Counting Galois representations by spectral trace formula

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The question of estimating the dimension of the space of holomorphic cusp form of weight one (in terms of the level), originally raised by Serre, has received the attention of several number theorists but the current record is still far from the truth we expect. This problem is equivalent to counting Artin representations by Deligne-Serre theorem. Duke obtained the first improvement on the bound by using techniques from analytic number theory like large sieves and amplification method. Subsequently, Michel and Venkatesh improved upon Duke's bound by embedding the space of holomorphic forms into the whole space of all Maass cusp forms and using a Kuznetsov type formula which gives a sharper large sieve inequality. I shall explain the methods of Duke, Michel and Venkatesh and my contribution in the case of octahedral forms. I shall also show that a strong conjectural large sieve inequality will give the expected answer.