

Equidistribution results on the homogeneous varieties

by **Wenzhi Luo** (Ohio State University)

In these lectures, we'd like to survey some recent development on the mass equidistribution of cuspidal holomorphic Hecke-eigenforms on arithmetic hyperbolic surfaces, which is analogue of the ergodicity of Laplacian eigenfunctions on surfaces whose geodesic flow is ergodic, as well as its higher dimensional generalization to arithmetic quotients of Hermitian symmetric spaces of non-compact type, focusing on the Siegel modular varieties and Hilbert-Blumenthal varieties. Our approach is via the Selberg trace formula, through the Bergman kernel and the Selberg-Godement dimension formula. In the case of modular surface, we'll give applications of a remarkable relation (a la Jacquet, Harris-Kudla and Watson) between the equidistribution of eigenforms and the degree 8 triple product L-functions. Next we turn to more geometric equidistribution results. It is well-known that the closed geodesics on the modular surface, when collected according to the discriminants, are equidistributed with respect to the hyperbolic measure, by the works of Duke and Iwaniec. We study and evaluate asymptotically the variance of this distribution on the unit tangent bundle, and show it is equal to the classic variance of the geodesic flow as studied by Ratner, but twisted by an intriguing arithmetic invariant, the central value of certain L-function. Our approach is via Weil representation and the theta correspondence. Some of the above works are based on my various joint works with J.Cogdell, Z.Rudnick and P.Sarnak.