France-Korea Particle Physics mini-workshop on BSM physics in the LHC era

Lepton Flavour Violation and PMNS Matrix

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Contents:

- Neutrino masses and (B-L)
- New Physics at TeV scale
- Low energy effective Lagrangian with A4 symmetry
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- Constraints from SUSY
- Comparison with Minimal Flavour Violation

Motivation-1 :

From the theory view point the simplest and more appealing (though still unconfirmed) possibility to describe neutrino masses is the leading non-renormalizable SU(2)xU(1) invariant operator

Weinberg's list

$$L = L_{SM} + \frac{c_5}{\Lambda} L_5 + \frac{c_6}{\Lambda^2} L_6 + \ldots$$

[80 independent d=6 operators]
Λ= scale of new physics

a unique d=5 operator (up to flavour combinations)

$$\frac{\mathrm{L}_5}{\Lambda_L} = \frac{(\tilde{H}^+ l)(\tilde{H}^+ l)}{\Lambda_L} = \frac{1}{2}\frac{v^2}{\Lambda_L}\nu\nu + \ldots$$

$$m_\nu = y\frac{v^2}{\Lambda_L} \longleftrightarrow m_f = \frac{y_f}{\sqrt{2}} v$$

smallness of m_ν due to $\frac{v}{\Lambda_L} << 1$

$$m_\nu \approx \sqrt{|\Delta m^2_{32}|} \approx 0.05\ \mathrm{eV} \rightarrow \quad \Lambda_L \approx 10^{15}\ \mathrm{GeV}$$ not that far from GUT scale

the effective theory is "nearly" renormalizable
the first effect of New Physics: neutrino masses and mixing angles!

Motivation-2 :

L_5 violates B-L by two units

- B-L violated, in general, when attempting to unify particle interactions (GUTs)
- global quantum numbers expected to be violated at some level by quantum gravity effects

ν as a window on GUT physics

$\Lambda_L \approx 10^{15}\ GeV$ independent indication of a new physical threshold around the GUT scale

- many GUTs contain ν^c
- heavy ν^c exchange produces a specific version of L_5

$$\frac{L_5}{\Lambda_L} = -\frac{1}{2}(\tilde{H}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{H}^+ l) + h.c. + \ldots$$

see-saw mechanism

see-saw can enhance small mixing angles in M and in y_ν into the large ones observed in ν oscillations

interesting link to baryogenesis

- B-L violation welcome in baryognesis
- out-of-equilibrium, CP violating decay of ν^c can drive baryogenesis through leptogenesis

PNMS Matrix with A4 :

mechanism to generate TB mixing from A_4

keeps separate φ_T and φ_S at the LO

$G_f = A_4 \times Z_3 \times U(1)_{FN}$

explains why $m_e << m_\mu << m_\tau$

$\langle \varphi_T \rangle, \ldots$ $\qquad$ $\langle \varphi_S \rangle, \ldots$

G_T $\qquad$ G_S

$(m_e^+ m_e)$ diagonal $\qquad$ $U_{TB}^T \, m_\nu \, U_{TB} = (m_\nu)_{diag}$

$$L_{mass} = -e^c H^+ y_e(\varphi) l + \frac{(\tilde{H}^+ l) Y(\varphi) (\tilde{H}^+ l)}{\Lambda_L} + h.c. + \ldots$$

G_f-invariant

after G_f breaking from $\langle\varphi\rangle$, masses of charged leptons and of neutrinos are generated

$$m_l = \frac{v}{\sqrt{2}} y_e(\langle \varphi \rangle) \qquad\qquad m_\nu = \frac{v^2}{\Lambda_L} Y(\langle \varphi \rangle)$$

PMNS-2:

$$l = (3,\omega,0)$$
$$e^c = (1,\omega^2,+2)$$
$$\mu^c = (1'',\omega^2,+1)$$
$$\tau^c = (1',\omega^2,0)$$

can also be extended to the quark sector
[F, Hagedorn, Lin, Merlo 0702194,
Altarelli,F, Hagedorn 08020090]

$$\varphi \equiv \begin{cases} \varphi_T/\Lambda = (3,1,0) \\ \varphi_S/\Lambda = (3,\omega,0) \\ \xi/\Lambda = (1,\omega,0) \\ \vartheta/\Lambda = (1,1,-1) \end{cases}$$

symmetry breaking sector

the success of this program crucially depends on a correct vacuum alignment

$$\langle\varphi_T\rangle/\Lambda = (u,0,0) + O(u^2)$$
$$\langle\varphi_S\rangle/\Lambda \propto (u,u,u) + O(u^2)$$
$$\langle\xi\rangle/\Lambda \propto u + O(u^2)$$
$$\langle\vartheta\rangle/\Lambda \equiv t$$

$$y_e(\langle\varphi\rangle) = \begin{pmatrix} c_e t^2 u & 0 & 0 \\ 0 & c_\mu t\, u & 0 \\ 0 & 0 & c_\tau u \end{pmatrix} + O(u^2)$$

tau Yukawa coupling < 4π

$$0.001 < u < \vartheta_C^2$$
$$t \approx \vartheta_C^2$$

corrections to TB mixing

$\vartheta_c = 0.22$ Cabibbo angle

$$\vartheta_{13} = O(u)$$

from subleading corrections

New Physics at TeV scale-1:

without any extra assumptions

ν masses
ν oscillations
0νββ decay ?
p decay ?

extended gauge symmetry
additional d.o.f.
like ν^c,
superheavy gauge bosons,
...

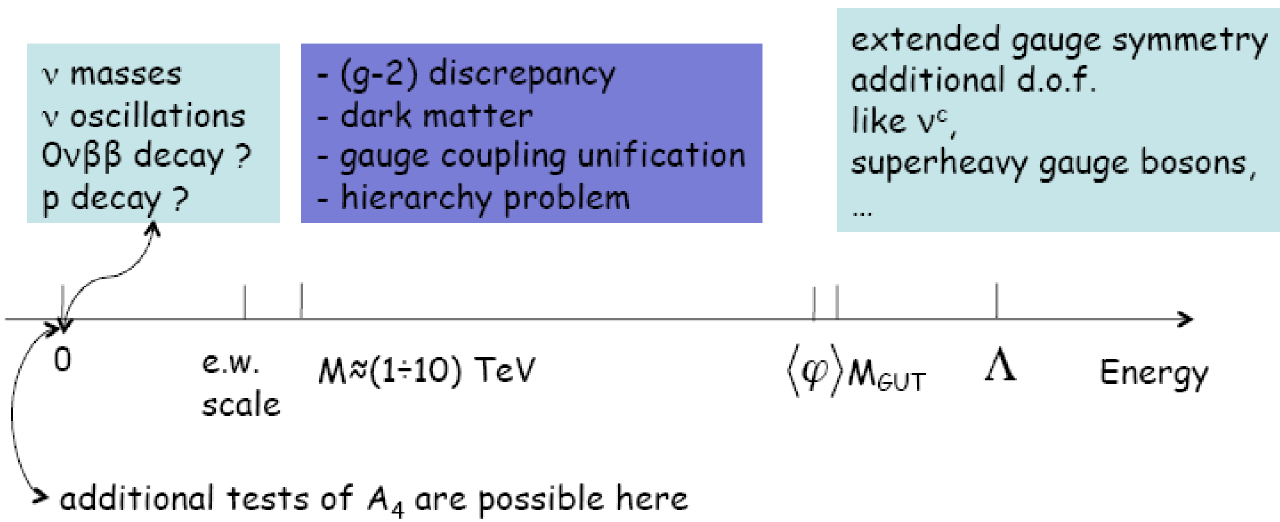
difficult to realize additional tests of the high-energy theory

e.g. the (type I) see-saw
depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those
describing $(L_{SM})+L_5$:
3 masses, 3 mixing angles
and 3 phases

New Physics at TeV Scale-2:

additional assumption:
there is new physics at a scale $M \approx (1 \div 10)$ TeV << $\langle\varphi\rangle$ <<Λ



the energy region close to M will be explored by LHC soon

Low energy Effective Lagrangian -1:

low-energy effective Lagrangian

at energies E<M, after integrating out the d.o.f. associated to the scale M

$$L_{eff} = i\frac{e}{M^2} e^c H^+ \left(\sigma^{\mu\nu} F_{\mu\nu}\right) \mathcal{M}(\langle\varphi\rangle) l + [4\text{ - fermion}] + h.c. + \ldots$$

L_{eff} local operator, still invariant under G_f [by treating <φ> as spurions]
[neglecting RGE effects, still controlled by <φ>, but not local in <φ>]

- effects with $1/M^2$ suppression can be observable
- flavor pattern in L_{eff} controlled (up to RGE effects) by the same SB parameters <φ> that control m_e and m_ν
- in the basis where charged leptons are diagonal

$\mathrm{Im}\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$	d_i	electric dipole moments
$\mathrm{Re}\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$	$a_i = \frac{(g-2)_i}{2}$	anomalous magnetic moments
$\left\|\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}\right\|^2 \quad (i \neq j)$	$R_{ij} = \frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)}$	LFV transitions $\mu \to e\gamma \quad \tau \to \mu\gamma \quad \tau \to e\gamma$
[4-fermion operators]	other LFV transitions	$\mu \to eee \quad \tau \to \mu\mu\mu \quad \tau \to eee \quad \ldots$

Experimental Bounds:

- bounds on the scale M, from the present limits on d_i, a_i, R_{ij},...
- correlations among d_i, a_i, R_{ij} and ϑ_{13} from the pattern $\langle\varphi\rangle$

charged lepton Yukawa couplings

$$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii} = \alpha_i (y_e)_{ii} + \ldots$$

O(1) (complex) coefficients

$d_e < 1.6 \times 10^{-27}\ e\ cm$	$M > 80\ TeV$
$d_\mu < 2.8 \times 10^{-19}\ e\ cm$	$M > 80\ GeV$
$\delta a_e < 3.8 \times 10^{-12}$	$M > 350\ GeV$
$\delta a_\mu \approx 30 \times 10^{-10}$	$M \approx 2.7\ TeV$

α approximately real?

[from recent review by Raidal et al 08011826]

[warning: relation between the scale M and new particle masses M' can be not trivial. In a weakly interacting theory g M/4π≈M']

LFV in A4 x.... -1:

$$\mathcal{M}(\langle\varphi\rangle) = \begin{pmatrix} O(t^2u) & O(t^2u^2) & O(t^2u^2) \\ O(tu^2) & O(tu) & O(tu^2) \\ O(u^2) & O(u^2) & O(u) \end{pmatrix}$$

in the basis where charged leptons are diagonal; operators contribute to both $\mathcal{M}_{ii}$ and $\mathcal{M}_{ij}$ (i≠j)

up to O(1) coefficients $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from ϑ_{13}

$\tau \to \mu\gamma \quad \tau \to e\gamma$ below expected future sensitivity

$$R_{\mu e} < 1.2\times 10^{-11} (10^{-13}) \Rightarrow \frac{u}{M^2} < 1.2\times 10^{-11} (1.1\times 10^{-12})\ GeV^{-2}$$

$$u > 0.001 \Rightarrow M > 10\,(30) \quad TeV$$

$$u \approx 0.05 \Rightarrow M > 70(200) \quad TeV$$

probably above the region of interest for the $(g-2)_\mu$ and for LHC
is this inescapable?

LFV in A4 x ... -2:

4-fermion operators

dominant LFV operators [no VEV/Λ suppression!]

$$\frac{1}{M^2}\bar{e}^c\bar{\tau}^c\mu^c\mu^c$$

$$\frac{1}{M^2}(\bar{l}\bar{l}ll)$$

stands for several independent contractions, such as $(\bar{l}\bar{l})(ll)$, $(\bar{l}\bar{l})'(ll)''$, $\left((\bar{l}\bar{l})_3(ll)_3\right)$...

selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = \pm 2$

$$\tau^- \to \mu^+ e^- e^- \qquad \tau^- \to e^+ \mu^- \mu^-$$

$$\left.\begin{array}{l} BR(\tau^- \to \mu^+ e^- e^-) \\ BR(\tau^- \to e^+ \mu^- \mu^-) \end{array}\right\} < 10^{-7} \quad \Rightarrow \quad \frac{1}{G_F^2 M^4} < 10^{-7}$$

$M > 15$ TeV

again, probably above the region of interest for the (g-2)$_\mu$ and for LHC any way out?

LFV -03:

$\mathcal{M}_{ij}$ (i≠j) from two sources

- NLO corrections to φ_T $\qquad \langle \varphi_T \rangle / \Lambda = (u,0,0) + O(u^2)$
- double flavon insertions of the type $\qquad \xi^+ \varphi_S, \ \xi \varphi_S^+$ [other combinations vanish]

in a SUSY version of the model, with SUSY softly broken, a chirality flip requires an insertion of φ_T, at the LO in the SUSY breaking parameters. Example:

$$\int d^2\theta_{SUSY}\ e^c h_d \left(\frac{\varphi_T}{\Lambda} l \right) \qquad \int d^2\theta_{SUSY}\ e^c h_d \left(\frac{\varphi_T}{\Lambda} l \right) \theta^2_{SUSY}\, m_{SUSY}$$

other insertions can give rise to a chirality flip, but are suppressed by powers of (m_{SUSY}/L)

$$\frac{1}{\Lambda} \int d^2\theta_{SUSY} d^2\overline{\theta}_{SUSY}\ e^c h_d \left(\frac{\xi^+ \varphi_S}{\Lambda^2} l \right) \theta^2_{SUSY} \overline{\theta}^2_{SUSY}\ m^2_{SUSY}$$

if the only sources of chirality flip are fermion and sfermion (LR) masses, then there is no contribution to M_{ij} (i≠j) from $\xi^+ \varphi_S, \ \xi \varphi_S^+$
[at LO in m_{SUSY}] and the main effect comes from φ_T alone
[we take this as a definition of SUSY case in the present context]

LFV-04:

$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}$ in $A_4 \times \ldots$

SUSY case

$$\mathcal{M}(\langle\varphi\rangle) = \begin{pmatrix} O(t^2u) & O(t^2u^2) & O(t^2u^2) \\ O(tu^3) & O(tu) & O(tu^2) \\ O(u^3) & O(u^3) & O(u) \end{pmatrix}$$

in the basis where charged leptons are diagonal

off-diagonal elements below the diagonal $\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}$ $(i > j)$
are down by a factor of O(u) compared to generic non-SUSY case

up to O(1) coefficients $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from ϑ_{13}

$$R_{\mu e} < 1.2\times10^{-11}(10^{-13}) \Rightarrow \frac{u^2}{M^2} < 1.2\times10^{-11}(1.1\times10^{-12})\ GeV^{-2}$$

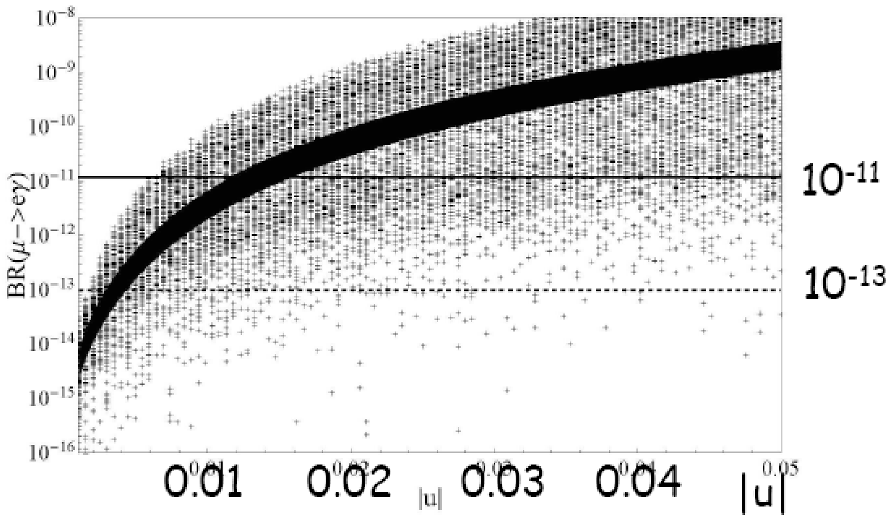
$u > 0.001 \Rightarrow M > 0.7(2)\quad TeV$

$u \approx 0.05 \quad \Rightarrow M > 14(48)\quad TeV$

O(1) coefficient

$$BR(\mu \to e\gamma) = \underbrace{\frac{12\pi^3\alpha_{em}}{G_F^2 m_\mu^4}\left(\delta a_\mu\right)^2}_{0.0014\times\left(\frac{\delta a_\mu}{30\times10^{-10}}\right)^2}\left[\gamma\vartheta_{13}\right]^4$$

LFV-05:



4-fermion operators (SUSY case)

in the limit (VEV/Λ)=0

- superpotential for matter fields vanishes
- Kahler potential becomes $SU(3)_L \times SU(3)_R$ symmetric
- soft masses are flavour diagonal

$$\frac{1}{M^2}\bar{e}^c\bar{\tau}^c\mu^c\mu^c \qquad \frac{1}{M^2}(\bar{l}\bar{l}ll)$$

should come with extra (VEV/Λ)^n suppression in the low-energy effective Lagrangian: lower bound on M relaxed

Minimal Flavour Violation -1:

Minimal Flavor Violation [MFV]

$$G_f = SU(3)_l \times SU(3)_{e^c} \times \dots$$

[D'Ambrosio, Giudice, Isidori, Strumia 2002
Cirigliano, Grinstein, Isidori, Wise 2005]

the largest G_f

$$l = (\bar{3},1) \qquad e^c = (1,3)$$

$$\varphi \equiv \begin{cases} y_e = (3,\bar{3}) \\ Y = (6,1) \end{cases}$$

G_f broken only by the
Yukawa coupling of L_{SM} and L_5

y_e and Y can be expressed in terms of lepton masses and mixing angles

$$y_e = \sqrt{2}\,\frac{m_e^{diag}}{v} \qquad Y = \frac{\Lambda_L}{v^2} U^* m_\nu^{diag} U^+$$

diagonal elements $\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$ are of the same size as in A_4x…
similar lower bounds on the scale M

MFV-02:

$$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij} = \beta\,(y_e Y^+ Y)_{ij} + \ldots$$

$$= \sqrt{2}\beta \frac{(m_l)_{ii}}{v}\frac{\Lambda_L^2}{v^4}\left[\Delta m_{sol}^2 U_{i2}U_{j2}^* \pm \Delta m_{atm}^2 U_{i3}U_{j3}^*\right] + \ldots$$

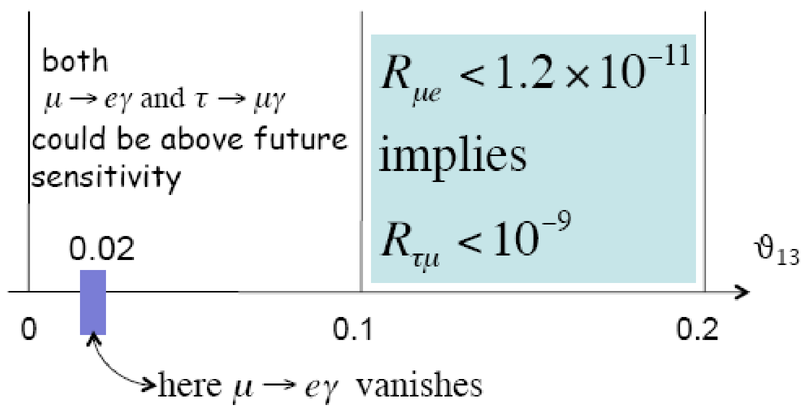
+ for normal hierarchy
- for inverted hierarchy

a positive signal at MEG $10^{-11} < R_{\mu e} < 10^{-13} \div 10^{-14}$ always be accommodated [but for a small interval around $\vartheta_{13} \approx 0.02$ where $R_{\mu e}=0$]

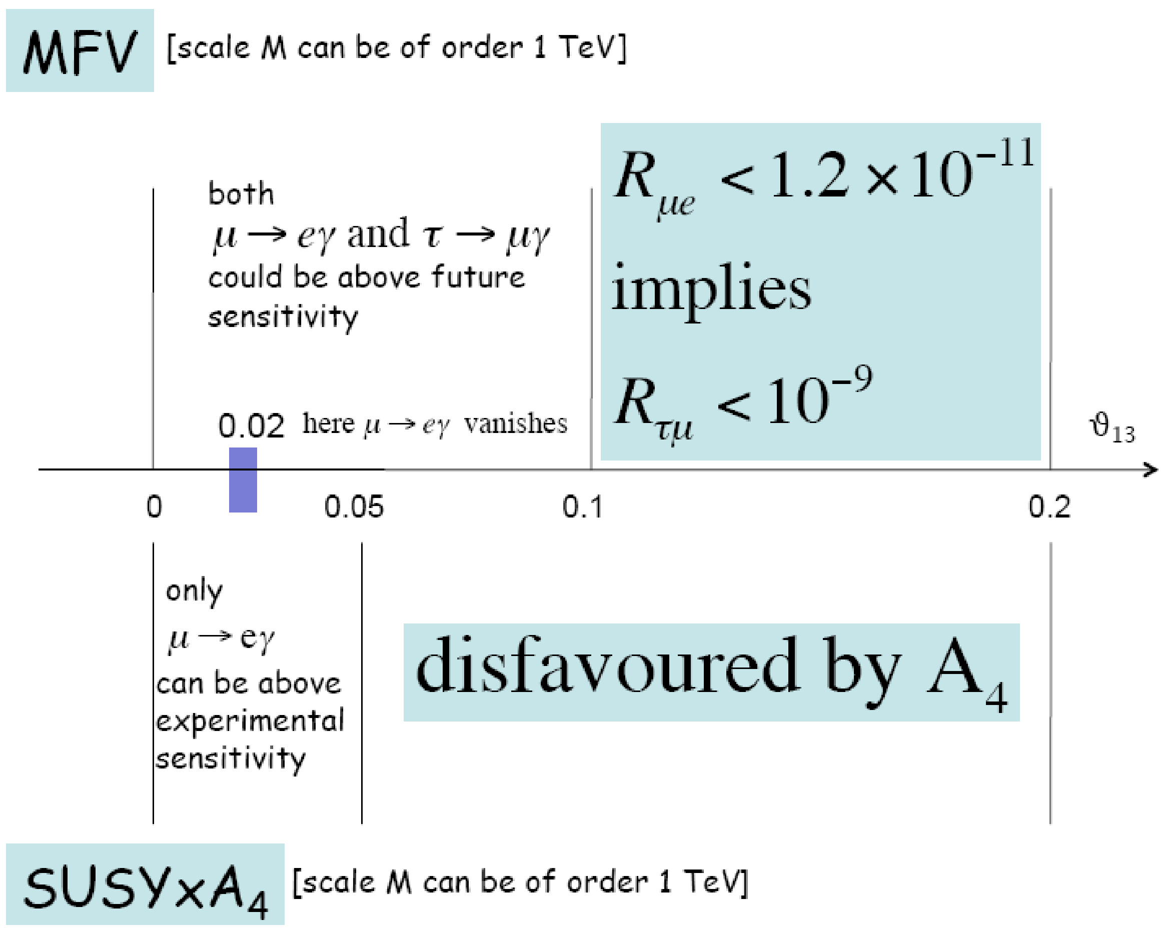
non-observation of R_{ij} can be accommodated by lowering Λ_L

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left|\frac{2}{3}r \pm \sqrt{2}\sin\vartheta_{13}e^{i\delta}\right|^2 < 1 \qquad r \equiv \frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}$$

[Cirigliano, Grinstein, Isidori, Wise 2005]



Comparison with MFV:



Conclusion:

- additional tests of A_4 models from LFV

  generic prediction

  $$R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$$ independently from ϑ_{13} (cfr MFV)

  $\tau \to \mu\gamma \quad \tau \to e\gamma$ below expected future sensitivity

- in the generic, non-SUSY, case

$$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} \propto \left(\frac{u}{M^2}\right)^2$$

0.001 < u < 0.05 requires M above 10 TeV

$\tau^- \to \mu^+ e^- e^- \quad \tau^- \to e^+ \mu^- \mu^-$ M above 15 TeV

no match with M fitting $(g\text{-}2)_\mu$

- in the SUSY, case

$$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} \propto \left(\frac{u^2}{M^2}\right)^2$$

M can be much smaller, in the range of interest for $(g\text{-}2)_\mu$

$\tau^- \to \mu^+ e^- e^- \quad \tau^- \to e^+ \mu^- \mu^-$ bound on M relaxed

O(1) coefficient

$$BR(\mu \to e\gamma) = 0.0014 \times \left(\frac{\delta a_\mu}{30 \times 10^{-10}}\right)^2 [\gamma\vartheta_{13}]^4$$