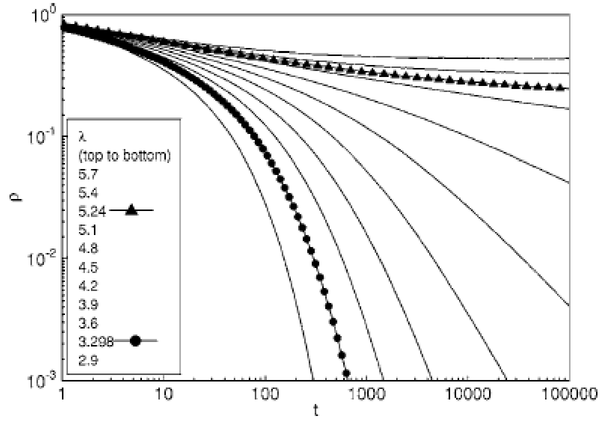


FIG. 2. Overview of the time evolution of the density for a system of 10^6 sites with $p=0.3$ and $c=0.2$. The clean critical point $\lambda_c^0 \approx 3.298$ and the dirty critical point $\lambda_c \approx 5.24$ are specially marked.

At clean critical point

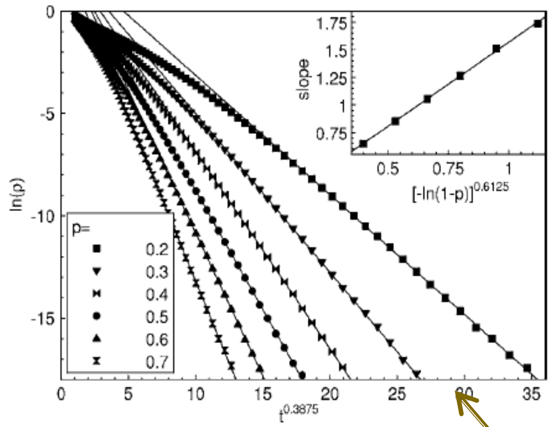


FIG. 3. Time evolution of the density at the clean critical point $\lambda_c^0=3.298$ for systems of 10^7 sites with $c=0.2$ and several p. The straight lines are fits to the stretched exponential $\ln\rho(t)\sim -Et^{d/(d+z)}$ predicted in Eq. (22) with $d=1$ and the clean $z=1.580$. Inset: Decay constant E vs $\tilde{p}^{z/(d+z)}$.

In the Griffith region

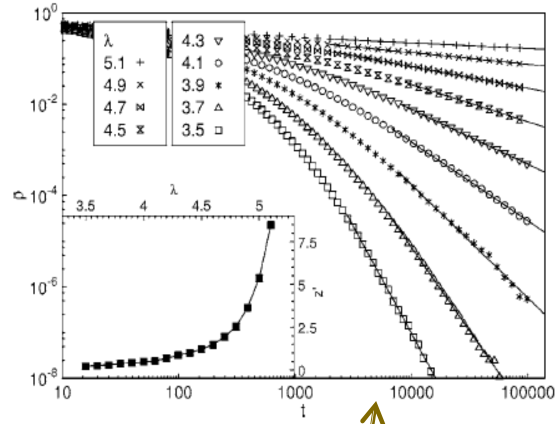


FIG. 4. Log-log plot of the density time evolution in the Griffiths region for systems with $p=0.3$, $c=0.2$, and several birth rates λ. The system sizes are 10^7 sites for λ=3.5, 3.7, and 10^6 sites for the other λ values. The straight lines are fits to the power law $\rho(t)\sim t^{-1/z'}$ predicted in Eq. (24). Inset: dynamical exponent z' vs birth rate λ.

- At clean critical point, stretched exponential decay $\ln\rho(t) \sim -\tilde{p}^{d/(d+z)}\, t^{d/(d+z)}$
- In the critical region λ(0) < λ < λ(p), nonuniversal power law $\rho(t) \sim t^{-1/z'}$

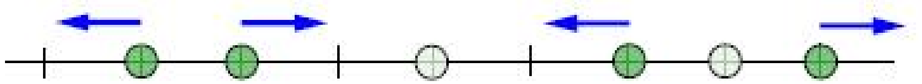
● Diluted conserved lattice gas (DCLG) model

Each lattice site is occupied by at most one particle.

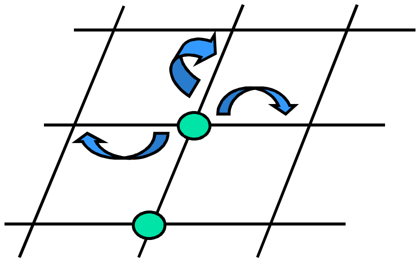
Active particles : particles with at least one particle on nearest-neighbor sites
Inactive particles : isolated particles surrounded by empty sites

Active particles attempt to hop to one of nearest-neighbor empty sites by repulsive short-range interaction.

[1d version]



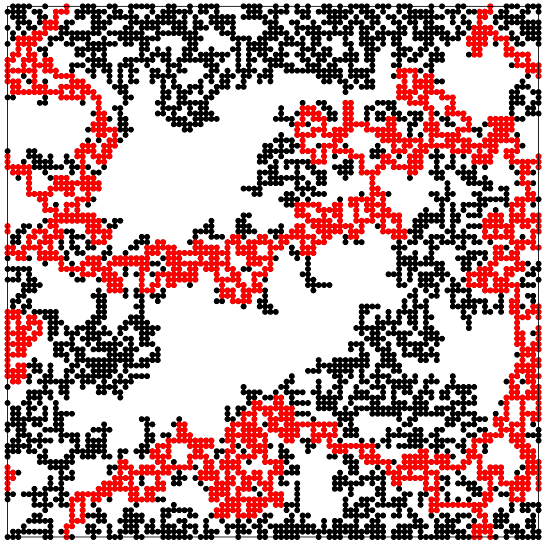
[2d version]



Control parameter : density of particles ρ
Order parameter : steady-state density of active particles n_{ss}

Disordered lattice :

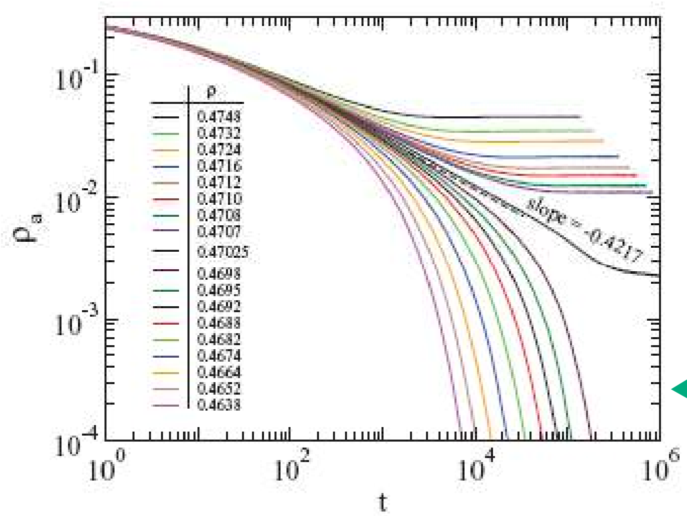
Infinite percolation network in 2D, 3D and 4D with the density of disorder (diluted sites) x.



On a 2D percolation network ($x<x_c$)

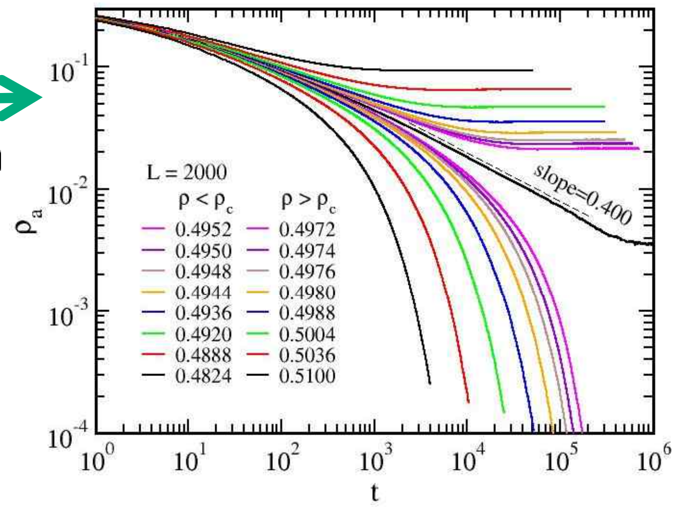
Phys. Rev E 84, 0411123 (2011)

Active particle densities:



$\rho_a(t)$ for x **=0.35**

$\rho_c \simeq 0.4961$; $\theta \simeq 0.400$



For x = **0.3**

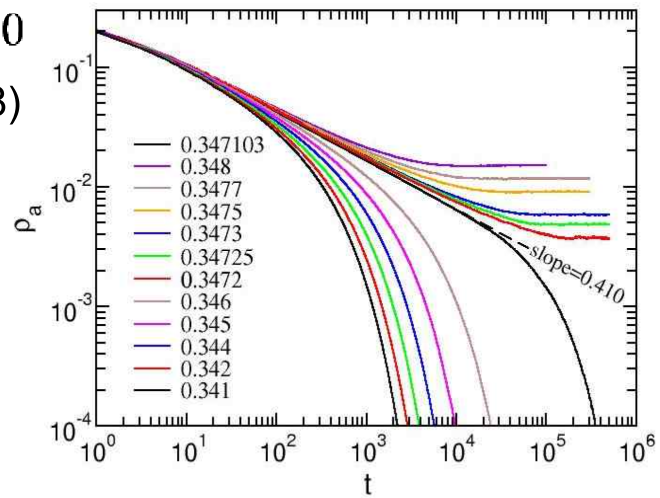
$$n_a(t) \sim \begin{cases} n_{ss}, & \text{for } \rho > \rho_c \\ t^{-\theta}, & \text{for } \rho = \rho_c \\ 0, & \text{for } \rho < \rho_c \end{cases}$$

$\rho_c = 0.47025$, with $\theta = 0.422$

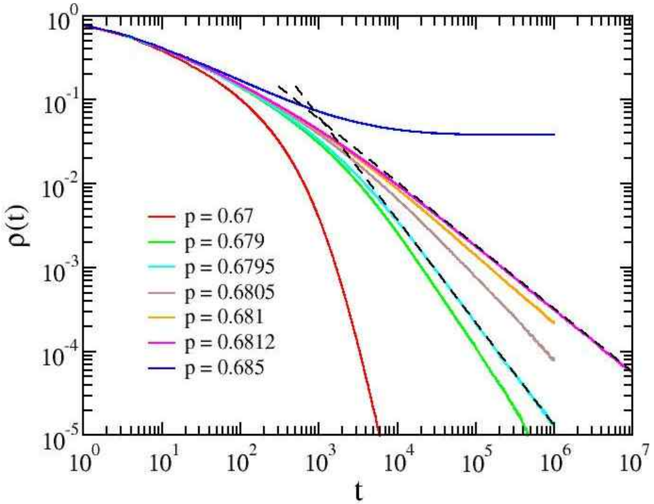
cff) on a regular square lattice

$\rho_c = 0.347103$, $\theta \simeq 0.410$

PRE78, 040103(R) (2008)



cff) CP model for x =0.35



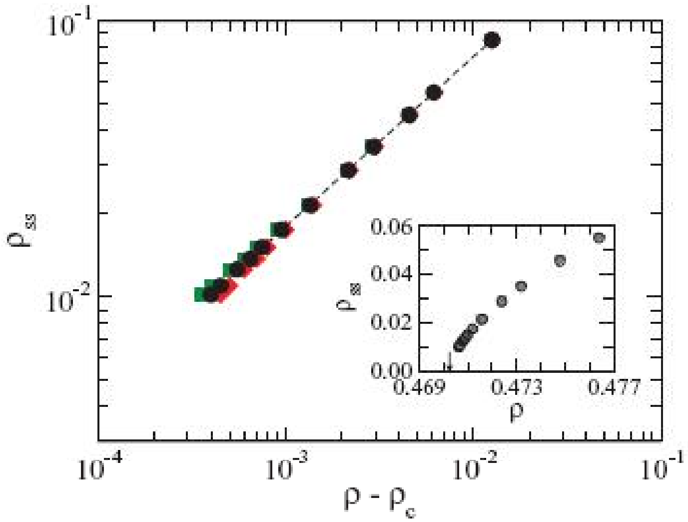
Order parameters: (x = **0.3**)

Saturated densities in the $t \to \infty$ limit.

$$n_{ss} \sim (\rho - \rho_c)^{\beta} \ \text{for} \ \rho > \rho_c$$

$$\text{with } \beta = 0.615(7)$$

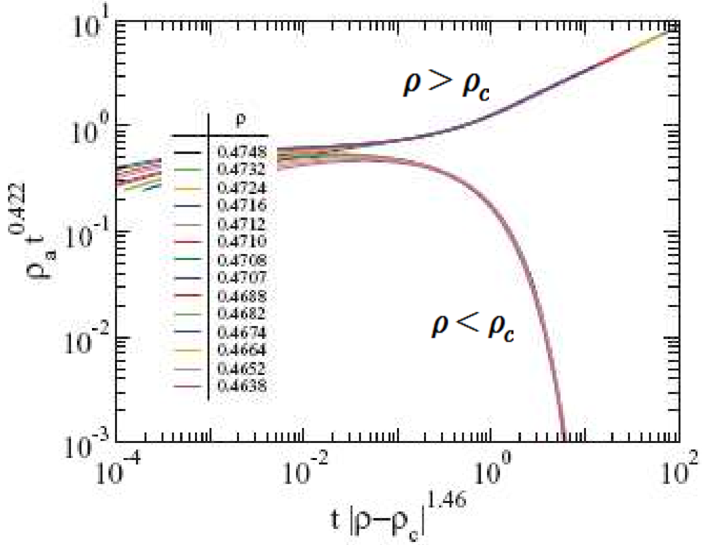
cff.) 2D (x =0) values : $\theta \simeq 0.42, \ \ \beta \simeq 0.64$



Off-critical scaling :

$$n_a(t,\,\rho) = t^{-\theta}F(t/\tau) = t^{-\theta}F(t|\rho-\rho_c|^{\nu_\parallel})$$

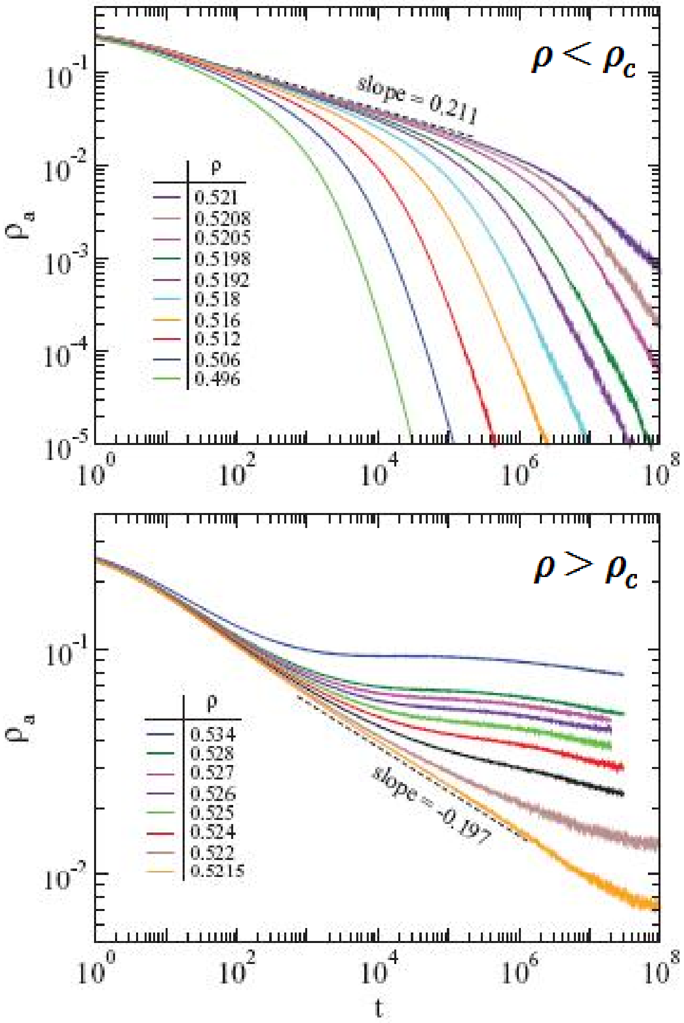
$$\nu_\parallel = \beta/\theta = 0.615/0.422 \simeq 1.46$$



Universal critical behavior for $x < x_c$ was observed with no Griffith phase.

Disorder appears to be irrelevant for the CLG model.

2D percolation network at $x=x_c$:



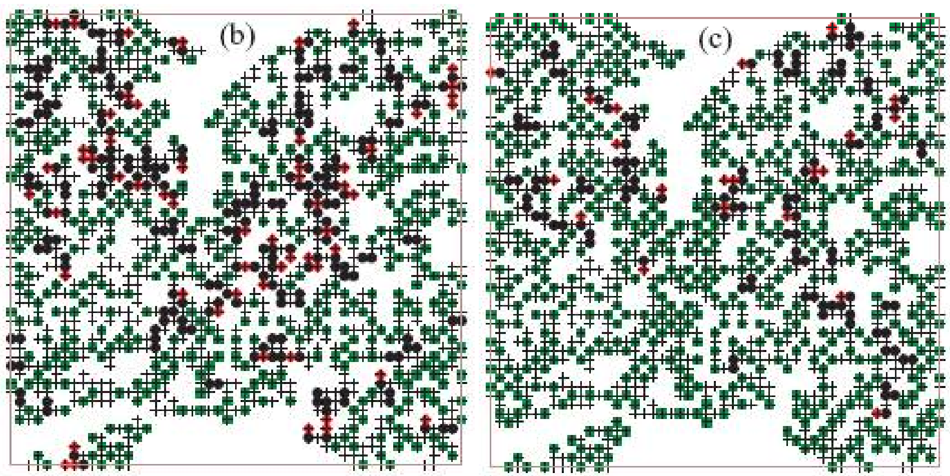
Active particle density :

$$0.5210 \leqq \rho_c \leqq 0.5215$$

$$n_a(\rho = \rho_c) \sim t^{-0.201(5)}$$

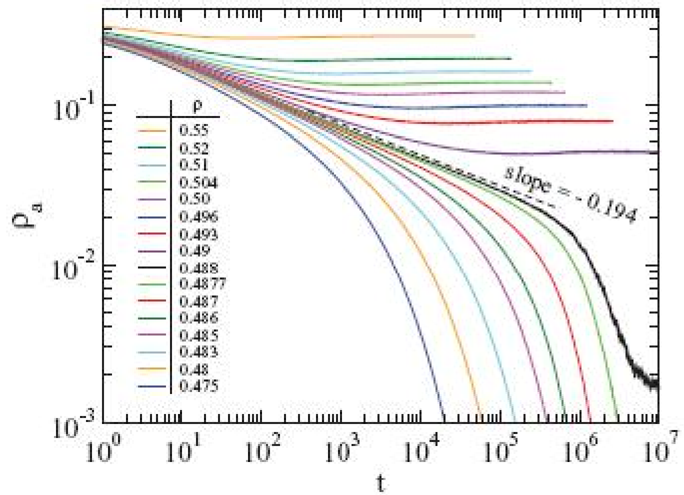
Data for both $\rho<\rho_c$ and $\rho>\rho_c$ appears to yield non-universal power-law behaviors, with powers depending on the particle density ρ.

Cause of nonuniversal behavior :
trapping effect of particles on dead ends and dangling ends



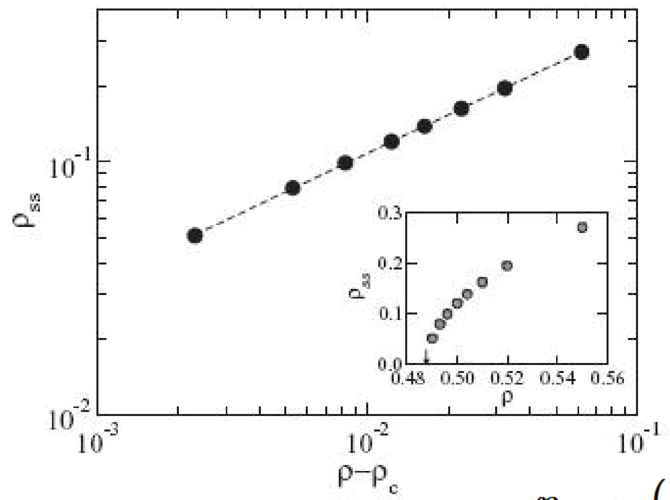
On a backbone network in 2D $x=x_c$:

Density of active particles :



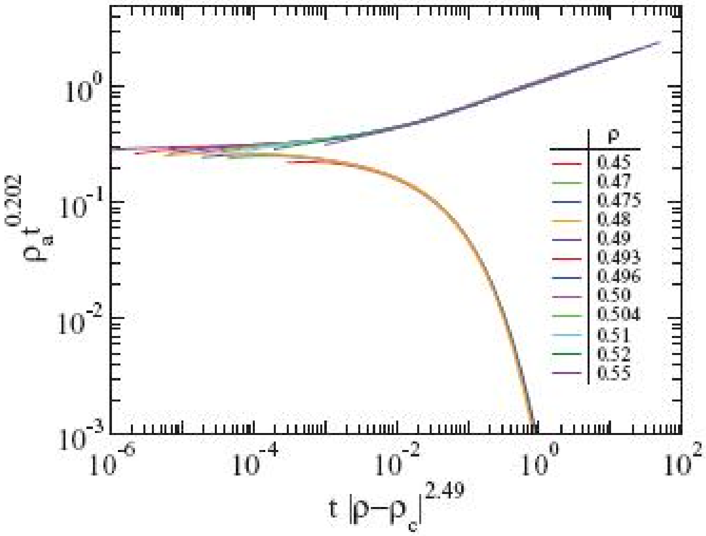
$\rho_c = 0.4877; \; \theta = 0.194$

Order parameter :



$n_{ss} \sim (\rho - \rho_c)^{0.502}$

Off-critical scaling plot :



Universal critical behavior with new sets of critical exponents only at x_c !!

Therefore, the pure fixed point is stable.

The nonuniversal power-law decrease observed on an infinite network was caused by the particles on dead ends and dangling blobs and, eliminating those sites, universal behavior was observed.

● Conserved threshold transfer process (CTTP)

Motivation :

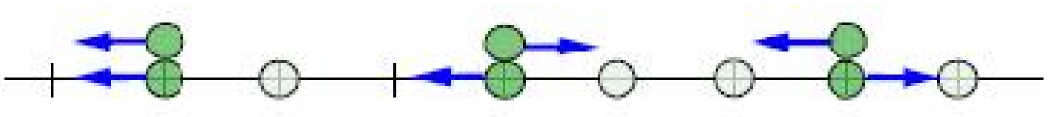
Trapping on dead ends is less significant for the CTTP model. Then, is the nonuniversal power-law behavior still observable on a critical percolation network?

Model description

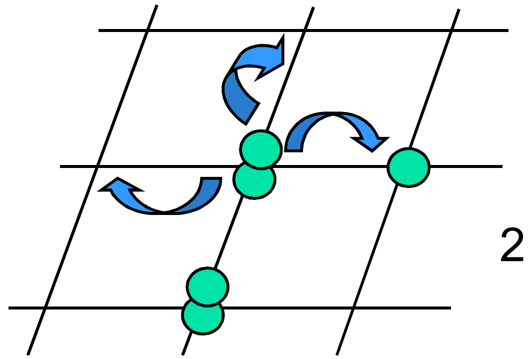
Each lattice site is either empty, singly occupied, or doubly occupied.

Doubly occupied sites are considered to be active sites and empty and singly occupied sites are inactive sites.

At each time step, both particles on each active site hop to nearest-neighbor sites.



1D version



2D version

Results on a 2D infinite network ($x<x_c$)

Active site densities :

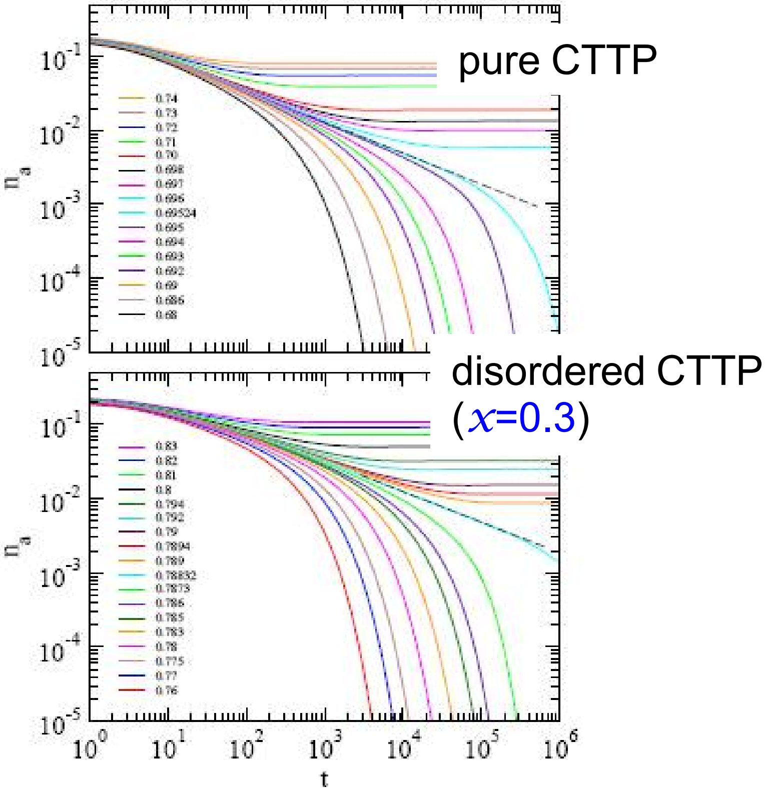


FIG. 2: Active-site densities as a function of the evolution time for the CTTP model on (a) a pure lattice and (b) on a diluted lattice of the disorder concentration $x = 0.3$. The data with the regression fits are at the determined critical densities.

Order parameter :

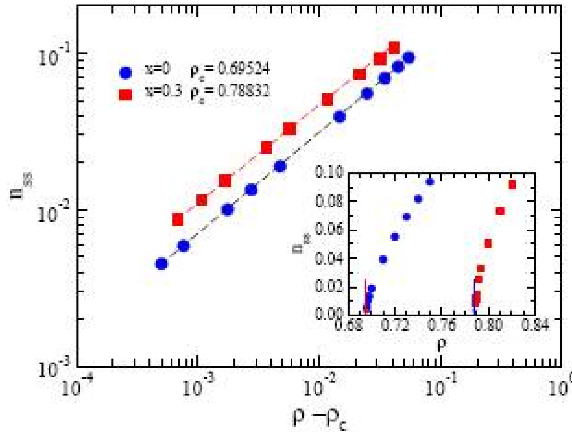


FIG. 3: The steady-state density of active sites as a function of the distance from criticality for the disorder concentration $x = 0$ (i.e., on a pure lattice) (circles) and $x = 0.3$ (squares). The inset shows the data on a normal scale.

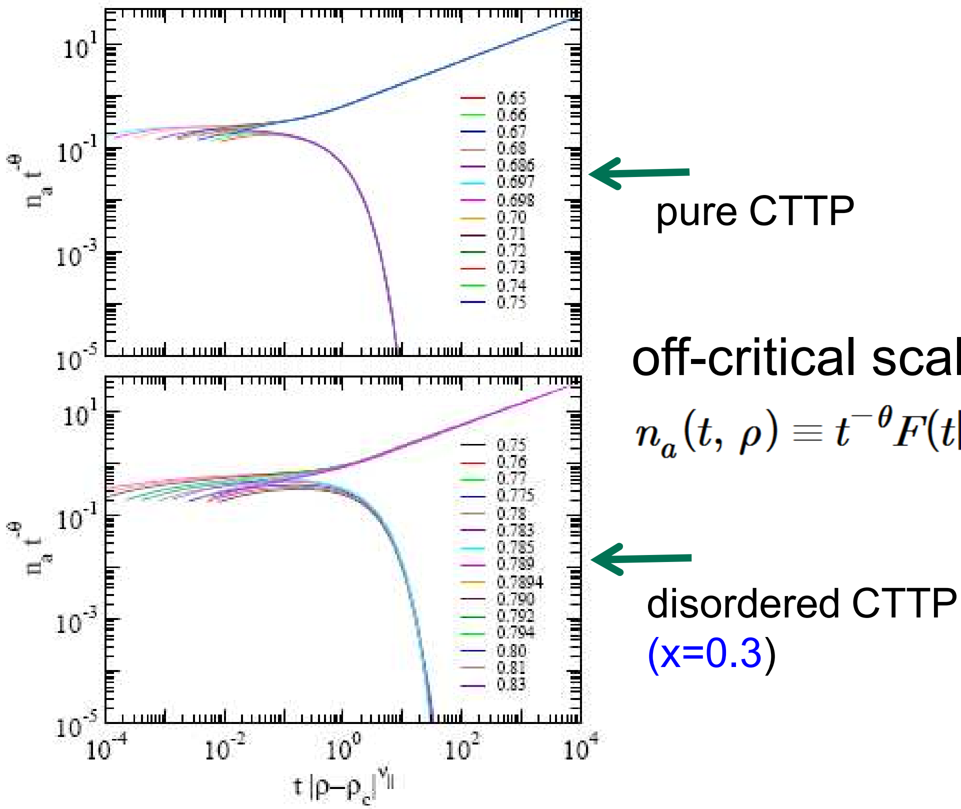


FIG. 4: The off-critical scaling for the data on a pure lattice (top) and those on a disordered lattice of $x = 0.3$.

off-critical scaling

$$n_a(t,\,\rho) \equiv t^{-\theta} F(t|\rho-\rho_c|^{\nu_\parallel})$$

Results for $x=x_c$:

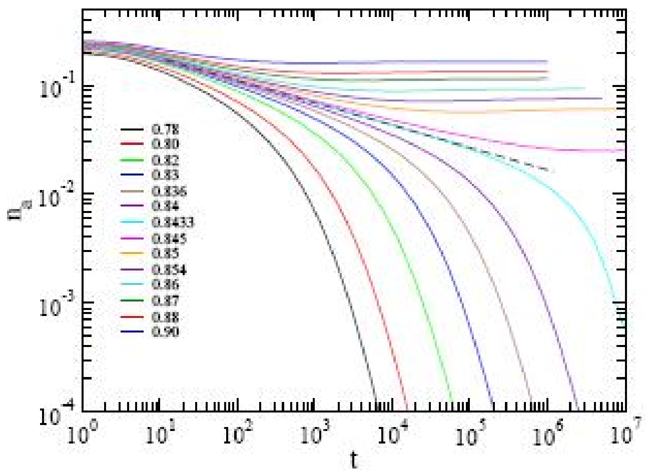


FIG. 5: (color online) Active-site densities as a function of the evolution time for the CTTP model on a critical percolation network generated at $x_c = 0.4073$. The data with the regression fits are at the determined critical densities. The numbers in the legend are the particle densities with the same order as the data from bottom to top.

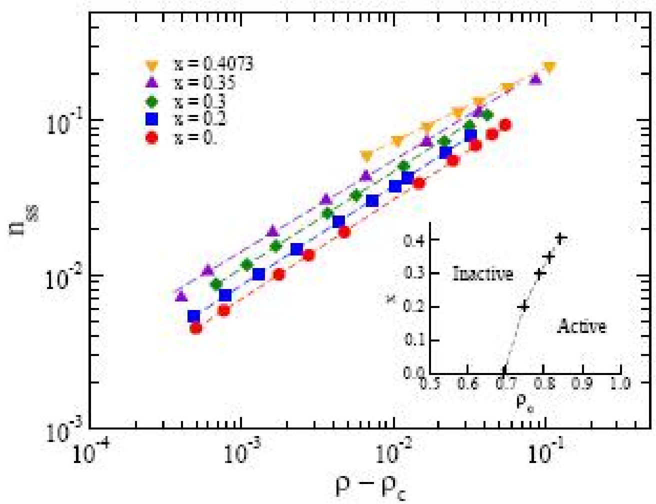
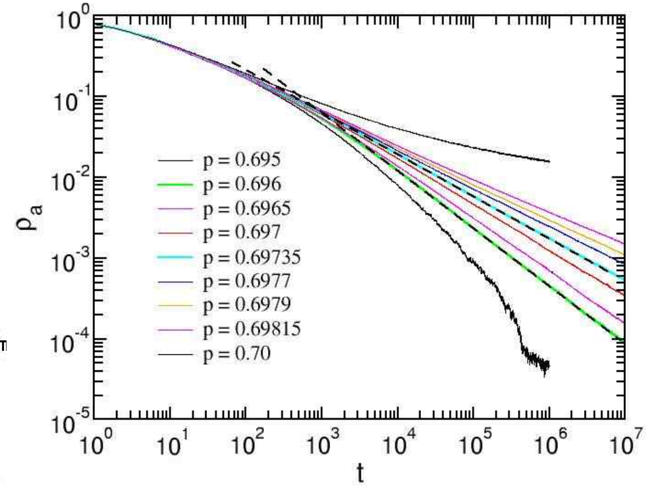


FIG. 7: (color online) The steady-state densities of active sites as a function of the distance from criticality, $\rho - \rho_c$, for the CTTP model on a disordered lattice of, from bottom to top $x = 0$ (i.e., on a pure lattice), $x = 0.2$, $x = 0.3$, $x = 0.4$, and at x_c.

cff) CP model for $x = x_c$



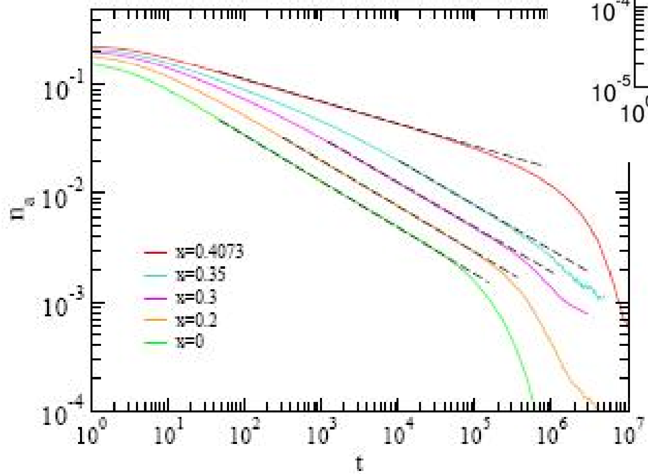


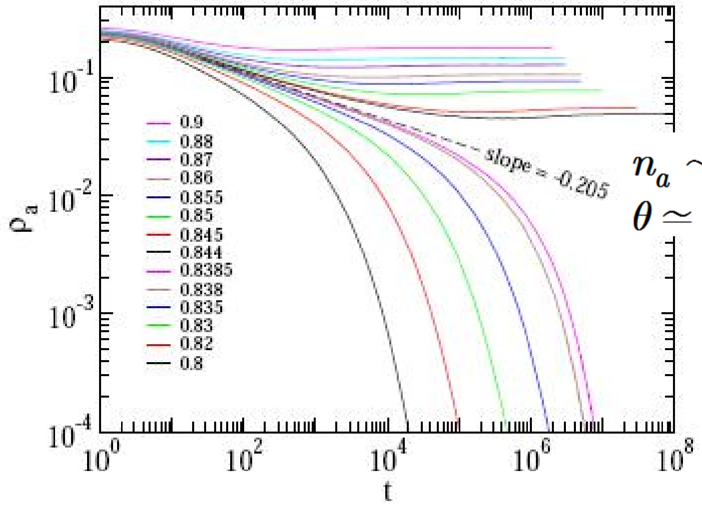
FIG. 6: (color online) Active-site densities as a function of the evolution time for the CTTP model on a disordered lattice of, from bottom to top $x = 0$ (i.e., on a pure lattice), $x = 0.2$, $x = 0.3$, $x = 0.4$, and at x_c. The dashed lines are the regression fits. The regression slope shows abrupt change at the critical concentration.

Estimates :

$x\,(1-p)$	ρ_c	θ	β
0	0.69524	0.423	0.641
0.2	0.744771	0.419	0.639
0.3	0.78832	0.421	0.625
0.35	0.81337	0.418	0.588
0.4073	0.8433	0.205	0.465

Results on a backbone network :

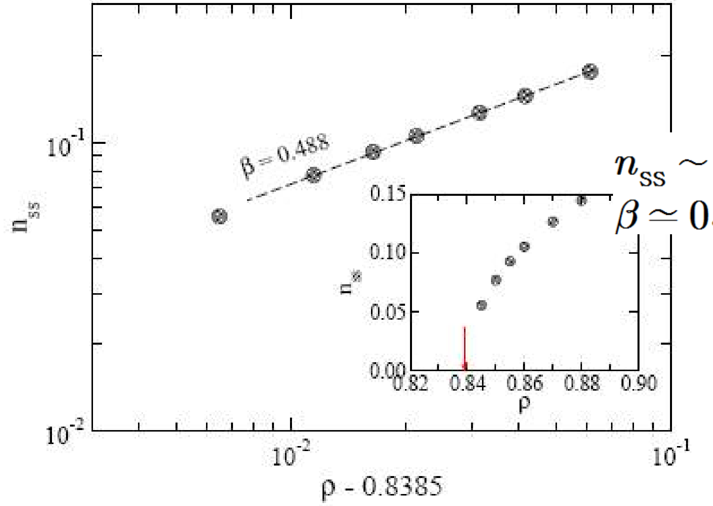
Active site density :



$$n_a \sim t^{-\theta}$$
$$\theta \simeq 0.205$$

FIG. 8: (color online) Active-site densities as a function of the evolution time for the CTTP model on a percolation backbone network. The numbers on the legend are the densities of particles in the same order as the data, and the data with a regression are for the determined critical density.

Order parameter :



$$n_{ss} \sim (\rho - \rho_c)^{\beta}$$
$$\beta \simeq 0.489$$

FIG. 9: (color online) The steady-state densities plotted on a double logarithmic scale against the distance from criticality for the CTTP model on a percolation backbone network. Data in the inset are on a normal scale.

Results on a backbone network appears to be similar to those on an infinite network. (not yet conclusive!)

Therefore, dead ends and dangling blobs on an infinite network appear to yield null effect on the APT in the CTTP model

● **Results** in higher dimensions :

On a 3D network

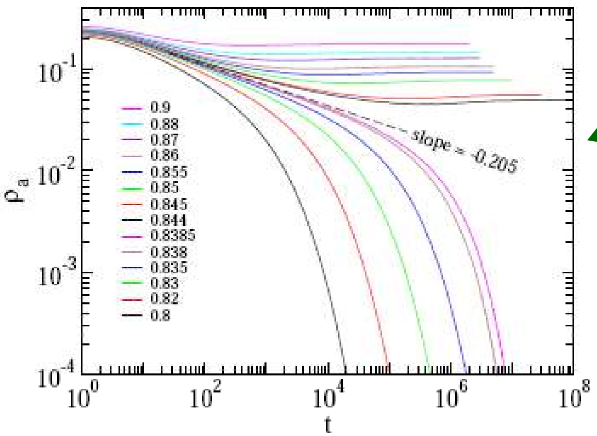


FIG. 8: (color online) Active-site densities as a function of the evolution time for the CTTP model on a percolation backbone network. The numbers on the legend are the densities of particles in the same order as the data, and the data with a regression are for the determined critical density.

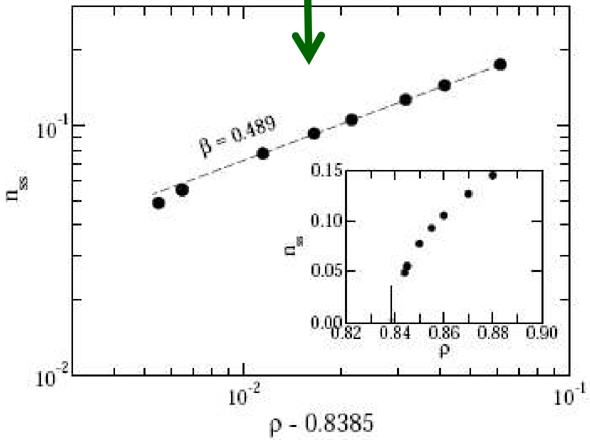


FIG. 9: (color online) The steady-state densities plotted on a double logarithmic scale against the distance from criticality for the CTTP model on a percolation backbone network. Data in the inset are on a normal scale.

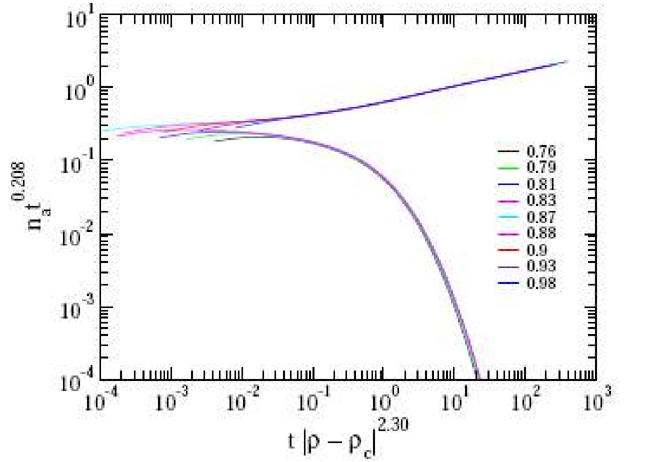


FIG. 12: The off-critical scaling for the data on a 3D critical percolation network.

2D, 3D & 4D

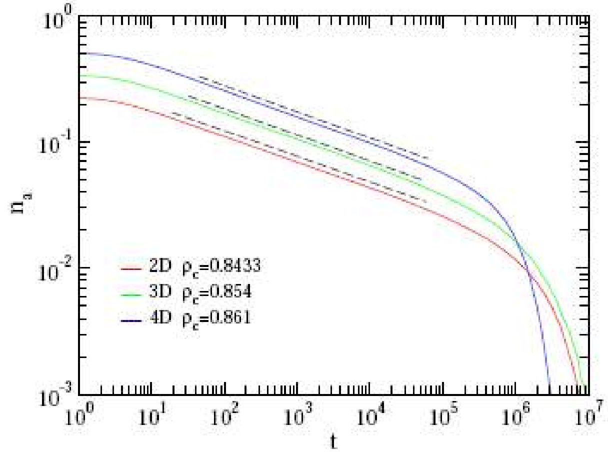


FIG. 13: (color online) Active-site densities against the evolution time for the CTTP model on a critical percolation network generated at $x = x_c$ in, from bottom to top, two, three, and four dimensions. The regression slopes are similar for three cases.

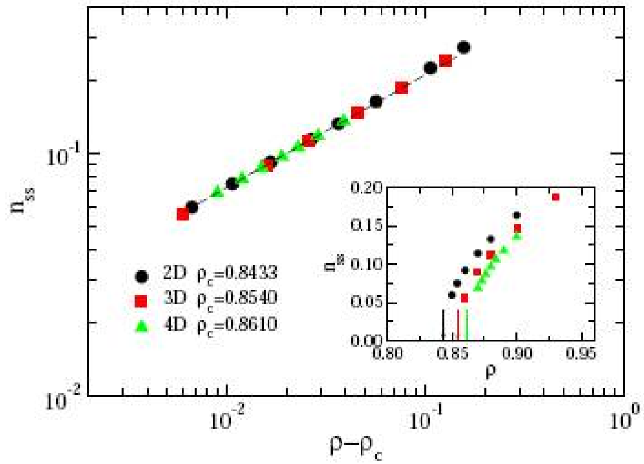


FIG. 14: (color online) The steady-state densities against the distance from criticality on a double logarithmic scale for the CTTP model on a critical percolation network in from botton to top, two, three, and four dimensions.

Black : 2D
Red : 3D
Green : 4D

Summary

- **The disorder appeared to be irrelevant** as long as the concentration of disorder was less than critical concentration (thus, pure fixed point is stable).

  Nonuniversal power-law region does not exist for the CLG and the CTTP models and, thus, CLG and CTTP models are clearly in different universality class from DP.

  Critical behavior on an infinite percolation network appears to be super-universal, i.e., it does not depend on the substrate fractal dimension.

- In one dimension, the disordered Manna model is on going, however, indication of nonuniversal power-law decrease in the active site density is not yet observed.

Preliminary results in 1D :

x : disorder concentration

p : probability of hopping from or to the disordered sites.