Measuring and Controlling Out of Equilibrium Fluctuations

Masaki Sano[1]
Shoichi Toyabe[2]
Ryo Suzuki1, Yuta Hirayama[1]

[1]Department of Physics, The University of Tokyo
[2]Ludwig-Maximilians University München

Outline

- Maxwell's demon
  - Introduction
- Demonstration of Information-Energy conversion

  (Toyabe, Sagawa, Ueda, Muneyuki, Sano, Nature Physics, 6, 988, (2010))
- Role of information in different systems
  - Laser cooling
  - Molecular motors
  - Chemotaxis of bacteria

Demons in Thermodynamics

Why does the arrow of time exist?

- Boltzmann's H theorem

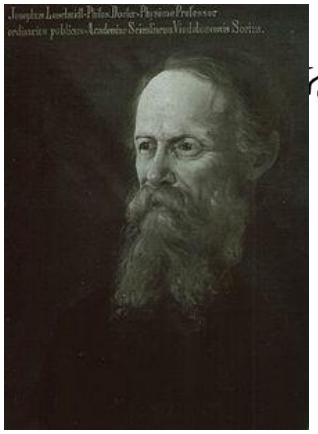
Increase of entropy

$$\frac{d}{dt}H = \frac{d}{dt}(\sum_i p_i \ln p_i) \leq 0$$

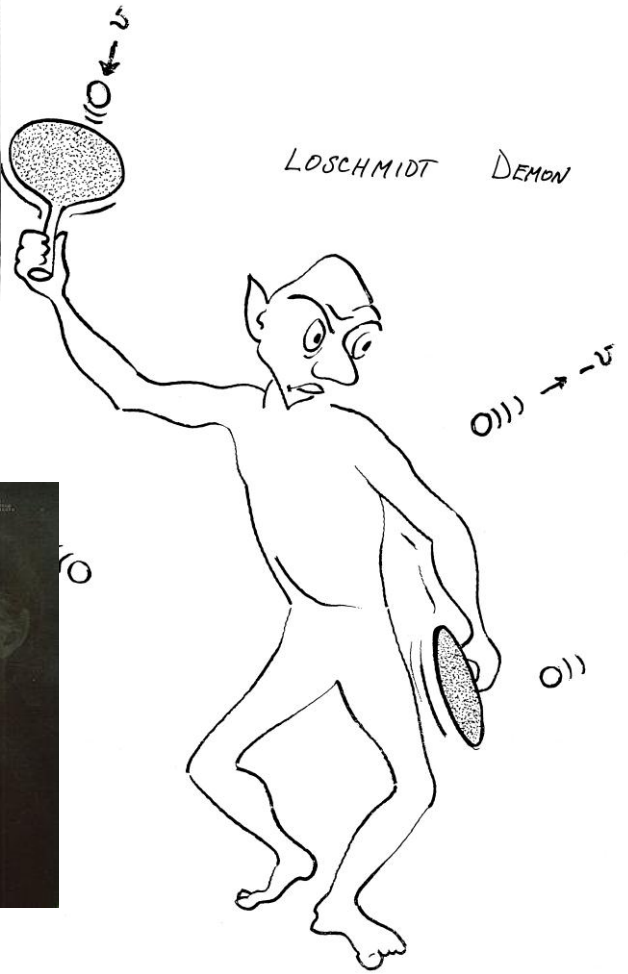
L. Boltzmann

- Loschmidt's demon

Entropy will decrease if the velocities of all atoms are reversed.



Y. Loschmidt



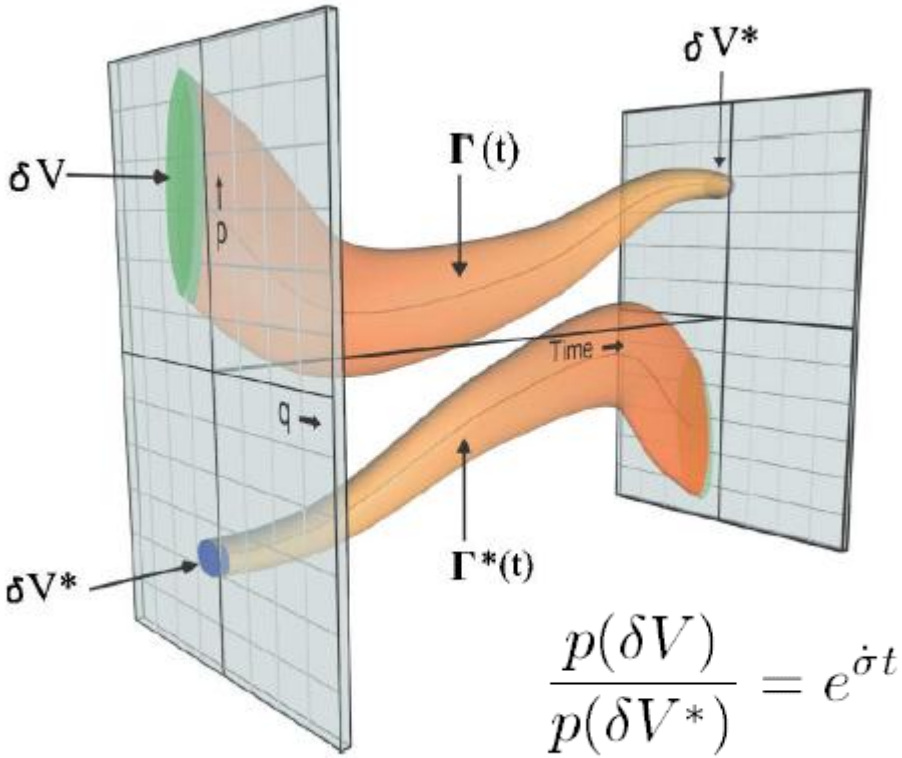
New Theories in Statistical Mechanics

Fluctuation Theorem

- Second law of thermodynamics
- Fluctuation dissipation theorem

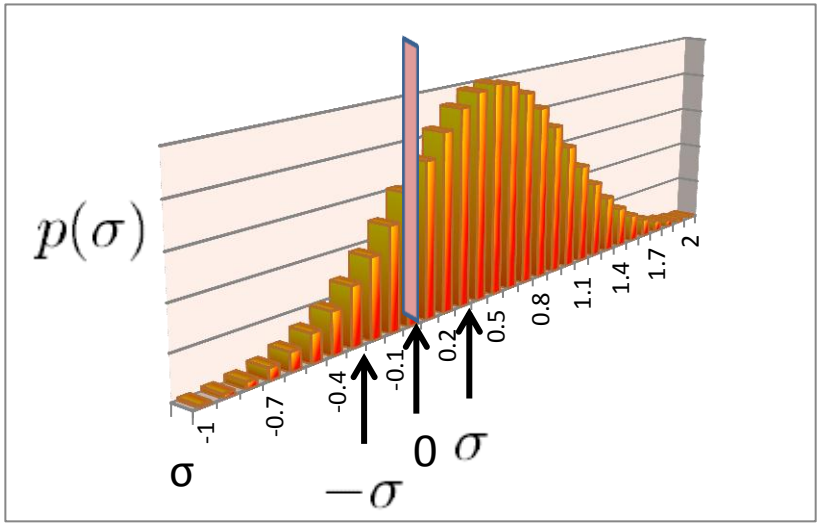
$$\frac{p(\sigma)}{p(-\sigma)} = e^{\sigma}$$

Evans, 1993
Gallavotti, Cohen



$$\frac{p(\delta V)}{p(\delta V^*)} = e^{\dot{\sigma} t}$$

Entropy production: $\sigma = \beta(W - \Delta F)$



Initial conditions producing positive entropy production is much more frequent than the negative entropy production

Confirmed for driven Brownian particles, electric current, etc.

Maxwell's demon

- Violation of the second law of thermodynamics (1871)

James Clerk Maxwell (1831-1879)

Opening & closing door do not perform work to atoms.

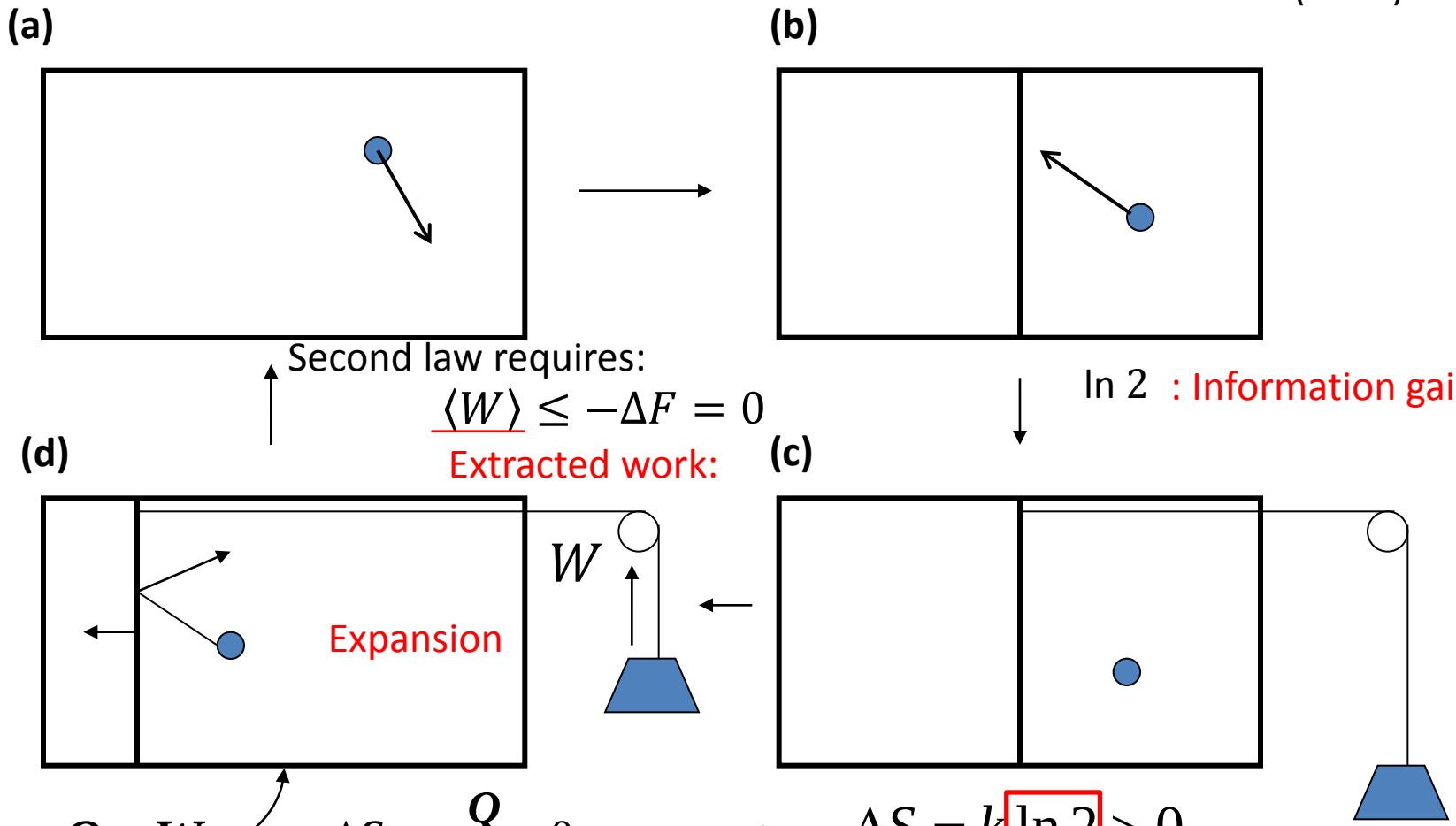
⇒ 2nd law really violate？

⇒ controversial state lasted more than 150 years.

Maxwell's Demon and Szilard Engine

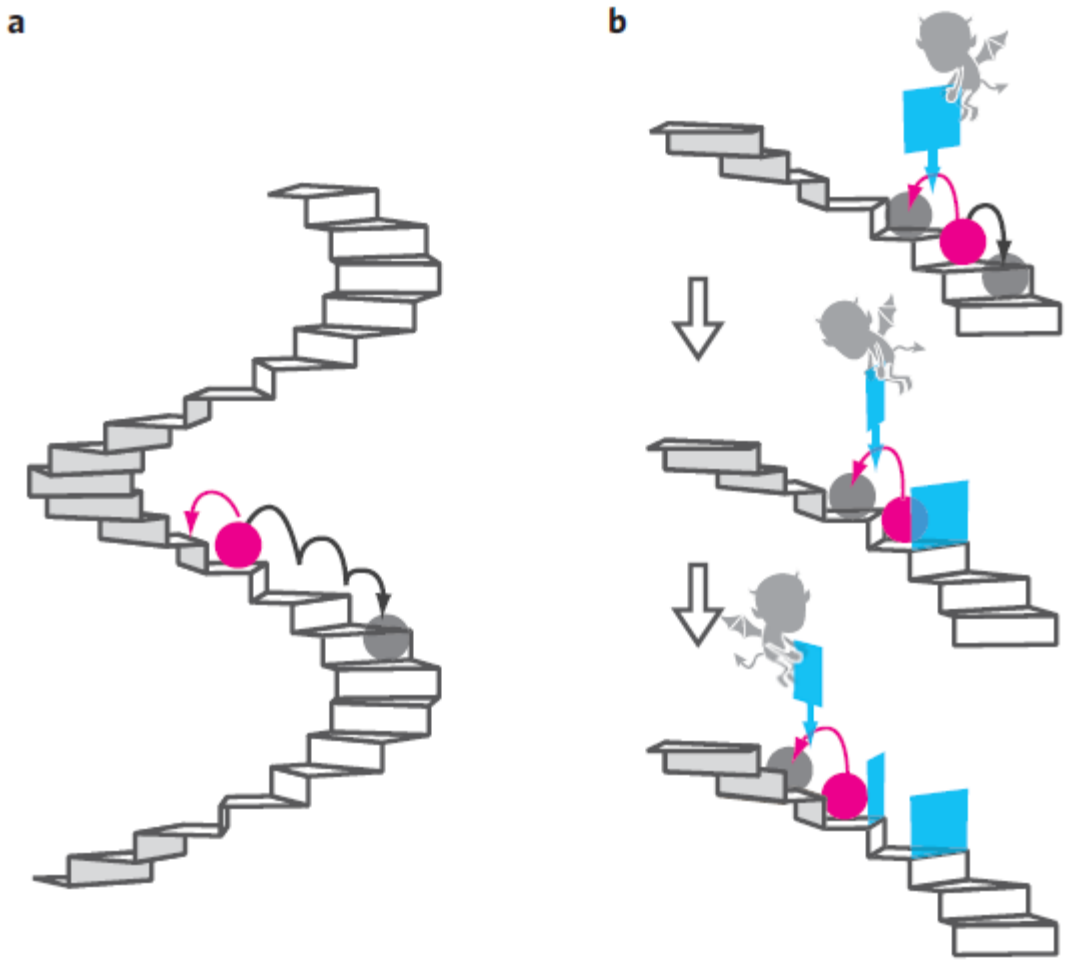
The simplest and analyzable Maxwell's demon

Szilard (1929)



However we gain, $W = k_B T \ln 2$: Information gain → decrease of entropy

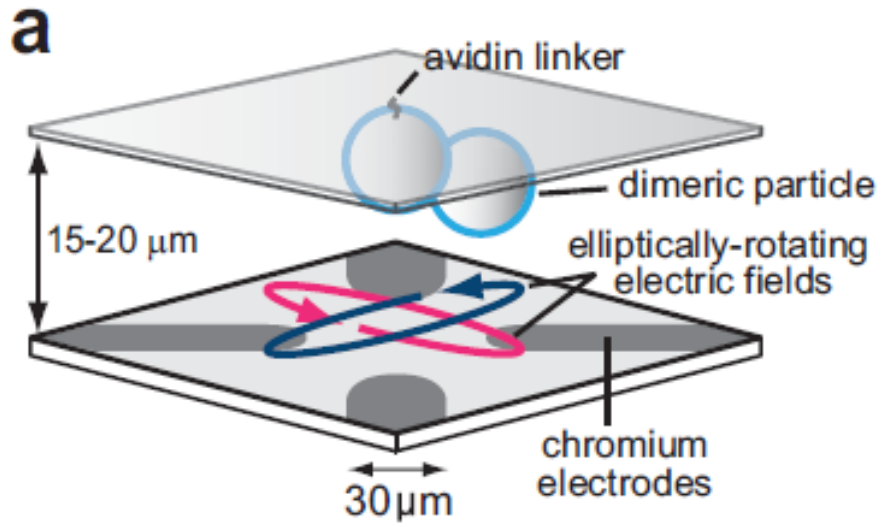
Schematic illustration of the experiment

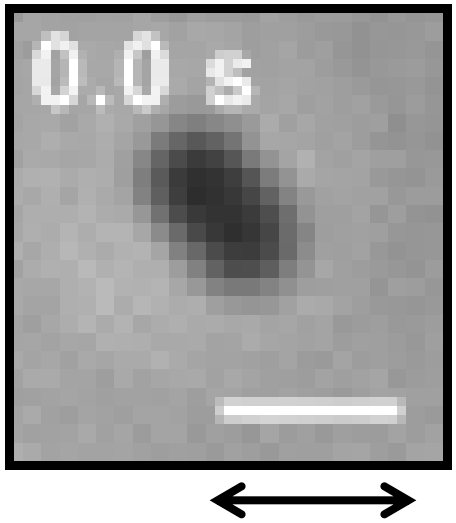


Toyabe, Sagawa, Ueda, Muneyuki, Sano, Nature Physics, 6**,** 988**,** (2010))

Experimental Setup

- Dimeric polystyrene particle (300nm) is linked on the substrate with a biotin.
- Particles exhibit a rotational Brownian motion.

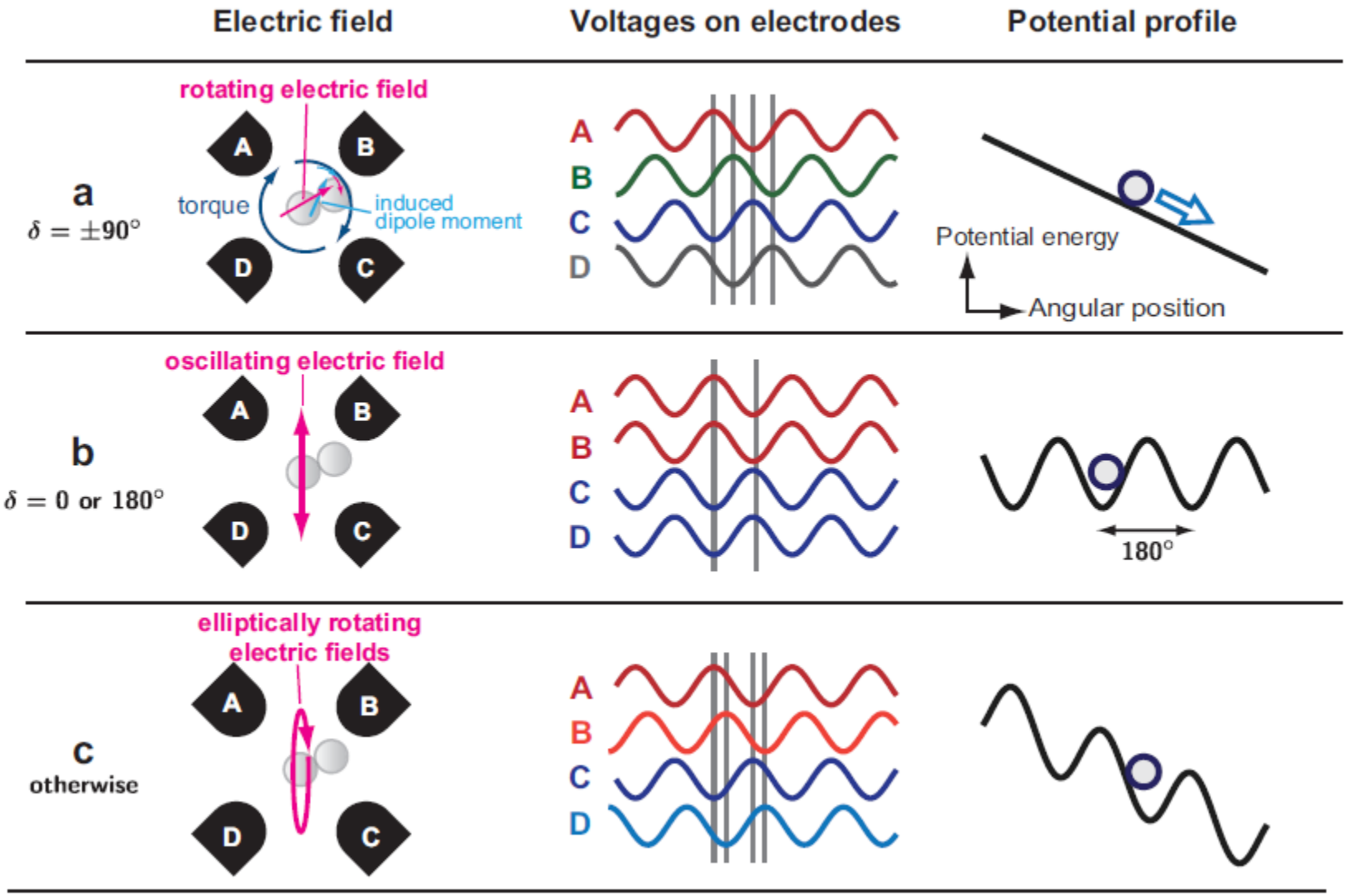




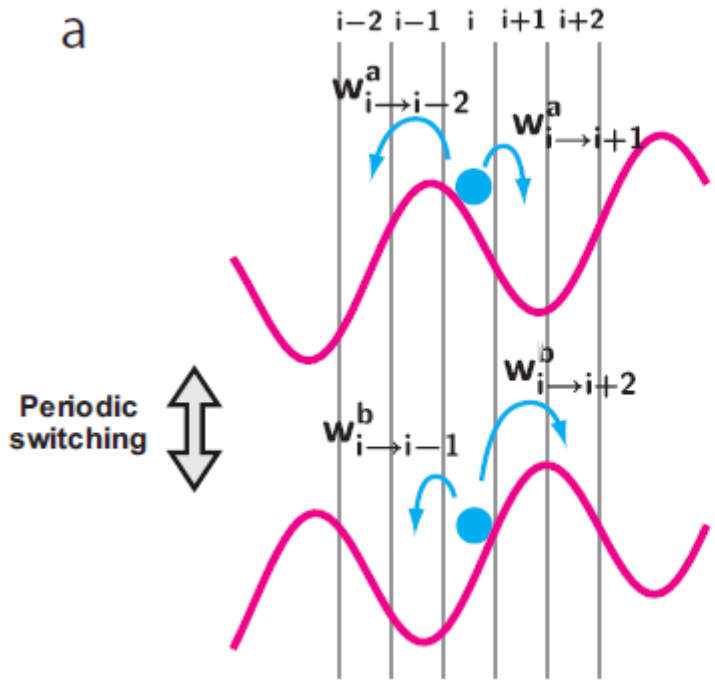
1 μm (1/1000 mm)

- Quadrant electrodes are patterned on the substrate

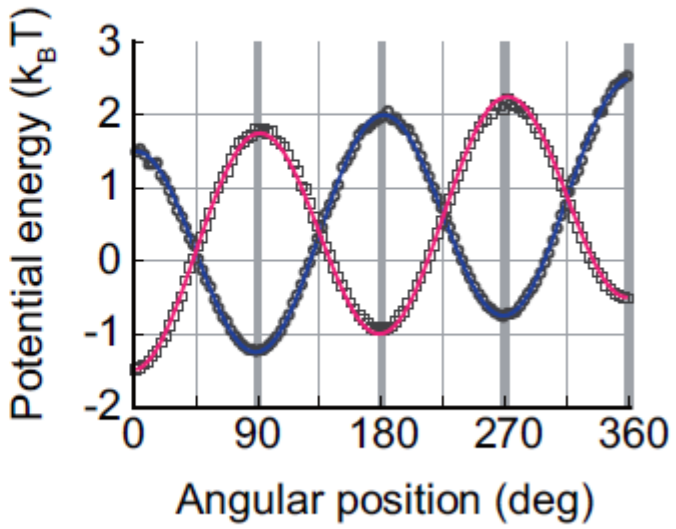
How to produce a spiral-stair-like potential



Estimating a potential function



b Measured potential function

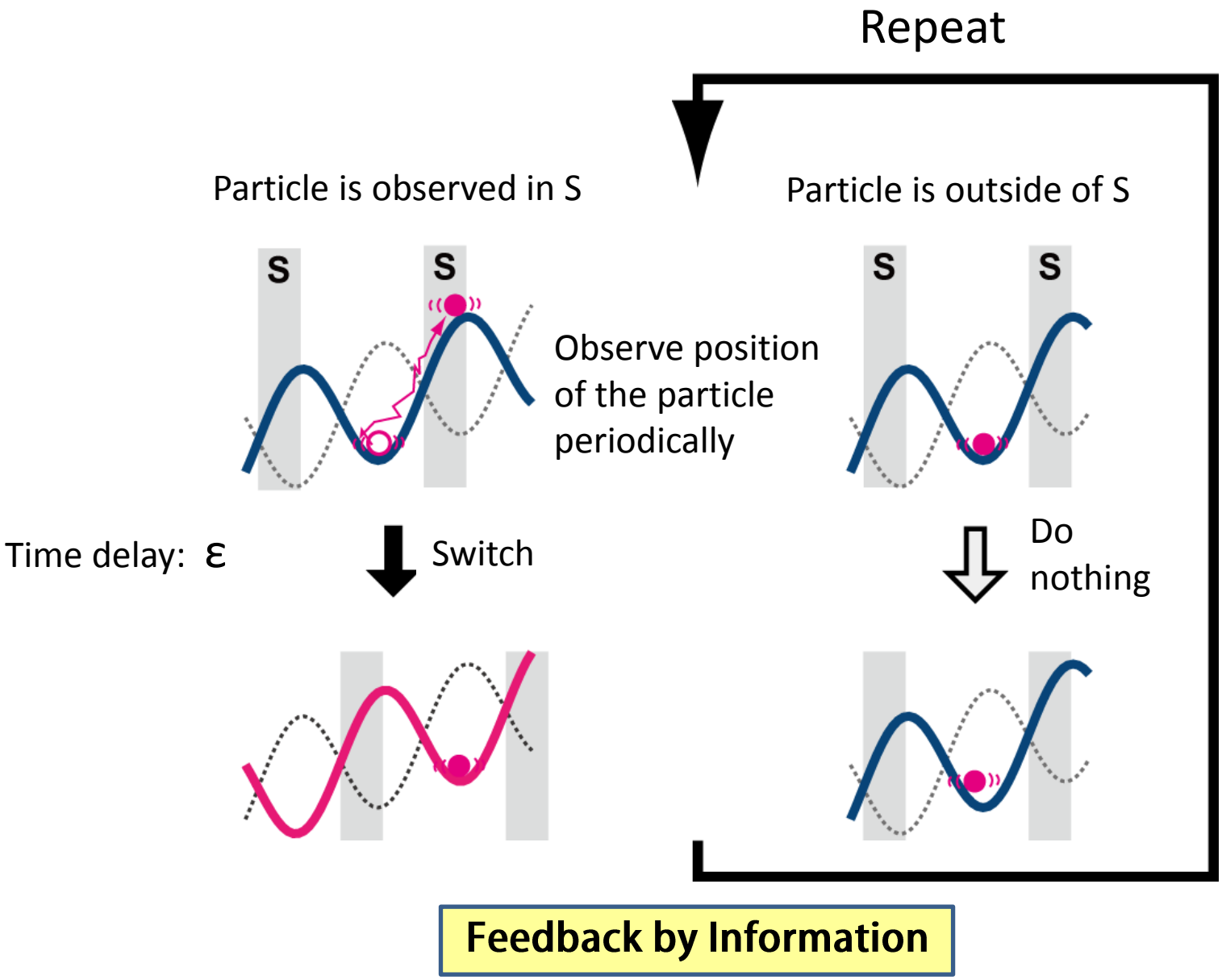


$$\varepsilon^2(\{U_i\}) \equiv \sum_{i<i} \sqrt{n_{i\to j} n_{j\to i}} \left[\Delta U_{i\to j} - \Delta U'_{i\to j}\right]^2,$$

where $\Delta U'_{i\to j} \equiv k_B T \left[\ln w_{j\to i} - \ln w_{i\to j}\right]$

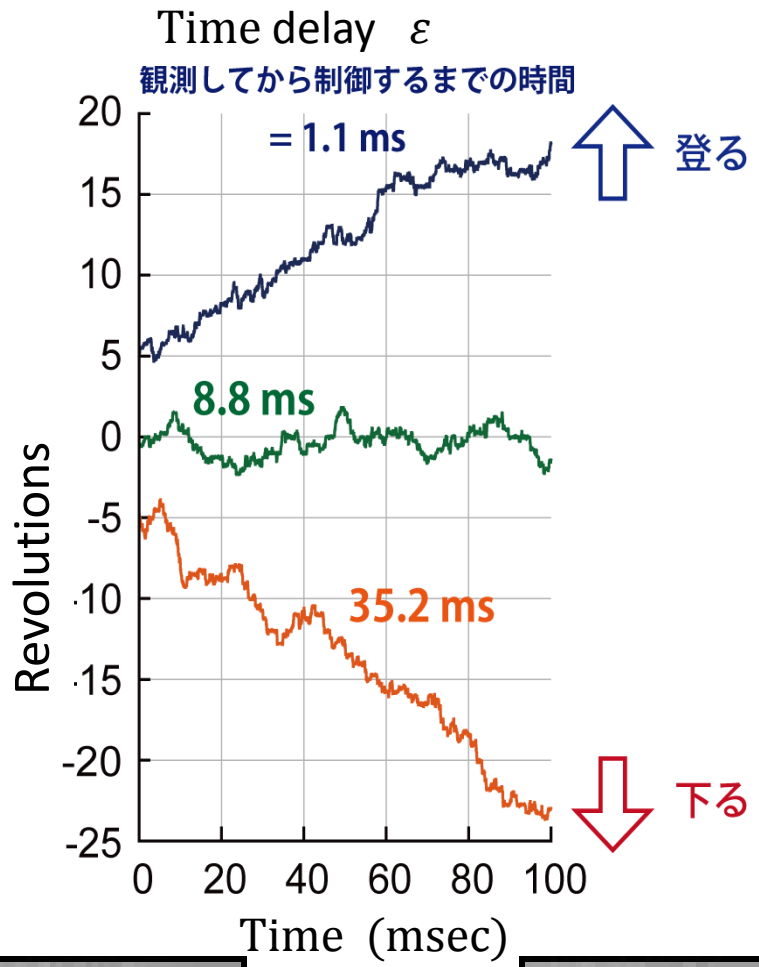
Minimize ε^2

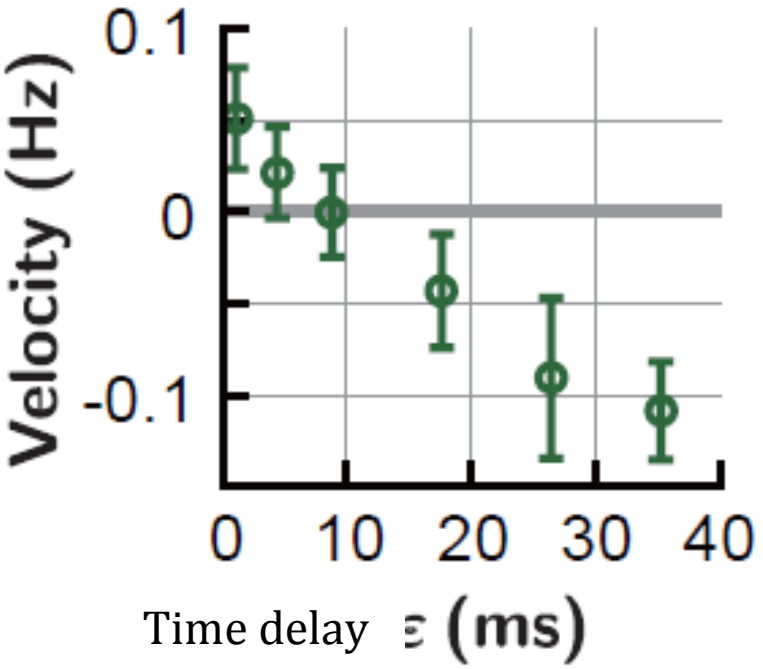
Feedback control based on information contents

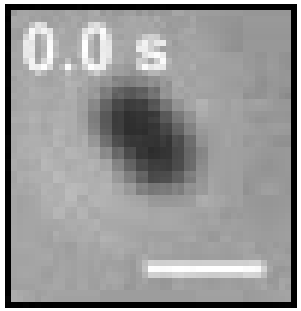


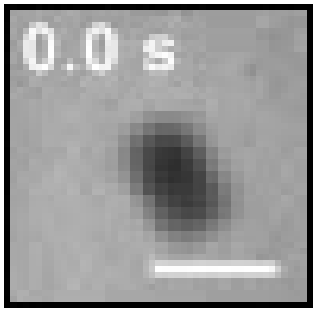
Feedback by Information

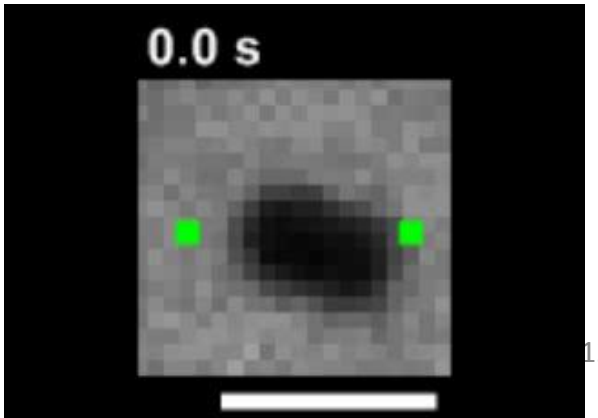
Trajectories under feedback control





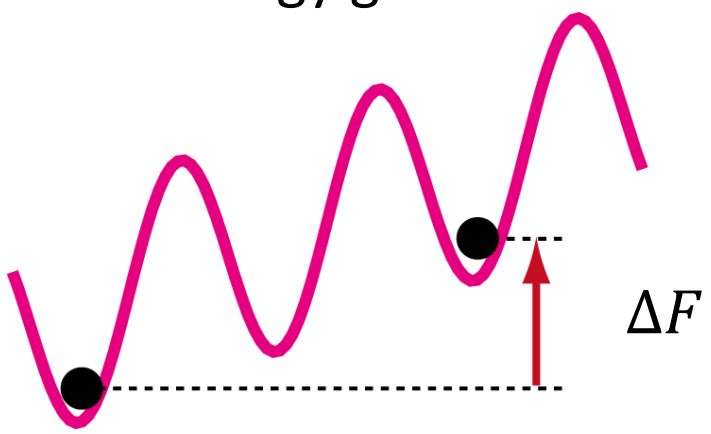




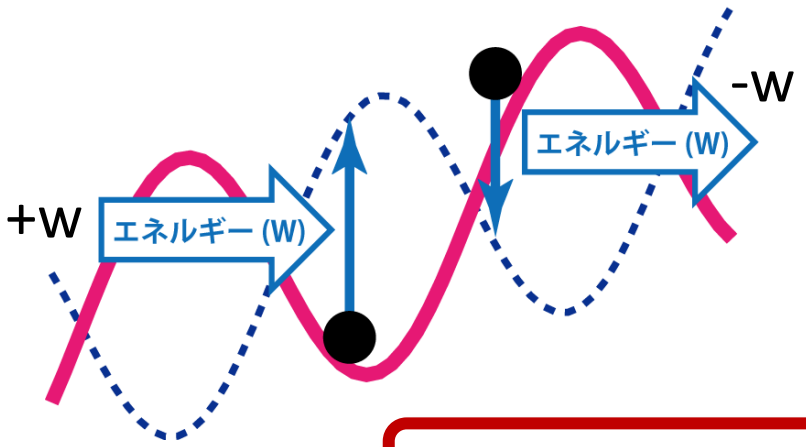


Calculation of Free Energy

- Free energy gain

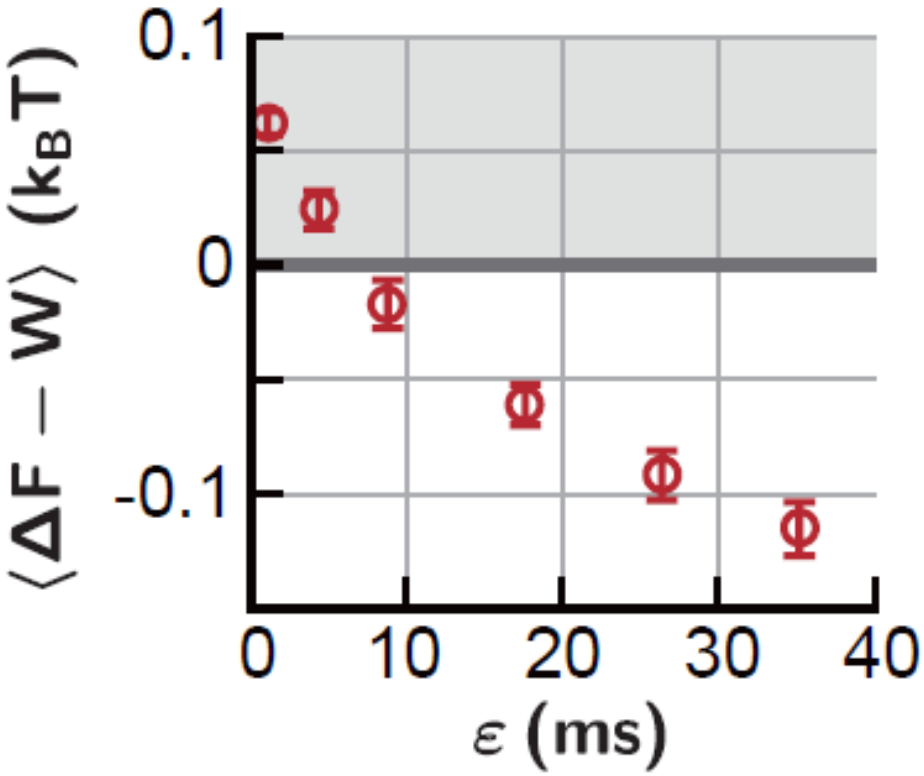


- Work done to the particle



W :Work performed to the system

F :Free energy gain of the system

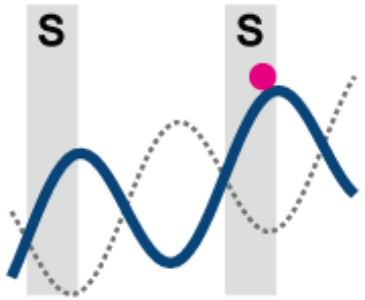


Calculate（ΔF-W）

Efficiency of Information-Energy Conversion

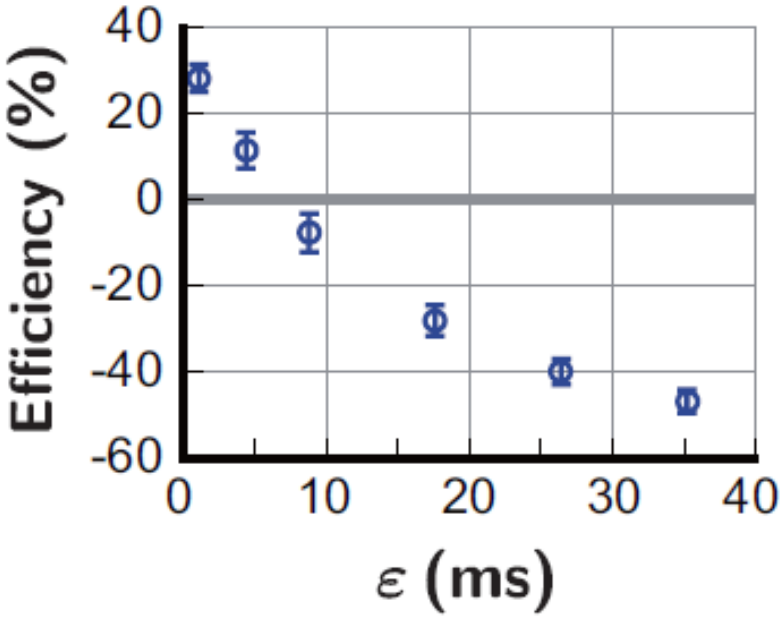
Information gained by the observation:

$$I = -p \ln p \ - (1-p)\ln((1-p)$$



Efficiency of Information-Energy Conversion :

$$\vartheta = \frac{\Delta F - W}{k_B T I}$$



28% Efficiency

$$\frac{p(\sigma)}{p(-\sigma)} = e^{\sigma}$$

Fluctuation Theorem → **Jarzynski equality**

W :Work performed to the system

F :Free energy gain of the system

$$\left\langle e^{(\Delta F - W)/k_{BT}} \right\rangle = 1$$

2nd law of thermodynamics: $\langle W \rangle - \Delta F \geq 0$

Generalized Jarzynski equality including information

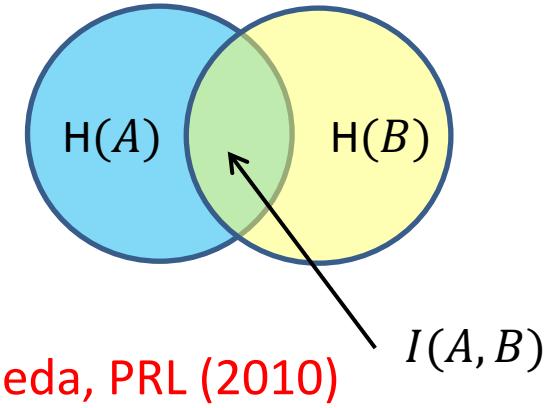
$$\langle W \rangle \geq \Delta F - kT\langle I \rangle$$

I : mutual information

measurement and control have errors

Correspondingly generalized Jarzynski equality:

$$\left\langle e^{(\Delta F - W)/k_B T} \right\rangle = \gamma$$



Sagawa & Ueda, PRL (2010)

Experimental test of generalized Jarzynski equality

- Entropy Production

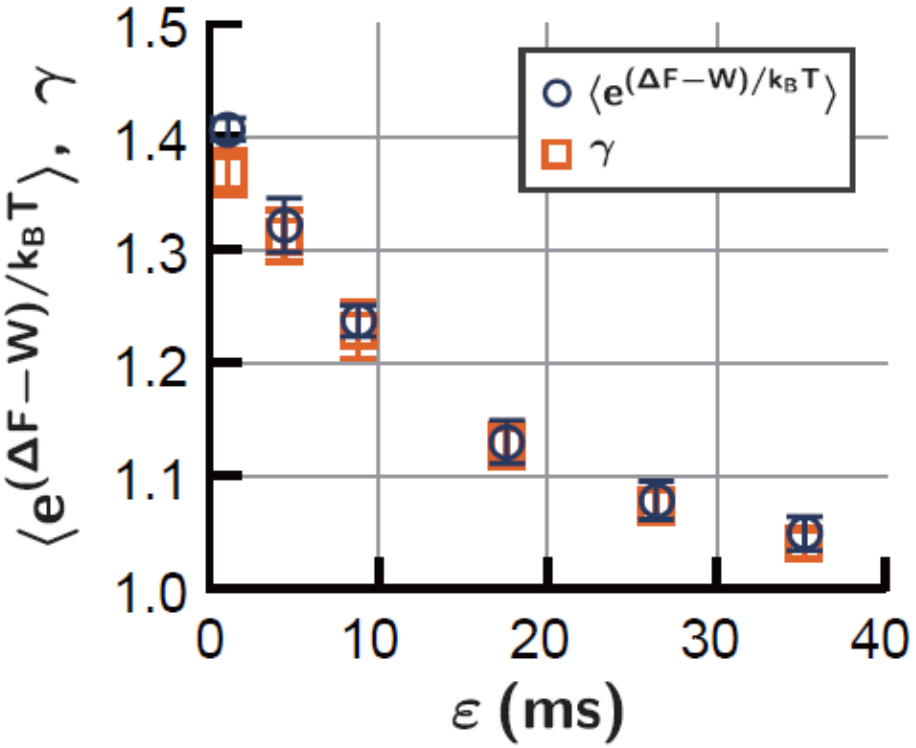
$$\left\langle e^{\beta(\Delta F - W)} \right\rangle = \gamma$$

generalized Jarzynski equality

Sagawa, Ueda, PRL (2010)

Feedback Efficacy

- **A fundamental principle to relate energy and information feedback**

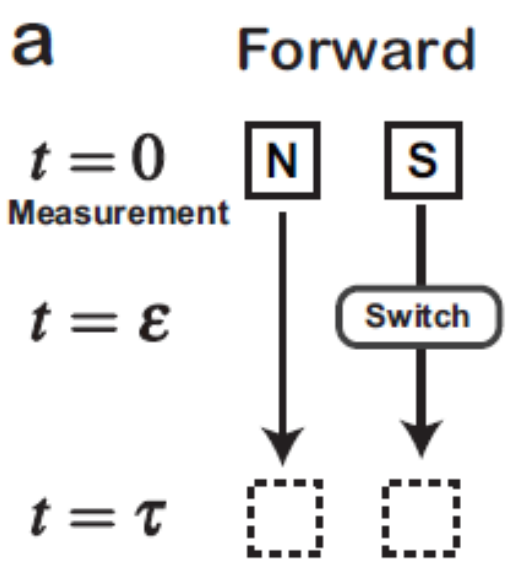


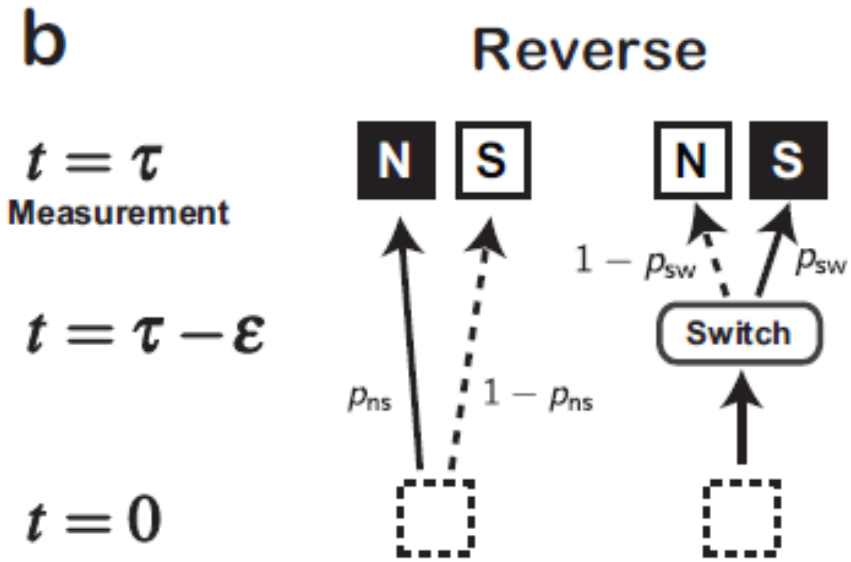
Agrees within measurement accuracy

Toyabe, Sagawa, Ueda, Muneyuki, Sano,

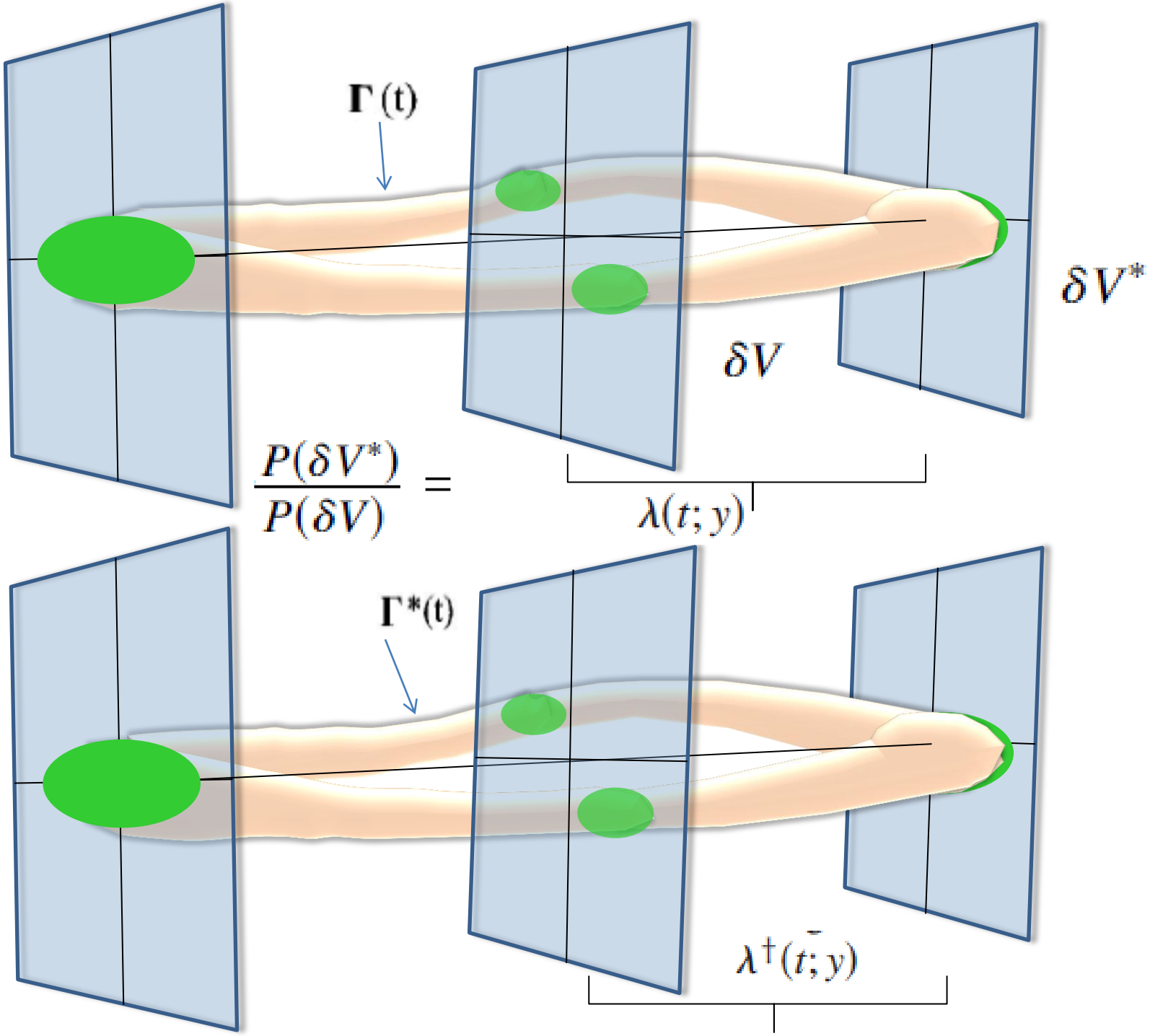
Nature Physics, 6, 988, (2010))

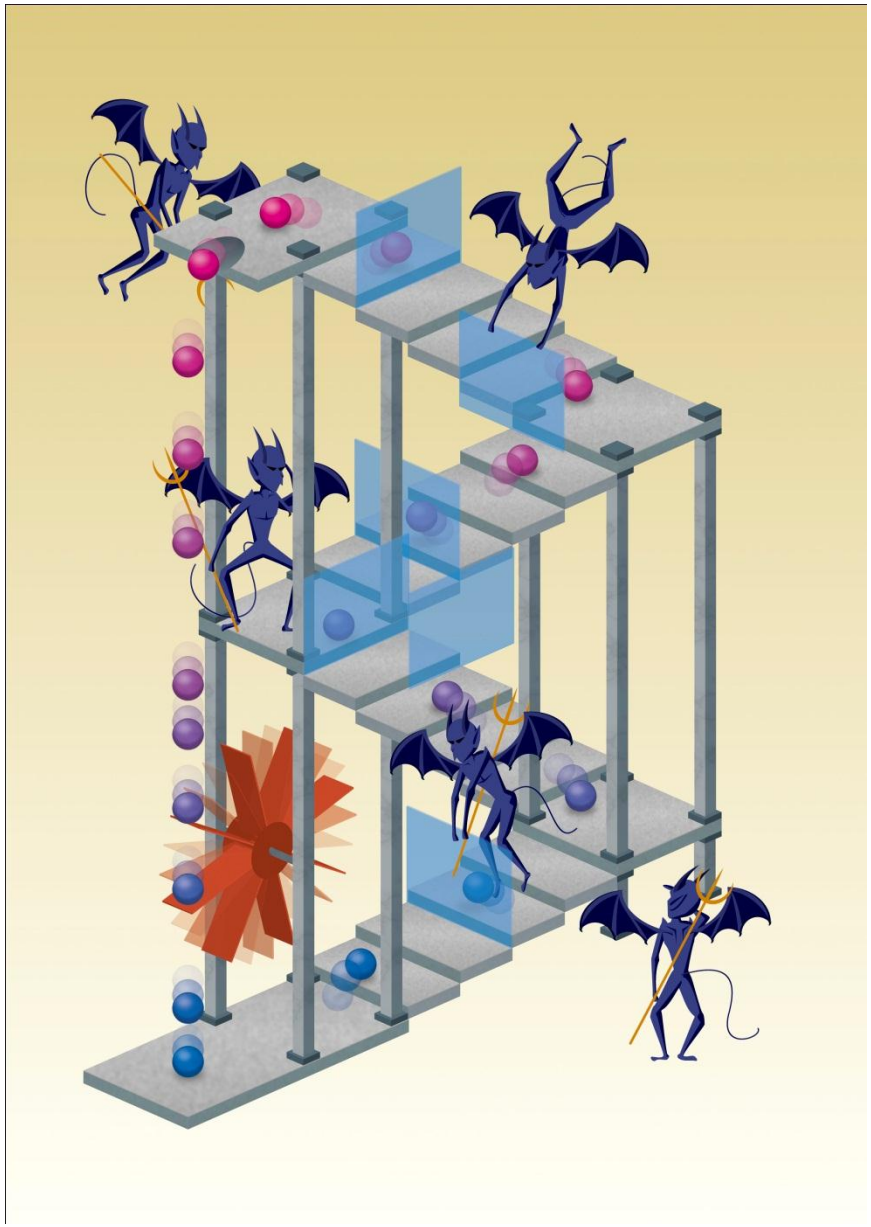
How to measure the feedback efficacy





$$\gamma = p_{sw} + p_{ns} \leq\geq 1$$


Γ(t)
δV*
δV
P(δV*) / P(δV) =
λ(t; y)
Γ*(t)
λ†(t̄; y)

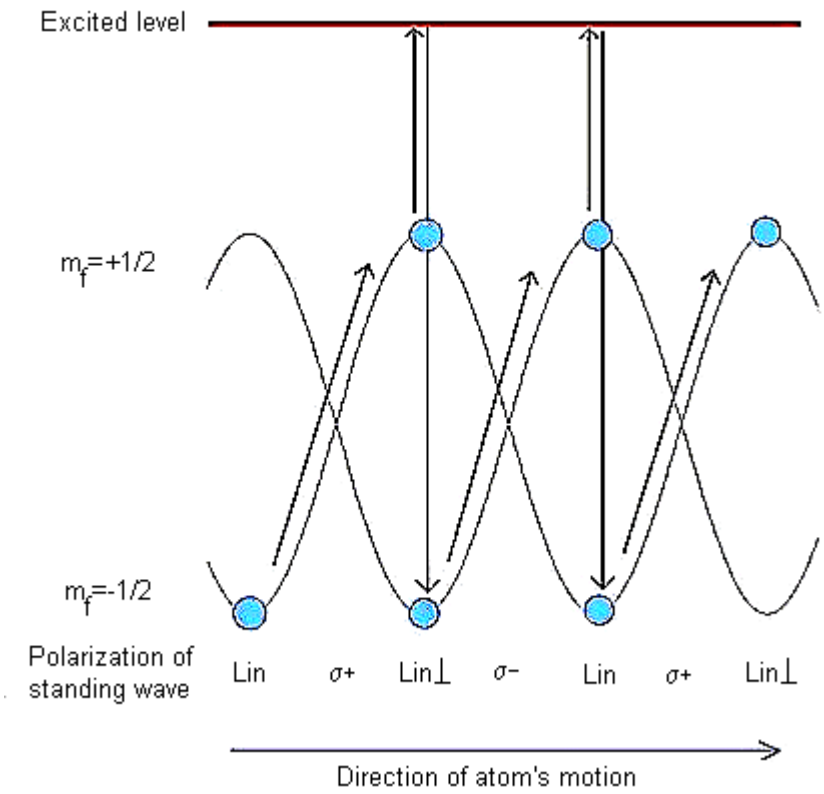


Toyabe, Sagawa, Ueda, Muneyuki, Sano, Nature Physics 6, 988–992 (2010)

see also Nature News, News&Views, NewScientist, TheEconomist, LiveScience, PhysicsWorld, IEEE spectrum, USA Today, CBS, Fox, etc

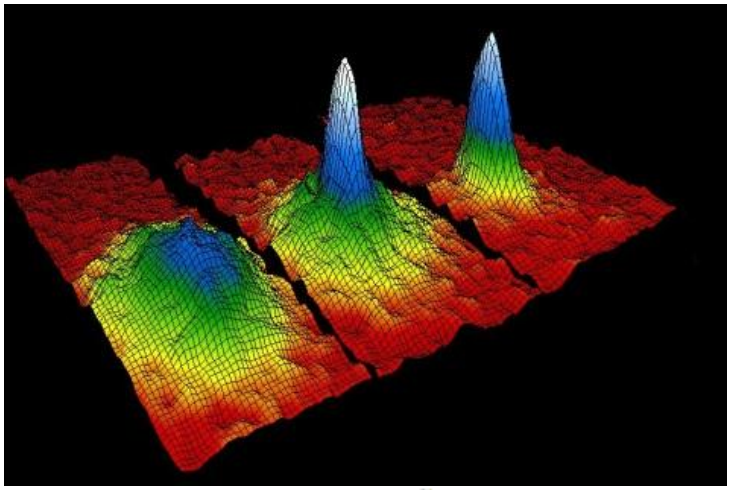
Laser Cooling: Sisyphus Cooling

Polarization Gradient Cooling

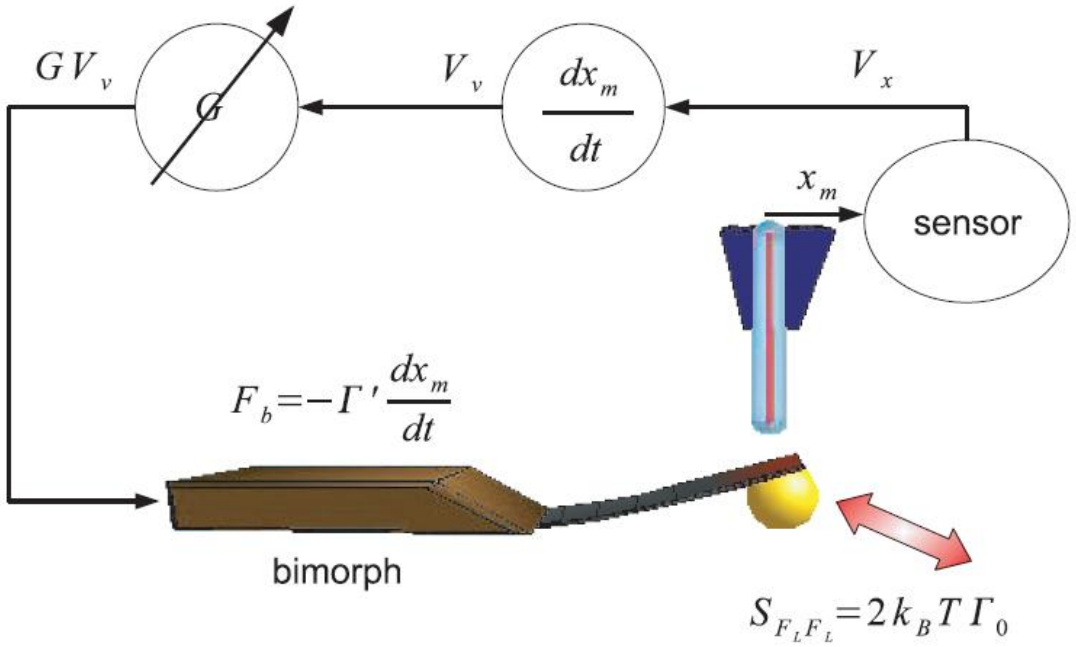


Cohen-Tannoudji

http://www.nobelprize.org/nobel_prizes/physics/laureates/1997/



Cooling limit with feedback control



G. Jourdan et al., Nanotechnology, (2007)

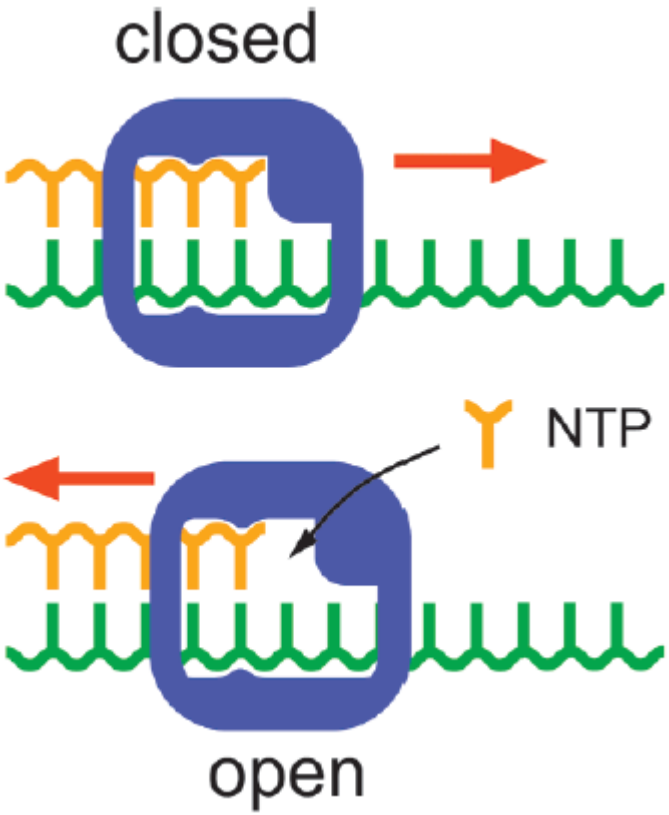
Reducing thermal fluctuations of lever of Atomic Force Microscope (AFM)

What is the limit of cooling?

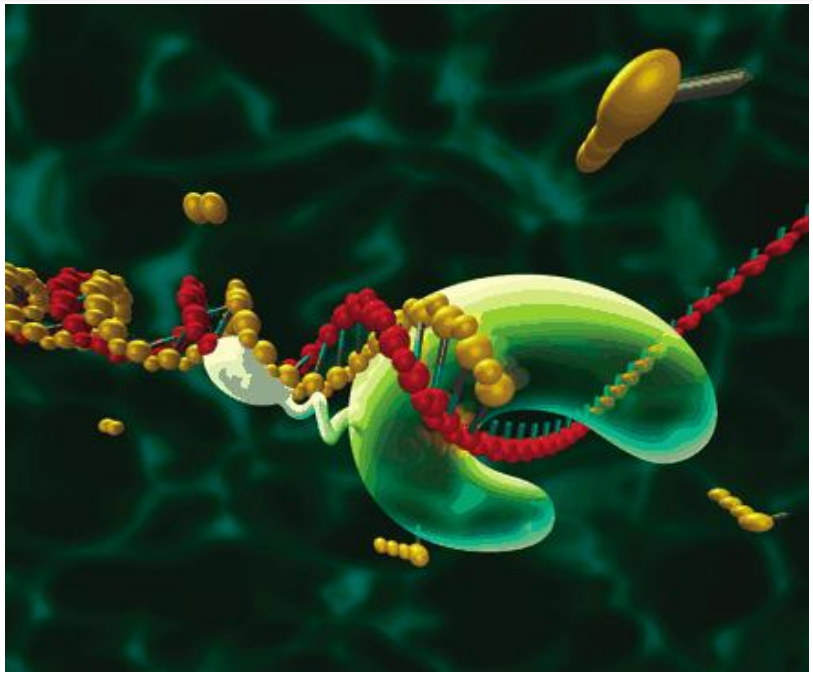
Cooling Limit: $$\frac{T - T_{\rm eff}}{T} \geq \frac{\sum_i \langle I_i \rangle_0}{\tau} t_r .$$

S. Ito and M. Sano, PRE (2011)

Are molecular motors Maxwell's demon?

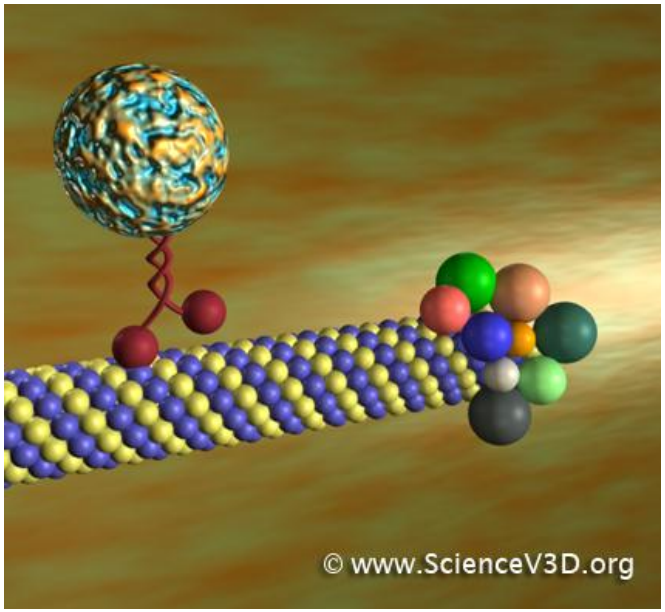


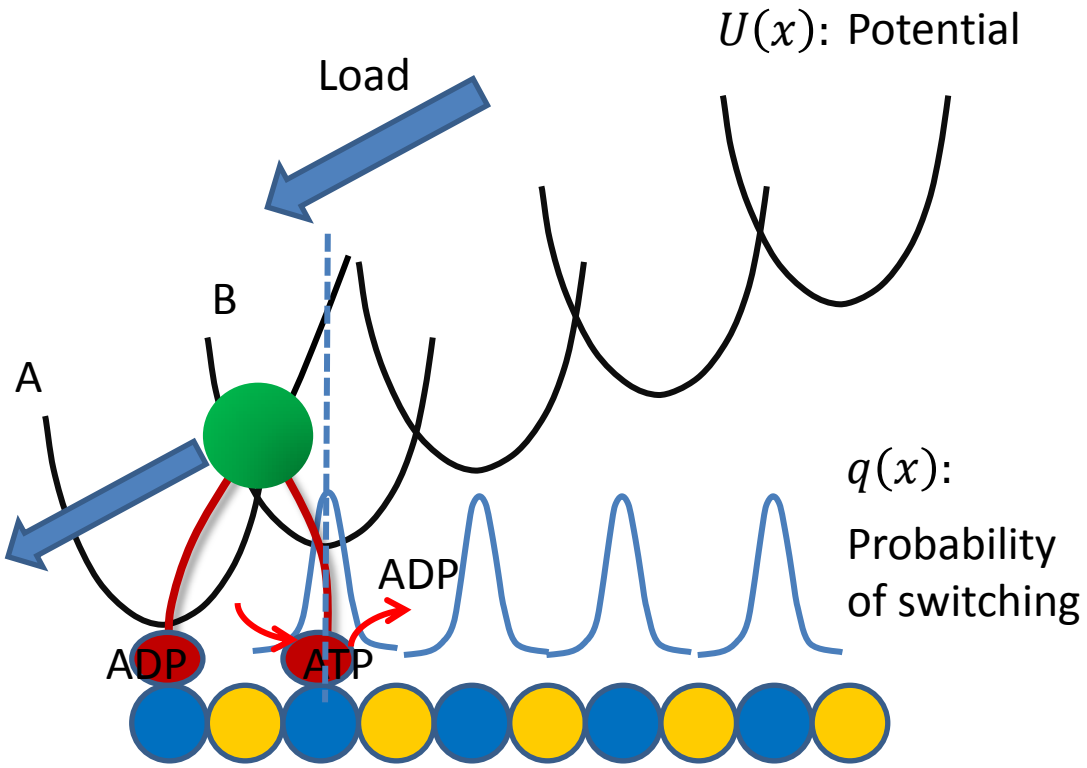
DNA Polymerase behaves like a thermal ratchet



- **Polymerase binds to DNA**
- **each process is reversible**
- **Moves back and forth by thermal fluctuations**
- **NTP can bind in the open space**
- **NTP concentration is higher than equilibrium**

Are molecular motors Maxwell's demon?





$$p_A(x) = \frac{e^{-\beta U_A(x)}}{Z}$$

$$\left\langle e^{-\beta(W-\Delta F)} \right\rangle = \gamma = \int q(x) \frac{p_A(x)}{p_B(x)} dx$$

Probability of switching

If the switching position is much shifted to forward direction, γ can exceed 1, otherwise not.

Summary

- Maxwell's demon
- Demonstration of Information-Energy conversion

  (Toyabe, Sagawa, Ueda, Muneyuki, Sano, Nature Physics, 6**,** 988**,** (2010),

  for detailed check with laser tweezers, Suzuki, Hiraraya, MS in preparation.)

- Possible roles of information feedback in other systems were discussed.