The 10th Summer Institute for Theoretical Physics

2010년 8월 1일 – 8월 7일

Introduction to Effective Field Theory

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Lecture 1

강의계획

- 1시간 30분 x 6회 = 9시간
- Effective theory: general aspect (3 ~~lectures~~ hours)
- Soft-collinear effective theory at tree level (3 ~~lectures~~ hours)
- SCET with loops (3 ~~lectures~~ hours)
- Explicit examples and problems

What are Effective Theories?

- Rich physics at every scale:
  From the age of the universe to the lifetime of W bosons, or even at smaller scales
- Separate regions in a parameter space:
  different appropriate description of important physics.

1. Relevant physics differs in different regions.
2. No single description useful in every parameter space.

If there are large or small parameters, put them infinity or zero. The rest can be treated as perturbations.

$(1-x)^{-\frac{1}{2}} = 1 + \frac{x}{2} + O(x^2)$

1. relativity vs. classical physics

$$T = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = \frac{1}{2}mv^2 + \cdots$$

2. quantum mechanics vs. classical physics $[x, p] = i\hbar \rightarrow 0$

3. multipole expansion in electromagnetism

$$\frac{1}{|\vec{R} - \vec{r}|} = \sum_{n=0}^{\infty} \frac{P_n(\cos\theta)}{R}\left(\frac{r}{R}\right)^n, \quad R \gg r$$

4. hydrogen atom

$$H_0 = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{r}$$ ← Leading term

$$H_{\text{hf}} = \frac{4}{3}\alpha^4 mc^2 g_P \frac{m}{m_P} \frac{\mathbf{I} \cdot \mathbf{S}}{\hbar^2} \quad (n = 1)$$

At leading order, proton spin decoupled. $SU(2)$ Symmetry

SU(2) breaking terms $\frac{1}{m_P}$ effect

In particle physics, the parameter is distance or energy.

What are Effective Field Theories?

- In relativistic quantum theory, particles are created or annihilated.
- Including the quantum fluctuations, it becomes effective field theory.
- If some particles are heavy, eliminate them to make an effective theory.

But life is not that simple because of the fluctuation,
i. e., renormalization effects.

1. Eliminating heavy particles produce nonrenormalizable interactions.

2. The interactions are nonlocal.
   $\rightarrow$ Replace the nonlocal interactions to local interactions so that the effective theory becomes valid.

3. This modifies the high-energy behavior of the theory.

4. Matching: The physics should be the same at the boundary. (e. g., coupling constant)

1. Top-down approach

(1) Suppose we have a complete renormalizable theory at high energy.

(2) Start from the heaviest particle M, integrate it out, calculating the matching condition.

(3) Use RG to scale down to the next heaviest particle M'.

(4) We have a sequence of effective field theories.

2. Bottom-up approach

(1) Describe physics at a given scale E, to a given accuracy δ.

(2) Some renormalizable, some nonrenormalizable. But we can describe physics with a finite number of parameters.

- finite number of parameters for each dimension $\sim \frac{1}{M^k}$
- coefficients $\left(\frac{E}{M}\right)^k \approx \delta \longrightarrow k \approx \frac{\ln(1/\delta)}{\ln(M/E)}$

If we continue, what will happen? Nobody cares since the very process of constructing effective theories at each scale E is doing science.

Matching, Renormalization

- Why do we use effective field theory?

1. Easy to see relevant physics:
   Deal with the particles and interactions we know. Speculate high-energy physics later.
2. We can use mass-independent scheme like $\overline{\mathrm{MS}}$.
3. It renders infrared finite after matching.

- Examples: chiral Lagrangian, heavy quark effective field theory, nonrelativistic QCD, soft-collinear effective theory, etc..

- Large logs: $\ln E/\mu$ always appears. In some scheme $\ln m/\mu$ can appear. $\longrightarrow$ Choose a good renormalization scheme.

- Mass-independent scheme

(β functions are independent of μ.) - dimensional regularization with $\overline{\mathrm{MS}}$.

- Dimensional regularization modifies the high-energy behavior.

$$I = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 + A^2)^n} \longrightarrow \mu^\epsilon \int \frac{d^{4-\epsilon} l}{(2\pi)^{4-\epsilon}} \frac{1}{(l^2 + A^2)^n}$$

Details of dimensional regularization

$$I = \int [dx] \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 + A^2)^\alpha} \quad \longrightarrow I_\delta = c(\delta) \int [dx] \frac{d^{4+\delta} l}{\mu^\delta (2\pi)^{4+\delta}} \frac{1}{(l_\delta^2 + l^2 + A^2)^\alpha}$$

$$\lim_{\delta \to 0} c(\delta) = 1$$

We can do the integral as a whole as

$$\int \frac{d^n l}{(2\pi)^n} \frac{(l^2)^\beta}{(l^2 - A^2)^\alpha} = \frac{i}{(4\pi)^{n/2}} (-1)^{\alpha+\beta} (A^2)^{\beta-\alpha+n/2} \frac{\Gamma(\beta + n/2)\Gamma(\alpha - \beta - n/2)}{\Gamma(n/2)\Gamma(\alpha)}$$

Rewrite the integral as

$$c(\delta) \int [dx] \frac{d^\delta l}{(2\pi\mu)^\delta} \frac{d^4 l}{(2\pi)^4} \frac{1}{(l_\delta^2 + l^2 + A^2)^\alpha} = \int [dx] \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 + A^2)^\alpha} r(\delta) \left(\frac{l^2 + A^2}{4\pi\mu^2} \right)^{\delta/2}$$

ρ

$$r(\delta) = c(\delta) \frac{\Gamma(\alpha - \delta/2)}{\Gamma(\alpha)}$$

$$\rho^{\delta/2} \approx 1 \quad \text{for} \quad |\ln \rho| \ll \frac{1}{\delta}$$

For small δ , low-energy physics is not changed.
Modification of physics at short distances

It can change the physics at large distances, but we can avoid infrared divergences by matching.

General scheme of matching

Large scale ϕ_i, χ

$\mathcal{L}_H(\chi, \phi) + \mathcal{L}(\phi)$

renormalization group

$\mu = M$

particle mass

MATCHING

$\mathcal{L}(\phi) + \delta\mathcal{L}(\phi)$

renormalization group

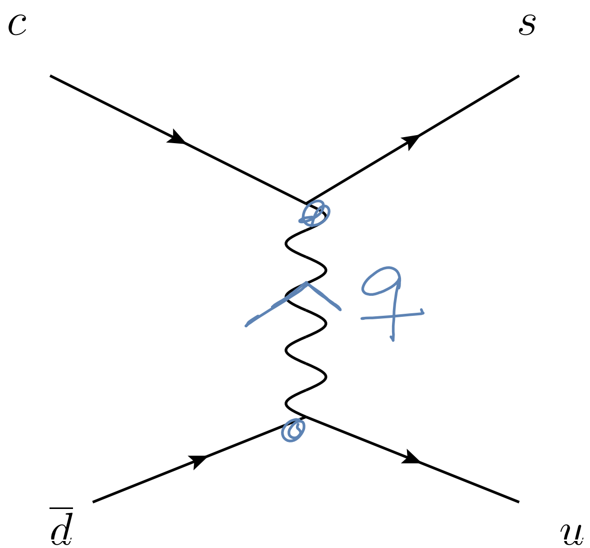
Low energy ϕ_i

$(\bar{u}d)_{V-A} = \bar{u}\gamma_\mu(1-\gamma_5)d$

Example: Weak decays

$c \to s u \bar{d}$

Tree level



$$\mathcal{M}_0 = V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} \otimes \left(\frac{i g_W}{2\sqrt{2}}\right)^2 \frac{-i}{q^2 - m_W^2}$$

$$= -i \frac{g_W^2}{8 m_W^2} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + O\left(\frac{q^2}{m_W^2}\right)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2}$$

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}_i c_i)_{V-A} (\bar{u}_j d_j)_{V-A}$$

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left(C_1 O_1 + C_2 O_2 \right)$$

One-loop

$$O_1 = (\bar{s}_i c_j)_{V-A} (\bar{u}_j d_i)_{V-A}$$

$$O_2 = (\bar{s}_i c_i)_{V-A} (\bar{u}_j d_j)_{V-A}$$

Full QCD

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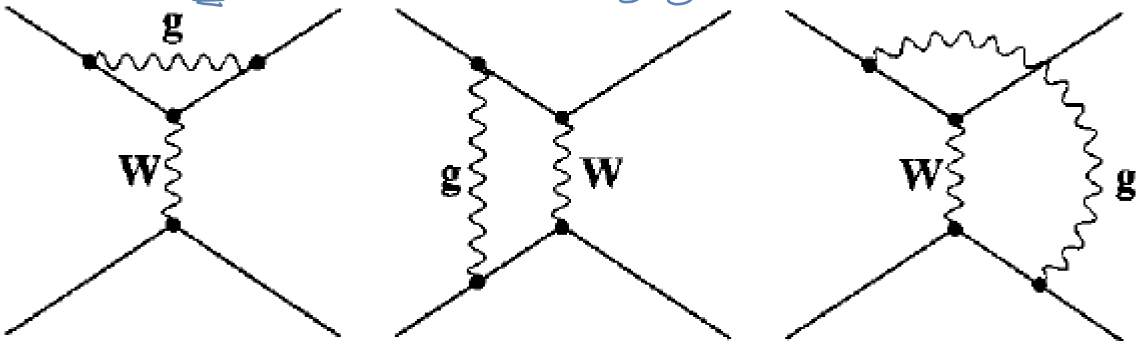
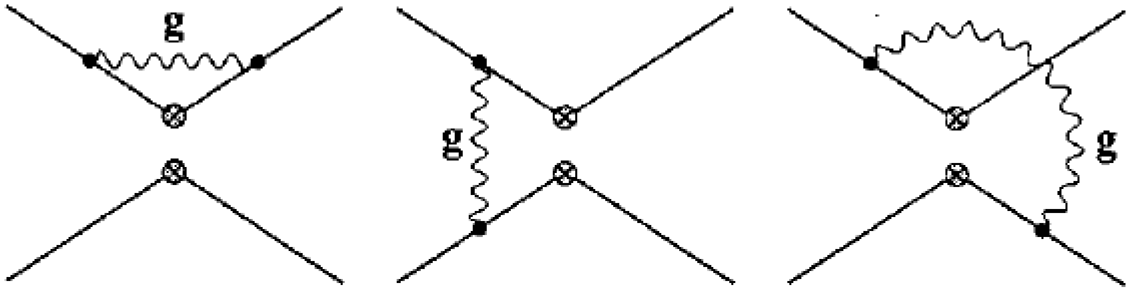
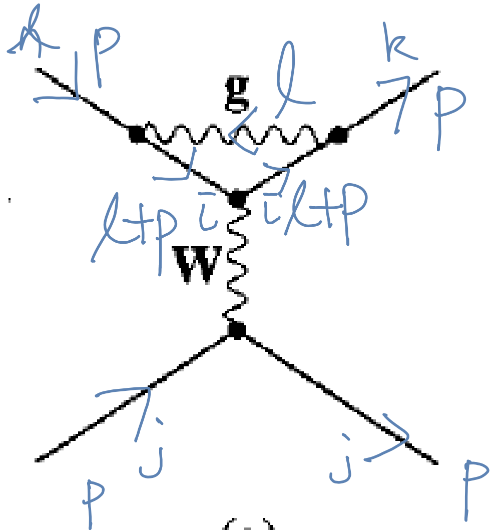


(a) (b) (c)

Effective theory



(a) (b) (c)



(a)

$$D = 4 - 2\varepsilon$$

$$d^D l = \frac{d^D l}{(2\pi)^D}$$

$$(T_a)_{ki}(T_a)_{il}$$

$$= (T_a^2)_{kl}$$

$$= C_F\,\delta_{kl}$$

$$C_F = \frac{N^2-1}{2N}$$

$$\mathcal{M}_a = \int d^D l\, \bar{s}_k\, i g \gamma_\alpha (T_a)_{ki} \frac{i}{\not l + \not p} \frac{i g_W}{2\sqrt{2}} \gamma_\mu (1-\gamma_5) \frac{i}{\not l + \not p}$$

$$\otimes\, i g \gamma^\alpha (T_a)_{il} c_l \cdot \bar{u}_j \frac{i g_W}{2\sqrt{2}} \gamma^\mu (1-\gamma_5) d_j$$

$$\otimes\, \frac{-i}{l^2} \frac{+i}{m_W^2}$$

$$= -\frac{G_F}{\sqrt{2}} g^2 (\bar{u}_j d_j)_{V-A}$$

$$\otimes \int d^D l \frac{1}{l^2} \frac{1}{[(l+p)^2]^2}$$

$$\otimes\, \bar{s}_k \gamma_\alpha (T_a)_{ki} (\not l + \not p) \gamma_\mu (1-\gamma_5) (\not l + \not p) \gamma^\alpha (T_a)_{il} c_l$$

$$2\int_0^1 dx\, x \frac{1}{((l+xp)^2 - A^2)^3}$$

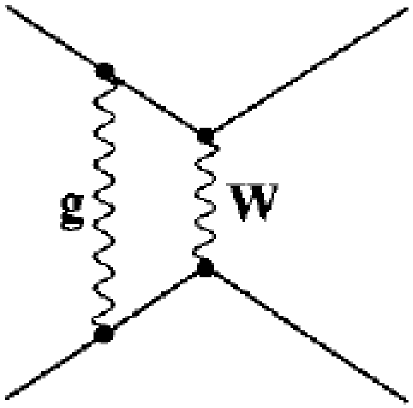
$$\frac{1}{AB^2} = \int_0^1 dx\, x \frac{1}{((1-x)A + xB)^3}$$

$$(1-x)\,l^2 + x(l+p)^2 = l^2 + 2x\, l\cdot p + x p^2 = (l+xp)^2 - \underbrace{(-x(1-x)p^2)}_{A^2}$$

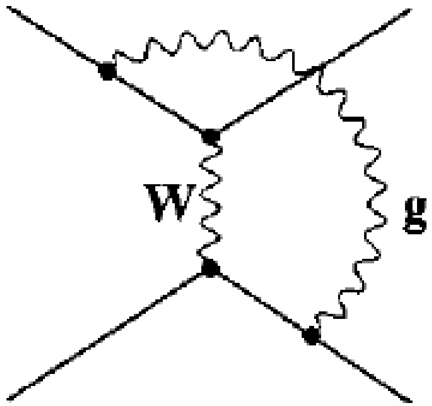
$$= + \frac{i}{16\pi^2}\left[\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\right]$$

$$\mathcal{M}_a = -i\frac{\alpha_s}{4\pi} C_F \frac{G_F}{\sqrt{2}} (\bar{s}_i c_i)_{V-A} (\bar{u}_j d_j)_{V-A}$$

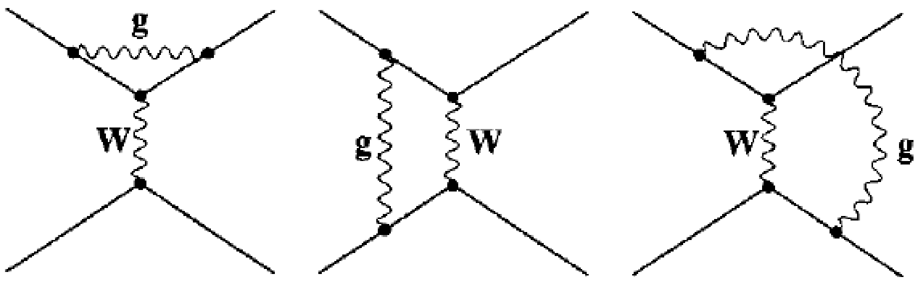
$$\otimes \ln\frac{\mu^2}{-p^2}$$



(b)

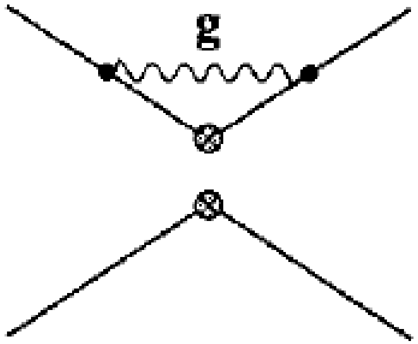


(c)

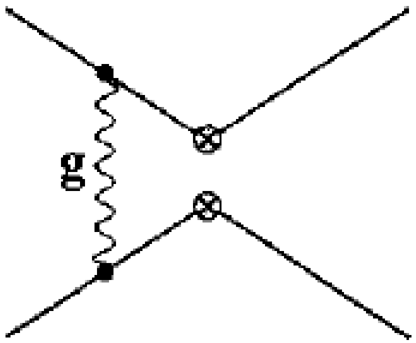


(a) (b) (c)

$$M_{\text{full}} = -i\frac{G_F}{\sqrt{2}}V_{cs}^* V_{ud}\Big[\Big(1 + 2C_F\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2}\Big)\mathcal{O}_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{m_W^2}{-p^2}\mathcal{O}_2 - 3\frac{\alpha_s}{4\pi}\ln\frac{m_W^2}{-p^2}\mathcal{O}_1\Big]$$



(a)



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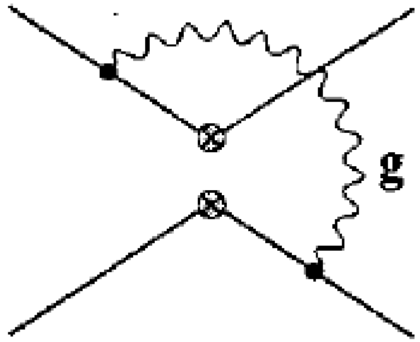

(c)