Applications of holographic QCD

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SITP

Muon g-2
Why muon anomalous magnetic moment?
The current status of muon anomalous magnetic moment.

Holographic Baryons

Conclusion

Why muon anomalous magnetic moment?

- It provides the most stringent, sub-ppm test of SM: (E821 at BNL 2006)

$$a_\mu^{\rm exp} = \frac{g_\mu - 2}{2} = 11659208.0(5.4)(3.3) \times 10^{-10}$$

- Currently 3.2σ deviation with SM estimate:

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 302(63)(61) \times 10^{-11}$$

- Factor ~ 5 improvement or 5σ at FNAL by 2014(?)

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The current status of muon anomalous magnetic moment.

- Theoretical prediction:

$$a_\mu^{\rm th} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm strong} + a_\mu^{\rm new}$$

- QED contribution $a_\mu^{\rm QED} = 116584718.10(0.16) \times 10^{-11}$ at 4.5 loops (Kinoshta et al, 2008):

$$\begin{aligned} a_\mu^{\rm QED} &= \frac{\alpha}{2\pi} + 0.76585410(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050964(87)\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad + 130.8055(80)\left(\frac{\alpha}{\pi}\right)^4 + 663(20)\left(\frac{\alpha}{\pi}\right)^5 + \cdots \end{aligned}$$

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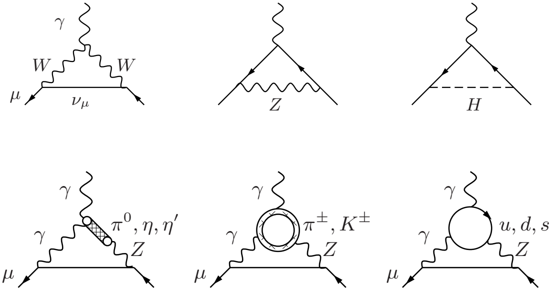
Weak corrections:

- Weak corrections at two loops : $a_\mu^{\mathrm{weak}} = 154(2) \times 10^{-11}$



(a) [L.D.] (b) [L.D.] (c) [S.D.]

Hadronic corrections:

- Hadronic corrections (Marciano 2008): Major source of uncertainty

$$a_\mu^{\mathrm{had}} = 116591778(2)(46)(40) \times 10^{-11}$$

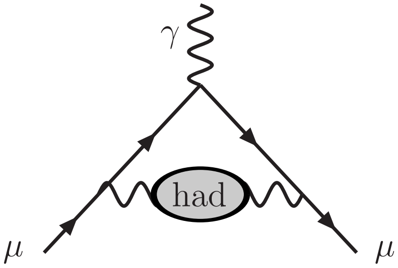


Figure: Leading hadronic contribution to $g-2$.

Higher Order Hadronic VP contributions:

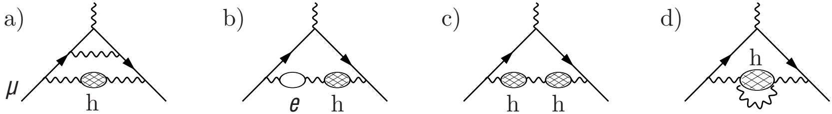
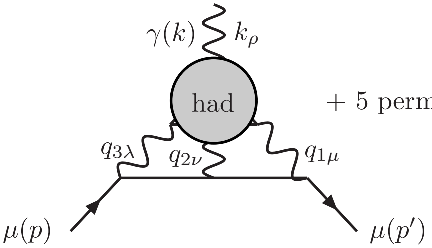


Figure: a)-c) involving LO vacuum polarization, d) involving HO vacuum polarization (FSR of hadrons).

Hadronic Light-by-Light corrections:



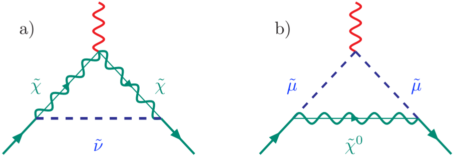
$+$ 5 permutations of the q_i

Figure: Hadronic contribution of the light–by–light scattering to the muon electromagnetic vertex.

New Physics corrections:

- New physics contributions (Lopez et al. '94): One-loop SUSY

$$a_\mu^{\rm SUSY} \simeq (sgn\,\mu)\,130 \times 10^{-11} \left(\frac{100\,{\rm GeV}}{m_{\rm SUSY}}\right)^2 \tan\beta$$



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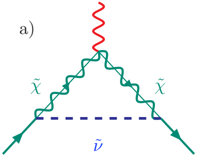

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Current Status of Muon Anomalous Mganetic Moment:

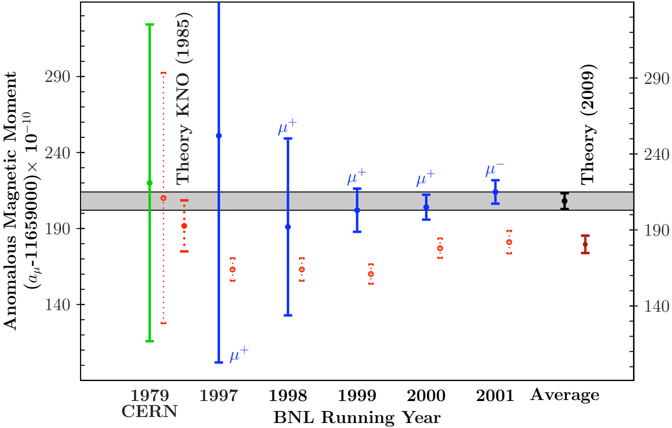

Figure: Results for the individual E821 measurements, together with the new world average and the theoretical prediction. (Jegerlehner and Nyffeler, Phys. Rep. 2009)

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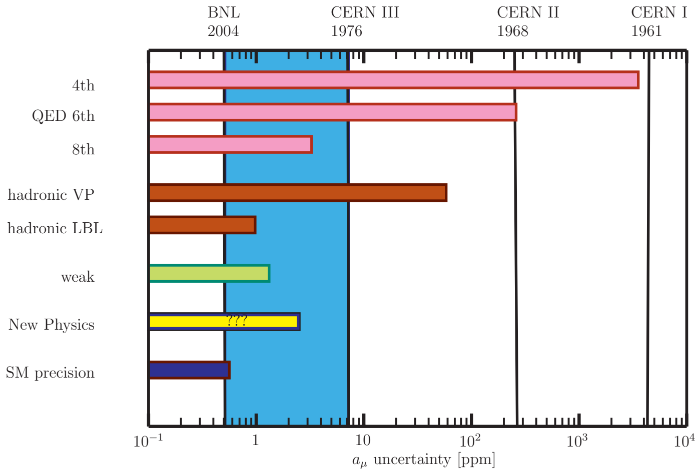

Figure: Sensitivity of $g-2$ experiments to various contributions. (Jegerlehner and Nyffeler, Phys. Rep. 2009)

Theory and experiment comparison (in units 10^{-11}):

Contribution	value	error	Reference
QED incl. 4.5 loops	116 584 718.1	0.2	KN '06('08)
Leading hadronic VP	6 903.0	52.6	J 2008
Subleading hadronic VP	-100.3	1.1	J 2006
Hadronic LBL	116.0	39.0	JN 2009
Weak incl. 2-loops	153.2	1.8	PPR 1995
Theory	116 591 790.0	64.6	
Experiment	116 592 080.0	63.0	BNL E821
Exp. - Theory. (3.2σ)	290.0	90.3	

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EKSS model (bottom-up approach)

- Construct $U(3)_L \times U(3)_R$ flavor gauge theory in a slice of AdS_5 ($\epsilon \leq z \leq z_m = (0.323)^{-1}$) as a model for hQCD:

4D	$\bar{q}_L \gamma^\mu t^a q_L$	$\bar{q}_R \gamma^\mu t^q q_R$	$\bar{q}_L^\alpha q_R^\beta$
5D	$A^a_{L\mu}$	$A_{R\mu}$	$\frac{2}{z} X^{\alpha\beta}$

$$S = \int d^5x \sqrt{g}\ \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}({F_L}^2 + {F_R}^2) \right\} + S_Y + S_{CS},$$

- Flavor-singlet bulk scalar, Y, dual to F^2 ($F\tilde{F}$), described by

$$S_Y = \int d^5x \sqrt{g} \left[\frac{1}{2}|DY|^2 - \frac{\kappa}{2}(Y^{N_f}\det(X) + \mathrm{h.c.}) \right].$$

- Finally we introduce CS term for QCD flavor anomaly:

$$S_{CS} = \frac{N_c}{24\pi^2} \int [\omega_5(A_L) - \omega_5(A_R)],$$

where $d\omega_5(A) = \mathrm{Tr} F^3$. (N.B. We need either bulk fermions or a counter term in IR to recover the gauge invariance at IR)

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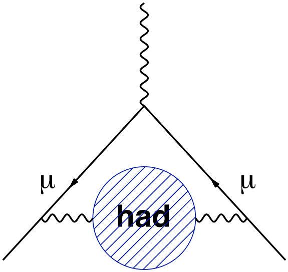
Holographic Calculation of Hadronic Leading Contribution

The HLO contribution is given as (Blum '03)

$$a_\mu^{\mathrm{HLO}} = 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2)\,\bar{\Pi}_{\mathrm{em}}^{\mathrm{had}}(Q^2)\,,$$

$$f(Q^2) = \frac{m_\mu^2 Q^2 Z^3 (1 - Q^2 Z)}{1 + m_\mu^2 Q^2 Z^2} \quad \text{and} \quad Z = -\frac{Q^2 - \sqrt{Q^4 + 4 m_\mu^2 Q^2}}{2 m_\mu^2 Q^2}\,.$$



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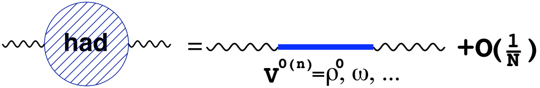
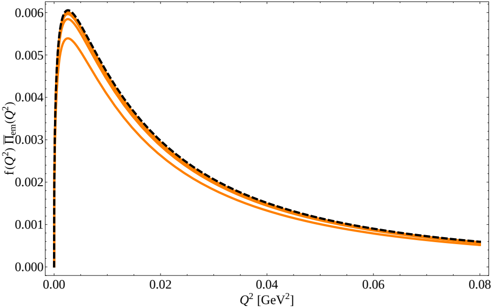


We have $\bar{\Pi}_V(Q^2) \simeq \sum_{n=1}^{4} \frac{Q^2 F_{V_n}^2}{(Q^2+M_{V_n}^2)M_{V_n}^4} + \mathcal{O}(Q^2/(M_{V_5}^2))$



Holographic Calculation of Hadronic Leading Contribution

We obtain (arXiv:0911.0560, done with D. Kim and S. Matsuzaki)

$$a_\mu^{\rm HLO}|^{N_f=2}_{\rm AdS/QCD} = 470.5 \times 10^{-10}, \tag{1}$$

which agrees, within 10% errors, with the currently updated value (BaBar 2009)

$$a_\mu^{\rm HLO}[\pi\pi]|_{\rm BABAR} = (514.1 \pm 3.8) \times 10^{-10}\,. \tag{2}$$

We expect that the discrepancy may be due to the $1/N_c$ corrections together with the isospin-breaking corrections.

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- For the hadronic LBL we need to calculate 4-point functions of flavor currents:

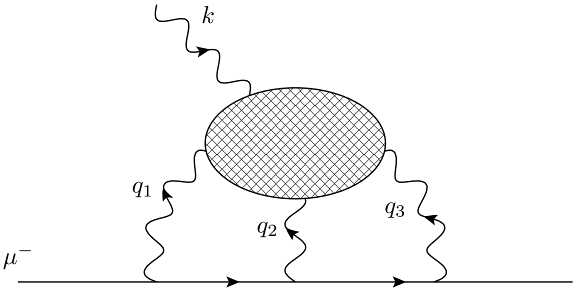


Figure: Hadronic Light-by-light corrections to muon $g-2$.

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- Since there is no quartic term for $A_{Q_{\rm em}}$ $(Q_{\rm em} = 1/2 + I_3)$, there is no 1PI 4-point function for the EM currents in hQCD:

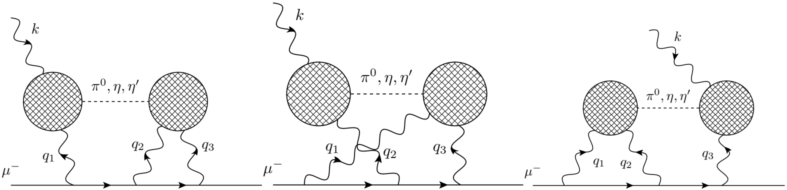


Figure: Light-by-light correction is dominated by the pseudo scalar mesons (and also axial vectors) exchange.

- Higher order terms like F^4 or F^2X^2 terms are α' suppressed.

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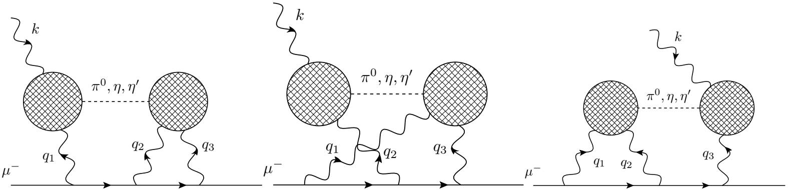

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Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- In hQCD the LBL diagram is dominated by VVA or VVP diagrams, which come from the CS term:

$$F_{\gamma^*\gamma^* P(A)}(q_1, q_2) = \frac{\delta^3}{\delta V(q_1)\delta V(q_2)\delta A(-q_1 - q_2)} S_{5D\mathrm{eff}} \quad (3)$$

where the gauge fields satisfy in the axial gauge, $V_5 = 0 = A_5$,

$$\left[\partial_z \left(\frac{1}{z}\partial_z V_\mu^{\hat{a}}(q,z)\right) + \frac{q^2}{z} V_\mu^{\hat{a}}(q,z)\right]_\perp = 0\,, \quad (4)$$

$$\left[\partial_z \left(\frac{1}{z}\partial_z A_\mu^{\hat{a}}\right) + \frac{q^2}{z} A_\mu^{\hat{a}} - \frac{g_5^2 v^2}{z^3} A_\mu^{\hat{a}}\right]_\perp = 0\,, \quad (5)$$

Holographic Calculation of HLBL (DKH+D.Kim, PLB '09)

- For two flavors the longitudinal components, $A^a_{\mu\|} = \partial_\mu \phi^a$, and the phase of bulk scalar X are related by EOM as

$$\partial_z \left(\frac{1}{z} \partial_z \phi^a \right) + \frac{g_5^2 v^2}{z^3} (\pi^a - \phi^a) = 0\,, \qquad (6)$$

$$-q^2 \partial_z \phi^a + \frac{g_5^2 v^2}{z^2} \partial_z \pi^a = 0\,. \qquad (7)$$

Anomalous pion form factors

- The anomalous FF is with $\psi^a(z) = \phi^a - \pi^a$ and $J_q = V(iq, z)$

$$F_{\pi\gamma^*\gamma^*} = \frac{N_c}{12\pi^2}\left[\psi(z_m)J(Q_1, z_m)J(Q_2, z_m) - \int_z \partial_z\psi J_{Q_1} J_{Q_2}\right].$$

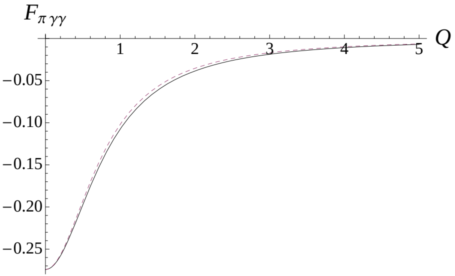


Figure: Anomalous pion form factor $F_{\pi\gamma^*\gamma^*}(Q^2, 0)$: dashed (VMD) and solid (AdS/QCD)

Anomalous pion form factors

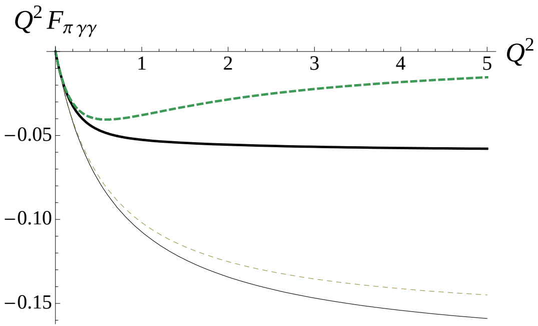
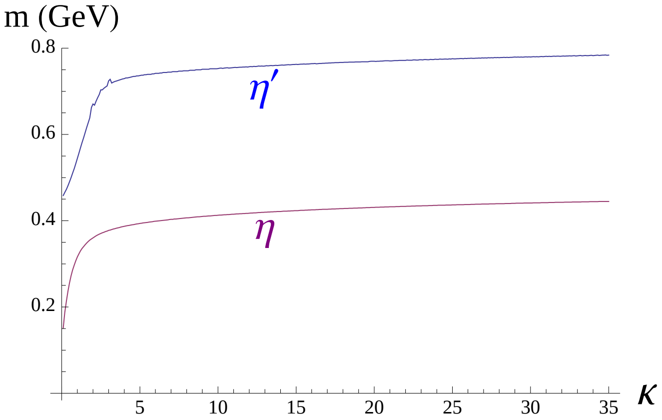


Figure: $F_{\pi\gamma^*\gamma}(Q^2, 0)$ for lower part; $F_{\pi\gamma^*\gamma^*}(Q^2, Q^2)$ for upper part (Brodsky-Lepage): solid line (AdS/QCD) and dashed line (VMD)

Anomalous form factors

- For η and η' we scan the parameter κ because of mixing ($m_q = 0.0022, m_s = 0.04$):



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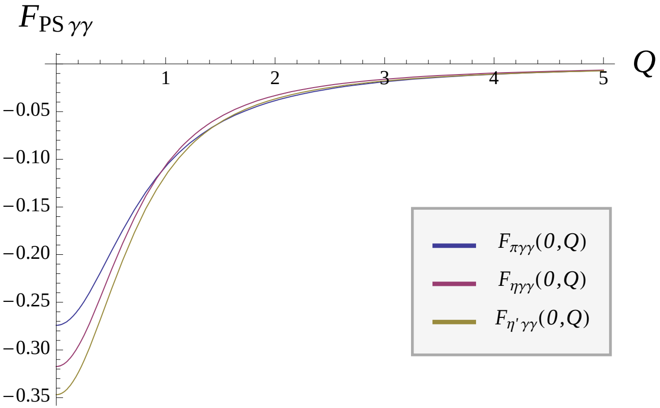

Hadronic LBL in hQCD (DKH+D.Kim, PLB '09)

- To calculate the hadronic LBL contribution to a_μ we expand the photon line as

$$J(-iQ,z) = V(q,z) = \sum_\rho \frac{-g_5 f_\rho \psi_\rho(z)}{q^2 - m_\rho^2 + i\epsilon}$$

Table: Muon $g-2$ results from the AdS/QCD in unit of 10^{-10}.

Vector modes	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	$a_\mu^{\rm PS}$
4	7.5	2.1	1.0	10.6
6	7.1	2.5	0.9	10.5
8	6.9	2.7	1.1	10.7

- In the LMD+V model (Nyffeler '09)

$$a_\mu^{\rm PS} = 9.9(1.6)\times 10^{-10}$$

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Vector modes	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	$a_\mu^{\rm PS}$
4	7.5	2.1	1.0	10.6
6	7.1	2.5	0.9	10.5
8	6.9	2.7	1.1	10.7

- In the LMD+V model (Nyffeler '09)

$$a_\mu^{\rm PS} = 9.9(1.6) \times 10^{-10}$$

Hadronic LBL in hQCD (DKH+D.Kim, PLB '09)

- To calculate the hadronic LBL contribution to a_μ we expand the photon line as

$$J(-iQ,z) = V(q,z) = \sum_\rho \frac{-g_5 f_\rho \psi_\rho(z)}{q^2 - m_\rho^2 + i\epsilon}$$

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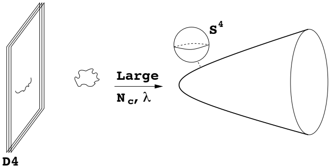
Holographic baryons

- N_c stack of $D4$ brane over $R^3 \times S^1$ describes pure $SU(N_c)$ YM. (Witten '98)

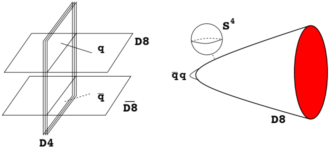


$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

with $R^3 = \pi g_s N_c l_s^3$ and $f(U) = 1 - U_{KK}^3/U^3$

Holographic baryons

- Adding flavors was done by Sakai-Sugimoto (2004) (Cf. Karch and Katz in $D3$-$D7$, probe approximation,'02).



- Spontaneous chiral symmetry breaking is geometrically realized:

$$SU(N_F)_L \times SU(N_F)_R \mapsto SU(N_F)_V \, . \qquad (8)$$

- Effective action on D8 is a $U(N_F)$ gauge theory,

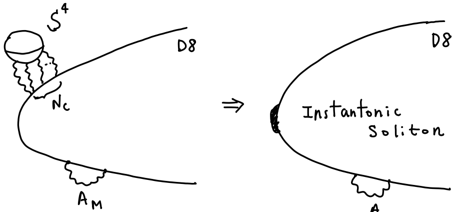
$$S_{D8} = -\mu_8 \int \mathrm{d}^9 x \, e^{-\phi} \sqrt{-\det\left(g_{MN} + 2\pi\alpha' F_{MN}\right)}$$
$$+\mu_8 \int \sum C_{p+1} \wedge \mathrm{Tr}\, e^{2\pi\alpha' F} ,$$

Holographic Baryons

- What are baryons in hQCD? It must be solitons:

$$m_{\mathrm{baryon}} \sim N_c \,. \tag{11}$$

- In SS model, D4 brane wrapping S^4 is the baryon vertex (Witten).
- $D4$ brane becomes instanton in $D8$ (Douglas '95).



Holographic Baryons

- In 5D YM there is a topologically conserved current, $d\,{}^*J = 0 = DF$,

$$J^M = \frac{1}{24\pi^2}\epsilon^{MNLPQ}\,\mathrm{tr}F_{NL}F_{PQ}\,. \tag{12}$$

- One can define the baryon current

$$B^\mu = \frac{1}{8\pi^2}\int \mathrm{d}z\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\nu\rho}F_{\sigma z}\,. \tag{13}$$

- In the gauge $A_z = 0$ one may write $U = \exp(2i\pi/f_\pi)$

$$A_\mu(x,z) = U^{-1}\partial_\mu U\psi_0(z) + \sum_{n\geq 1} B^{(n)}_\mu \psi_n(z)\,. \tag{14}$$

Then the baryon current becomes the Skyrme current

$$B^\mu = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\,U^{-1}\partial_\nu UU^{-1}\partial_\rho UU^{-1}\partial_\sigma U \tag{15}$$

Holographic Baryons

- Unlike Skyrme model we know that it has to be a baryon current, since the baryon carries the N_c unit of quark number in SS model

$$S_{CS}^{D4} = \int_{R\times S^4} C \wedge e^{F/2\pi} \sim N_c \int_R A\,. \qquad (16)$$

coupling to the instanton number density,

$$S_{CS}^{D8} = \frac{N_c}{24\pi^2}\int_{M^4\times R}\omega_5(A)\,, \quad \rho(x) = \frac{\delta S}{\delta A_0(x)} = \frac{N_c}{24\pi^2}\int \mathrm{d}z F\tilde{F}\,.$$

Holographic Baryons

- In the SS model the DBI action tends to shrink the solitons. In the conformally flat metric, the energy density becomes

$$-\int_{x,w} \frac{1}{4e^2(w)} \mathrm{Tr} F_{mn} F^{mn} + \mathrm{h.o.}, \quad \frac{1}{e^2(w)} \equiv \frac{\lambda N_c}{108\pi^3} \frac{M_{KK} U(w)}{U_{KK}} \,. \tag{17}$$

- A point-like instanton that is localized at $w = 0$ has

$$m_B^{(0)} \equiv \frac{4\pi^2}{e^2(0)} = \frac{\lambda N_c}{27\pi} M_{KK} \,. \tag{18}$$

- Since the instanton carries $U(1)$ charge, Coulomb repulsion prevents the instanton from collapsing: (HRYY '07; HSSY '07)

$$\rho_{baryon} \sim \frac{9.6}{M_{KK}\sqrt{\lambda}} \,, \tag{19}$$

where $M_{KK} \simeq 1\ \mathrm{GeV}$ is the UV cut-off of SS model.

Holographic Baryons

- At low energy the baryons are described as point-like bulk spinors,

$$\int_{x,w} \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_b(w) \bar{\mathcal{B}}\mathcal{B} + g_5(w) \frac{\rho^2_{baryon}}{e^2(w)} \bar{\mathcal{B}}\gamma^{mn} F_{mn} \mathcal{B} \right] - \int_{x,w} \frac{1}{4e^2(w)} \mathrm{Tr}\, F_{mn} F^{mn} + \cdots , \quad (20)$$

Holographic Baryons

- Hairy instantons: the spinor sources YM fields

$$\nabla^2 A_m^a = 2g_5(0)\rho^2_{baryon}\bar{\eta}^a_{mn}\partial_n\delta^{(4)}(x) \,, \tag{21}$$

whose solution goes as

$$A_m^a = -\frac{g_5(0)\rho^2_{baryon}}{2\pi^2}\bar{\eta}^a_{mn}\partial_n\frac{1}{r^2+w^2} \tag{22}$$

to compare with the 't Hooft ansatz

$$A_m^a = -\bar{\eta}^a_{mn}\partial_n \log\left(1+\frac{\rho^2}{r^2+w^2}\right) \simeq -\rho^2\bar{\eta}^a_{mn}\partial_n\frac{1}{r^2+w^2} \,, \tag{23}$$

- Including the quantum fluctuations to match the long-range instanton tail (Adkins+Nappi+Witten),

$$g_5(0) = \frac{2\pi^2}{3} \tag{24}$$

Holographic Baryons

- The Lagrangian is unique up to operators with two derivatives in the large N_c and large $\lambda = g_s^2 N_c$ and valid for $E < M_{KK}$.
- Though the coefficient of the Pauli term might be model dependent, the fact that it contains only the nonabelian part of the flavor symmetry is model-independent!
  $\longrightarrow$ The $U(1)$ coupling does not have the Pauli term.
- One immediate consequence of this is that the Pauli form factor

$$F_2^p(q^2) = -F_2^n(q^2) + \mathrm{h.o.}. \tag{25}$$

- Especially for instance $\mu_{\mathrm{an}}^p + \mu_{\mathrm{an}}^n = 0$, which is very close to the experimental value,

$$(\mu_{\mathrm{an}}^p + \mu_{\mathrm{an}}^n)_{\mathrm{exp}} = 1.79\mu_N - 1.91\mu_N = -0.12\mu_N \tag{26}$$

Holographic Baryons

- Vector couplings of baryons,

$$g_{\rm min}^{(n)} = \int_{-w_{max}}^{w_{max}} dw\, |f_L(w)|^2\, \psi_{(n)}(w)\,,$$

$$g_{\rm mag}^{(n)} = 2C\int_w dw \left(\frac{g_5(w)U(w)}{g_5(0)U_{KK}M_{KK}}\right) |f_L(w)|^2\, \partial_w \psi_{(n)}(w)\,.$$

- For SS model in the large N_c,

$$C = \frac{6}{\pi^2}\frac{\lambda N_c}{108\pi^3}(\rho_{baryon}M_{KK})^2 \simeq 0.18 N_c\,. \qquad (27)$$

- The axial coupling for the SS model with $\lambda N_c = 50$

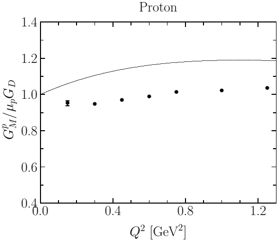
$$g_A \approx 1.30 - 1.31, \quad g_A^{\rm exp} = 1.2670 \pm 0.0035 \qquad (28)$$

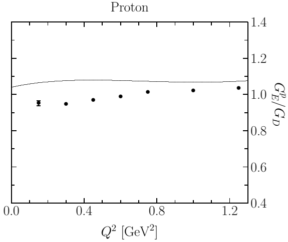
Holographic Baryons

- Using AdS/CFT correspondence we compute the form factors,

$$\langle p' | \, J^\mu(x) \, | p \rangle = e^{iqx} \, \bar{u}(p') \, \mathcal{O}^\mu(p, p') \, u(p), \quad q = p' - p$$

$$\mathcal{O}^\mu = \gamma^\mu \left[\frac{1}{2} F_1^S(q^2) + F_1^a(q^2) \tau^a \right] + \frac{\gamma^{\mu\nu}}{2m_B} q_\nu \left[F_2^S(q^2) + F_2^a(q^2) \tau^a \right] ,$$





Holographic Baryons

Hong-Inami-Yee model (2006):

- We need to introduce two bulk spinors:

$$S_{\rm kin} = \int_{z,x} \left[i\bar{N}_1\Gamma^M D_M N_1 + i\bar{N}_2\Gamma^M D_M N_2 - \frac{5}{2}\bar{N}_1 N_1 + \frac{5}{2}\bar{N}_2 N_2 \right] \quad ,$$

$$S_m = \int_{z,x} \left[-g\bar{N}_1 X N_2 - g\bar{N}_2 X^\dagger N_1 \right] ,$$

- 5d bulk metric and the covaraint derivative:

$$ds^2 = \frac{1}{z^2}\left(-dz^2 + \eta^{\mu\nu}dx_\mu dx_\nu\right) \quad \epsilon \le z \le z_m , \quad (29)$$

$$D_M = \partial_M + \frac{i}{4}\omega_M^{AB}\Gamma_{AB} - i(A^a)_M t^a , \quad (30)$$

- For two flavors, anomaly matching requires baryons to be massless when chiral symmetry is unbroken.

Holographic Baryons

- Mass should come from the Yukawa coupling for spin $1/2$:

$$\mathcal{L}_{\rm int} \ni -g\left(\bar{N}_2 X N_1 + \mathrm{h.c.}\right)\,, \tag{31}$$

where $X = \frac{1}{2} m\, z + \frac{1}{2}\sigma\, z^3,\ (\sigma = \langle \bar{q}\, q\rangle)\,.$

- To allow a left-handed zero mode for N_1, we impose

$$\lim_{\epsilon \to 0} N_{1L}(\epsilon) = 0\ (\text{normalizability}) \quad \text{and} \quad N_{1R}(z_m) = 0. \tag{32}$$

- The remaining boundary conditions for $N_{1L}(z_m)$ and $N_{1R}(\epsilon)$ are then determined by the equations of motion.
- The boundary conditions for N_2 are similar except for interchanging $L \leftrightarrow R$.

Holographic Baryons

- We now Fourier-transform the bulk spinor as

$$f_{1L,R}(p,z)\,\psi_{1L,R}(p) = \int \mathrm{d}^4x\, N_{1L,R}(x,z)e^{ip\cdot x}\,, \qquad (33)$$

where the 4D spinors satisfy with $\psi_{1L} = \gamma^5\psi_{1L}$ and $\psi_{1R} = -\gamma^5\psi_{1R}$

$$\not{p}\,\psi_{1L,R}(p) = |p|\,\psi_{1R,L}(p) \qquad (34)$$

- The K-K mode equations become with $M = 0$ and $\Delta = 9/2$

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{1}{2}g\sigma z^2 \\ -\frac{1}{2}g\sigma z^2 & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = -|p| \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix},$$

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Holographic Baryons

- We find the nucleon spectra:

z_m^{-1}	g	(p,n)	N1440	N1535	3rd	4th	5th	6th
0.33*	9.30	0.94*	2.14	2.24	3.25	3.30	4.35	4.36
0.21	3.80	0.94*	1.44*	1.50	2.08	2.12	2.72	2.75

Table: Numerical result for spin-$\frac{1}{2}$ baryon spectrum. $*$ indicates an input and we used $\sigma = (0.31\mathrm{GeV})^3$.

- We see a parity-doubling pattern in excited states with $M_{\frac{1}{2}^-} > M_{\frac{1}{2}^+}$, but difference gets smaller for excite states.

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Holographic Baryons

- The pion-nucleon coupling becomes smaller and smaller for excited states:

$z_m(\mathrm{GeV}^{-1})$	$g_{\pi NN}$	$g_{\pi NN_{1440}}$	$g_{\pi NN_{1535}}$
$(0.205)^{-1}$	13.5	1.94	2.67
$(0.33)^{-1}$	41.4	-	-

Table: Numerical result for pion-nucleon-nucleon couplings.

Conclusion

- Holographic QCD (hQCD) is an attempt to solve QCD under the holography principle.
- Recent advance in gauge/gravity duality provides models for hQCD.
- In the large N limit, hQCD becomes classical. By solving it at the classical level we get the nonperturbative properties of hadrons in the large N limit.
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Conclusion

- For instance there is 3.2σ deviation in muon g-2 between theory and experiment. Current theoretical error is estimated to be 6.5×10^{-10}. which is mainly due to QCD effects!
- By hQCD we find we find in hQCD

$$a_\mu^{\rm PS} = 10.7 \times 10^{-10}$$

upto $1/N$ and $1/\lambda$ corrections.

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