Universal order statistics of random walks

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We present an analytic study of the order statistics of a time series generated by the successive positions of a symmetric random walk of n steps with step lengths of finite variance \sigma^2. We show that the statistics of the gap d_{k,n}=M_{k,n} -M_{k+1,n} between the k-th and the (k+1)-th maximum of the time series becomes stationary, i.e, independent of n as n\to \infty and exhibits a rich, universal behavior. The mean stationary gap (in units of \sigma) exhibits a universal algebraic decay for large k, <d_{k,\infty}>/\sigma\sim 1/\sqrt{2\pi k}, independent of the details of the jump distribution. Moreover, the probability density (pdf) of the stationary gap exhibits scaling, Proba.(d_{k,\infty}=\delta)\simeq (\sqrt{k}/\sigma) P(\delta \sqrt{k}/\sigma), in the scaling regime when \delta\sim <d_{k,\infty}>\simeq \sigma/\sqrt{2\pi k}. The scaling function P(x) is universal and has an unexpected power law tail, P(x) \sim x^{-4} for large x. For \delta \gg <d_{k,\infty}> the scaling breaks down and the pdf gets cut-off in a nonuniversal way. Consequently, the moments of the gap exhibit an unusual multi-scaling behavior.

G. Schehr, S. N. Majumdar, Phys. Rev. Lett. **108**, 040601 (2012)