Lecture Series A

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1. Lattice-point counting, modular forms and hyperbolic geometry: From the Gauss circle problem to the hyperbolic lattice point problem.

The generating function for counting the number of lattice points inside a disc of radisu R is the Epstein zeta function. This function is invariant under the action of the group SL(2,Z). This group acts as a subgruop of the isometries of hyperbolic space. We study the geometry of hyperbolic space: metric, geodesics, horocycles. The analogue of the Gauss circle problem ends up being harder: to understand the error term in the hyperbolic lattice counting problem we need spectral theory. As a first step we introduce automorphic forms as functions on lattices.

2. The Theory of Eisenstein series.

The Eisenstein series and the Epstein zeta function are eigenfunctions of the hyperbolic Laplace operator. By separating variables we find their Fourier expansion. We study the analytic continuation of them for SL(2, Z) using the properties of the Riemann zeta function. We introduce Eisenstein series for general cofinite subgroups of SL(2, R), their functional equation and the scattering matrix. Here the analytic continuation is significantly more challenging.

3. The spectral decomposition and the Selberg trace formula.

The harmonic analysis of the space of square integrable automorphic functions requires the Eisenstein series and a new set of functions: the Maass cusp forms. We all these ingredients at hand, we introduce the pretrace formula and its application to the hyperbolic lattice counting problem. Without technical details the Selberg trace formula will be presented and (some of) its applications: counting cusp forms and counting closed geodesics.