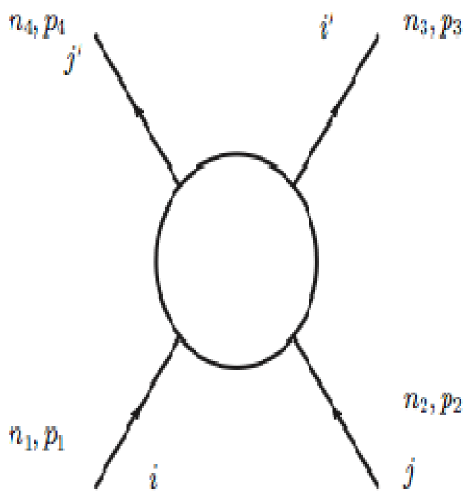
$$I_C = \tilde{I}_C - I_0 = -\frac{i}{16\pi^2}\left[-\frac{2}{\epsilon_{\mathrm{UV}}^2} - \frac{2}{\epsilon_{\mathrm{UV}}}\ln\frac{\mu^2}{-p^2} - \ln^2\frac{\mu^2}{-p^2}\right] + \cdots$$

Consider 2-2 scattering.



Current operators

$$J_n(x) = e^{i(\bar{n}\cdot p_1' - \bar{n}\cdot p_1)n\cdot x/2}\overline{\chi}_n(x)\frac{\not{\bar{n}}}{2}\chi_n(x),$$

$$J_{\bar{n}}(x) = e^{i(n\cdot p_2' - n\cdot p_2)\bar{n}\cdot x/2}\overline{\chi}_{\bar{n}}(x)\frac{\not{n}}{2}\chi_{\bar{n}}(x).$$

Now we consider

$$W = \int d^4x e^{iq\cdot x}\overline{\chi}_{n,i'}(x)\frac{\not{\bar{n}}}{2}\chi_{n,i}(x)\overline{\chi}_{\bar{n},j'}\frac{\not{n}}{2}\chi_{\bar{n},j}(0)G(x)_{ij}^{i'j'},$$

In qq scattering, the relevant Wilson lines are given as

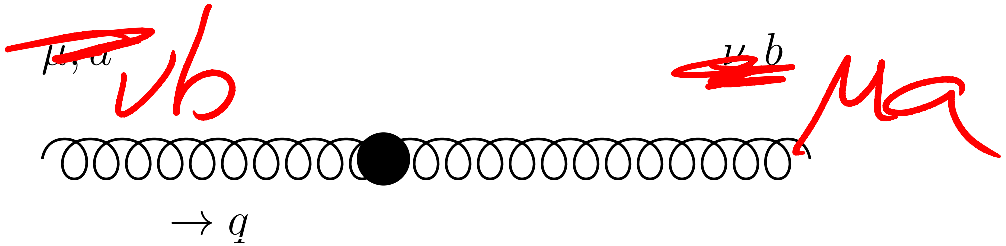
$$(O_2)^{i'i}_{j'j} = (\tilde{Y}^\dagger_{n_3} Y_{n_1})_{i'i} (\tilde{Y}^\dagger_{n_4} Y_{n_2})_{j'j}$$

$$(O_1)^{i'j}_{j'i} = (\tilde{Y}^\dagger_{n_3} Y_{n_2})_{i'j} (\tilde{Y}^\dagger_{n_4} Y_{n_1})_{j'i}$$

$$\begin{aligned}(O_2)^{i'i}_{j'j} = (\tilde{Y}^\dagger_{n_3} Y_{n_1})_{i'i} (\tilde{Y}^\dagger_{n_4} Y_{n_2})_{j'j} \quad &= \quad \left[\left(1 - g n_3 \cdot A \frac{1}{n_3 \cdot R^\dagger + i0}\right)\left(1 - \frac{1}{n_1 \cdot R + i0} g n_1 \cdot A\right)\right]_{i'i} \\ &\times \left[\left(1 - g n_4 \cdot A \frac{1}{n_4 \cdot R^\dagger + i0}\right)\left(1 - \frac{1}{n_2 \cdot R + i0} g n_2 \cdot A\right)\right]_{j'j}\end{aligned}$$

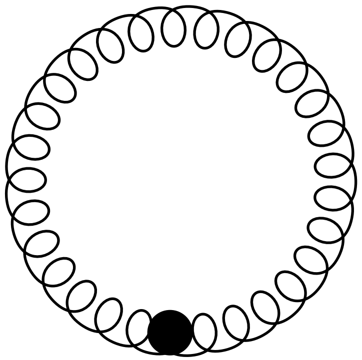
$$\left[\left(1-gn_3\cdot A\frac{1}{n_3\cdot R^\dagger+i0}\right)\left(1-\frac{1}{n_1\cdot R+i0}gn_1\cdot A\right)\right]_{i'i}$$
$$\times\left[\left(1-gn_4\cdot A\frac{1}{n_4\cdot R^\dagger+i0}\right)\left(1-\frac{1}{n_2\cdot R+i0}gn_2\cdot A\right)\right]_{j'j}$$

Feynman rule



$$\begin{aligned}
= \quad & g^2(T_a)_{i'i}(T_b)_{j'j}\left(\frac{n_3^\mu}{n_3\cdot q+i0}+\frac{n_1^\mu}{-n_1\cdot q+i0}\right)\left(\frac{n_4^\nu}{-n_4\cdot q+i0}+\frac{n_2^\nu}{n_2\cdot q+i0}\right)\\
+ \quad & g^2(T_aT_b)_{i'i}\delta_{j'j}\frac{n_3^\mu}{n_3\cdot q+i0}\frac{n_1^\nu}{n_1\cdot q+i0}+g^2\delta_{i'i}(T_aT_b)_{j'j}\frac{n_4^\mu}{n_4\cdot q+i0}+\frac{n_2^\nu}{n_2\cdot q+i0}\\
+ \quad & (a\leftrightarrow b,\mu\leftrightarrow\nu,q\to -q)
\end{aligned}$$

Radiative corrections



$$(K_2)^{i'i}_{j'j} = -\frac{ig^2}{2}\int \frac{d^D l}{(2\pi)^D}\frac{1}{l^2}$$

$$\times \quad \Big[2(T_a)_{i'i}(T_a)_{j'j}\Big((1,4)+(2,3)-(3,4)-(1,2)\Big)+C_F\delta_{i'i}\delta_{j'j}\Big((1,3)+(2,4)+(1,3)+(2,4)\Big)\Big],$$

where $\quad (i,j) = \dfrac{n_i \cdot n_j}{n_i \cdot l n_j \cdot l}.$

The basic integral is

$$
\begin{aligned}
J_{ij} &= \int \frac{d^D l}{(2\pi)^D} \frac{n_i \cdot n_j}{l^2 (n_i \cdot l - \lambda_i)(n_j \cdot l - \lambda_j)} \quad (\lambda_i = p_i^2 / \overline{n}_i \cdot p_i) \\
&= -\frac{i}{16\pi^2} 4 \int_0^\infty du\, dv \frac{n_i \cdot n_j}{(2uv n_i \cdot n_j + 2u\lambda_i + 2v\lambda_j)^{1+\epsilon}} \\
&= -\frac{i}{16\pi^2} \left(\frac{n_i \cdot n_j}{2}\right)^{-\epsilon} 2 \int_0^\infty du\, dv \left(uv + \frac{2u\lambda_i}{n_i \cdot n_j} + \frac{2v\lambda_j}{n_i \cdot n_j}\right)^{-1-\epsilon} \\
&= -\frac{i}{16\pi^2} \left(\frac{n_i \cdot n_j}{2}\right)^{-\epsilon} \frac{2}{\epsilon} \left(\frac{2\lambda_j}{n_i \cdot n_j}\right)^{-\epsilon} \int dv \frac{v^{-\epsilon}}{v + \frac{2\lambda_i}{n_i \cdot n_j}} \\
&= -\frac{i}{16\pi^2} \frac{2}{\epsilon^2} \left(\frac{2\lambda_i \lambda_j}{n_i \cdot n_j}\right)^{-\epsilon} = -\frac{i}{16\pi^2} \left(\frac{2}{\epsilon^2} + \frac{2\beta_{ij}}{\epsilon}\right).
\end{aligned}
$$

where $\beta_{ij} = \ln \frac{-2\sigma_{ij} p_i \cdot p_j \mu^2}{(-p_i^2)(-p_j^2)}$

Generalization to n-parton scattering

Jet function

$$J_q = 1 + \frac{\alpha_s}{4\pi} C_F \Big(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{\mu^2}{-p^2} + \frac{3}{2\epsilon} \Big)$$

$$S = 1 + \frac{\alpha_s}{4\pi} \sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \Big(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{-\sigma_{ij} n_i \cdot n_j \bar{n}_i \cdot p_i \bar{n}_j \cdot p_j \mu^2}{2(-p_i^2)(-p_j)^2} \Big)$$

$$\sum_i \mathbf{T}_i = 0$$

$$S \prod_i J_i = 1 - \frac{\alpha_s}{4\pi} \Big[\sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \Big(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{\mu^2}{-s_{ij}} \Big) + \sum_i \frac{\gamma_0^i}{2\epsilon} \Big]$$

$$\gamma_0^i = -3 C_F$$

Conclusion

- Effective theories are what we do science.
- Matching and running are basic ingredients.
- IR divergences are common in any set of effective theories.
- SCET offers a lot of possibilities in studying QCD effects in high-energy scattering.

The End

If you are doing everything well,
you are not doing enough.

- Howard Georgi